Distributed Control in Transportation and Supply Networks

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Joint work with
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1. **Stochastic Control in Inventory Networks**
   - Problem and Model
   - Basetock Policies as Local Heuristics
   - Model with Replenishment Lead Times

2. **Online Train Control in Railway Networks**
   - Multi-level approach
   - Online control of a station area

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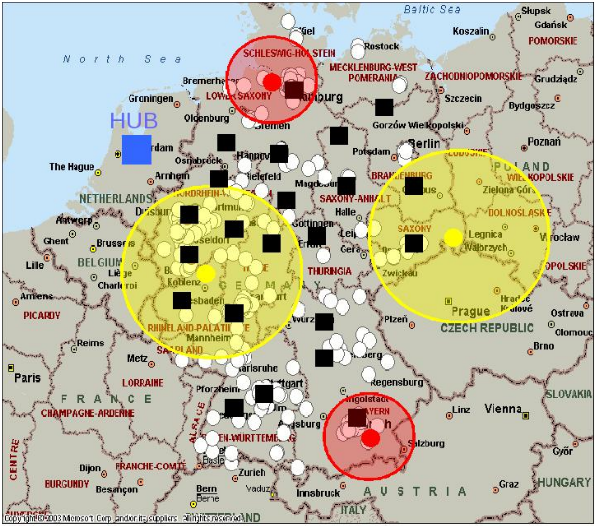
A spare parts supply network

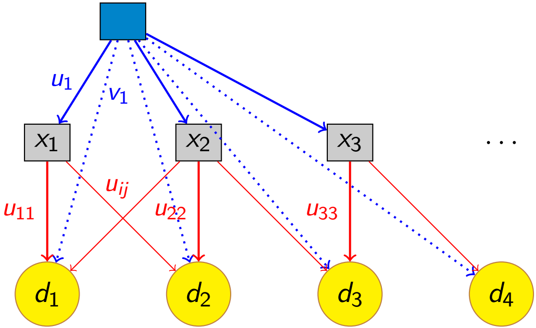


Stochastic control model

central warehouse

stocking points i

customer locations j



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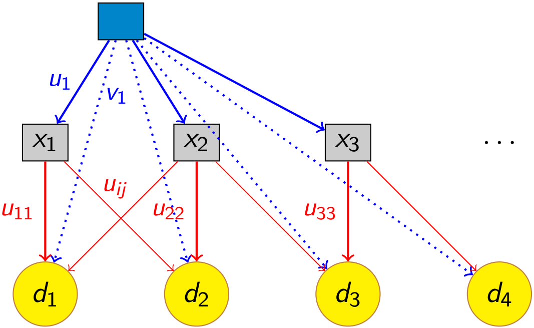

Model with linear dynamics and constraints

$$x_i(t+1) = x_i(t) + u_i(t) - \sum_{j \in \mathcal{C}_i} u_{ij}(t) \quad \text{and} \quad d_j(t+1) = D_j(t)$$

$$\text{where} \quad v_j(t) + \sum_{i \in \mathcal{S}_j} u_{ij}(t) = d_j(t) \quad \text{and} \quad \sum_{j \in C_i} u_{ij}(t) \le x_i(t)$$

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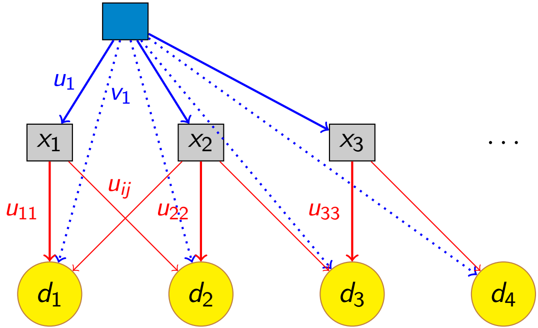
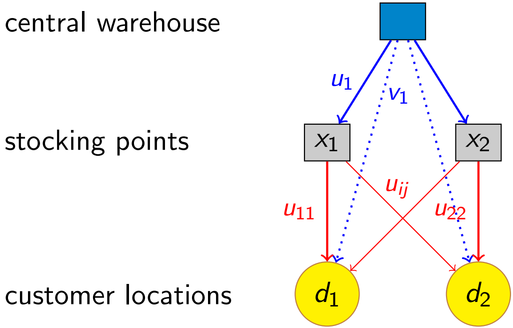


Immediate costs

$$g(\mathbf{x},\mathbf{u},\mathbf{v}) = \underbrace{\sum_i h_i x_i}_{\text{inventory costs}} + \underbrace{\sum_i \sum_{j \in \mathcal{C}_i} c_{ij} u_{ij}}_{\text{transportation costs}} + \underbrace{\sum_j m_j v_j}_{\text{emergency shipments}}$$

Simplified model

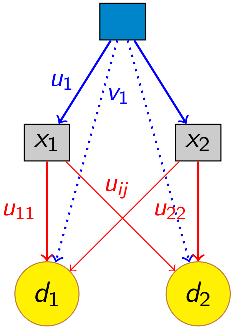


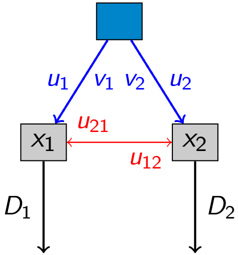
Simplified model

central warehouse

stocking points

customer locations





Simplified model

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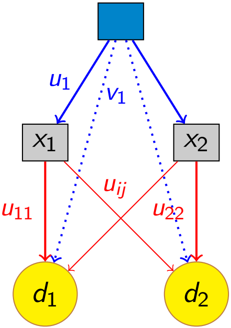
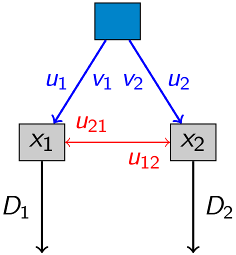
stocking points

customer locations





Simplified dynamics, constraints, and costs

$$x_1(t+1) = x_1(t) + u_1(t) - D_1(t) + u_{21}(t) + v_1(t)$$

where $u_{21}(t) + v_1(t) \geq -x_1(t)$

and cost $g_1(\mathbf{x}, \mathbf{u}, \mathbf{v}) = h_1(x_1 + u_{21} + v_1) + c_{21}\, u_{21} + m_1\, v_1$

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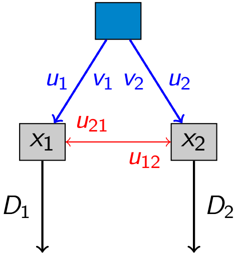
Basestock policies

Basestock policy

$$u_i(t) = \begin{cases} S_i - x_i(t) & \text{if } x_i \le S_i \\ 0 & \text{else} \end{cases}$$

S_i : basestock level



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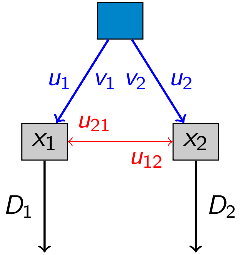

Case I: independent stocks without transshipment

Equivalent to single warehouse with lost sales.
Average cost per time step is

$$\lambda(S_i) = \mathbb{E}[m_i(D_i - S_i)^+] + \mathbb{E}[h_i(S_i - D_i)^+]$$

which is convex in S_i.

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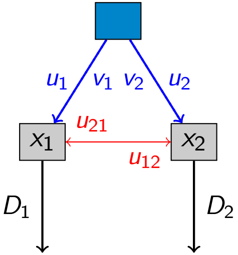


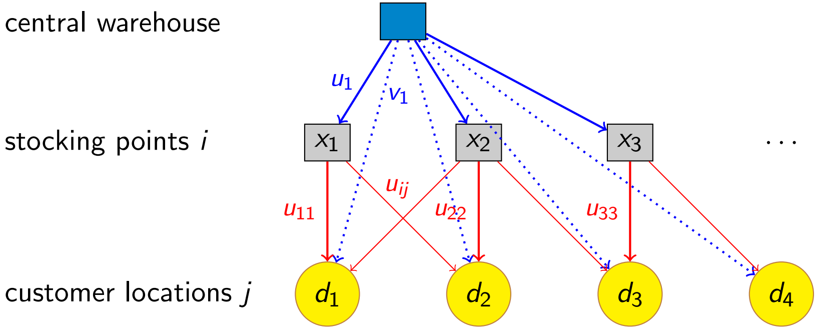
Case II: with transshipment

$$\begin{aligned} \lambda(S_1, S_2) = \min\ & \mathbb{E}[h_1 x_1^{(s)} + h_2 x_2^{(s)} + m_1 v_1^{(s)} + m_2 v_2^{(s)} + c_{12} u_{12}^{(s)} + c_{21} u_{21}^{(s)}] \\ \text{s.t. } & x_1^{(s)} = S_1 - d_1^{(s)} + v_1^{(s)} + u_{21}^{(s)} - u_{12}^{(s)} \quad \forall s \quad (\text{“scenarios”}) \\ & x_2^{(s)} = S_2 - d_2^{(s)} + v_2^{(s)} + u_{12}^{(s)} - u_{21}^{(s)} \quad \forall s \\ & x_1^{(s)}, x_2^{(s)}, v_1^{(s)}, v_2^{(s)}, u_{12}^{(s)}, u_{21}^{(s)} \geq 0 \quad \forall s \end{aligned}$$

Some numerical results

Data	Example 1					Example 2				
demand d	0	1	2	3	4	0	1	2	3	4
$\mathbb{P}[D_i = d]$	0.2	0.2	0.2	0.2	0.2	0.3	0.25	0.2	0.15	0.1
costs	$h = 1$ $c = 2$ $m = 10$					$h = 1$ $c = 2$ $m = 8$				
Solution										
no transshipment	$S_1 = S_2 = 4$ $\lambda^* = 4$					$S_1 = S_2 = 3$ $\lambda^* = 4.8$				
with transshipm.	$S_1 = 4, S_2 = 3$ $\lambda^* = 3.76$					$S_1 = S_2 = 3$ $\lambda^* = 3.75$				

Basestock policies - general case



Case II: with transshipment, general case

- Problem is a two-stage stochastic LP
- Recourse function is min-cost flow problem
  - with S_1, S_2 as parameter
- Average cost $\lambda(S_1, \ldots, S_n)$ is convex.

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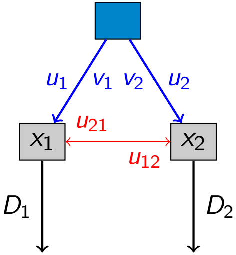
Lead times via state augmentation



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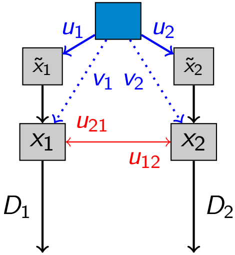

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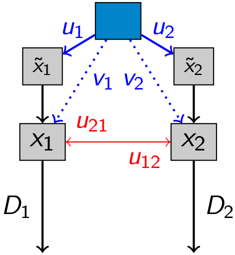


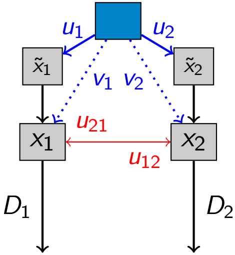
Augmented dynamics, constraints, and costs

$$x_1(t+1) = x_1(t) + \tilde{x}_1(t) - D_1(t) + u_{21}(t) + v_1(t)$$
$$\tilde{x}_1(t+1) = u_1(t)$$

where $u_{21}(t) + v_1(t) \geq -x_1(t)$

and cost $g_1(\mathbf{x}, \mathbf{u}, \mathbf{v}) = h_1(x_1 + u_{21} + v_1) + c_{21} u_{21} + m_1 v_1$

Lead times via state augmentation



What is the marginal value of an additional stock unit?

Augmented dynamics, constraints, and costs

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Parametric dynamic programming

Goal

We would like to find the differential cost function $d^*(x)$, which fulfills

$$\lambda^* + d^*(\mathbf{x}) = (Td^*)(\mathbf{x}) := \min_{\mathbf{u} \in \mathcal{U}(\mathbf{x})} \mathbb{E}\left[g(\mathbf{x}, \mathbf{u}, D) + d^*(f(\mathbf{x}, \mathbf{u}, D))\right]$$

for all $\mathbf{x}$.

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- → *Jones, Baric, Morari: Multiparametric Linear Programming with Applications to Control, 2007.*
- → *Diehl, Björnberg: Robust Dynamic Programming for Min-Max Model Predictive Control of Contrained Uncertain Systems, 2004.*
- → *de la Pena, Bemporad, Filippi: Robust Explicit MPC Based on Approximate Multiparametric Convex Programming, 2006.*
- → *Lincoln, Rantzer: Relaxing Dynamic Programming, 2006.*

Randomized relative value iteration

RRVI

1. Initialize $k := 0$, set $d_0(x) :\equiv 0$ and choose some $\hat{x}$
2. Evaluate $Td_k(\hat{x})$ and add plane to set of planes $\mathcal{V}_{k+1}$
3. Sample N points x, for each x
   - Evaluate $Td_k(x)$ and determine corresponding plane
   - Add plane to $\mathcal{V}_{k+1}$ if not redundant
4. Set $\tilde{d}_{k+1}(x)$ to maximum over planes
5. Set $d_{k+1}(x) := \tilde{d}_{k+1}(x) - \tilde{d}_{k+1}(\hat{x})$
6. Set $k := k + 1$ and repeat from 2.

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Lower bound

Every differential cost function $d(x)$ yields a lower bound

$$\underline{\lambda} = \min_x \; Td(x) - d(x)$$

Some more numerical results

Data	Example 2	
demand d / $\mathbb{P}[D_i = d]$	d: 0, 1, 2, 3, 4 $\mathbb{P}[D_i = d]$: 0.3, 0.25, 0.2, 0.15, 0.1	
costs	$h = 1$ $c = 2$ $m = 8$	
Solution	lead time 1	lead time 2
no transshipment	$S_1 = S_2 = 3$ $\lambda^* = 4.8$	$S_1 = S_2 = 5$ $\lambda^* = 6.745$
with transshipm. basestock	$S_1 = S_2 = 3$ $\lambda^* = 3.75$	$S_1 = 4, S_2 = 5$ $\lambda^* = 4.98$
with transshipm. RRVI		$\bar{\lambda} = 5.066$ $\underline{\lambda} = 4.95$

Conclusions

- Considered inventory-distribution problem (no lead time) is easy for any number of stocking points and customers
  - Basestock-policies are optimal
  - Basestock levels easy to determinine
- Lead time (time delays) can make problem hard
  - Value function dependent on augmented state
- Approximation of differential cost function yields
  - a policy (MPC by value function approximation)
  - a lower bound (to evaluate the quality of heuristics)

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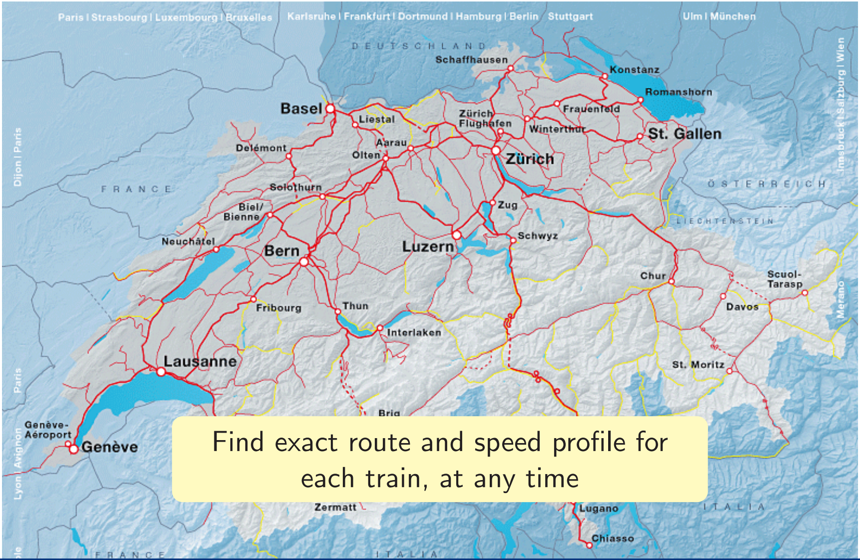
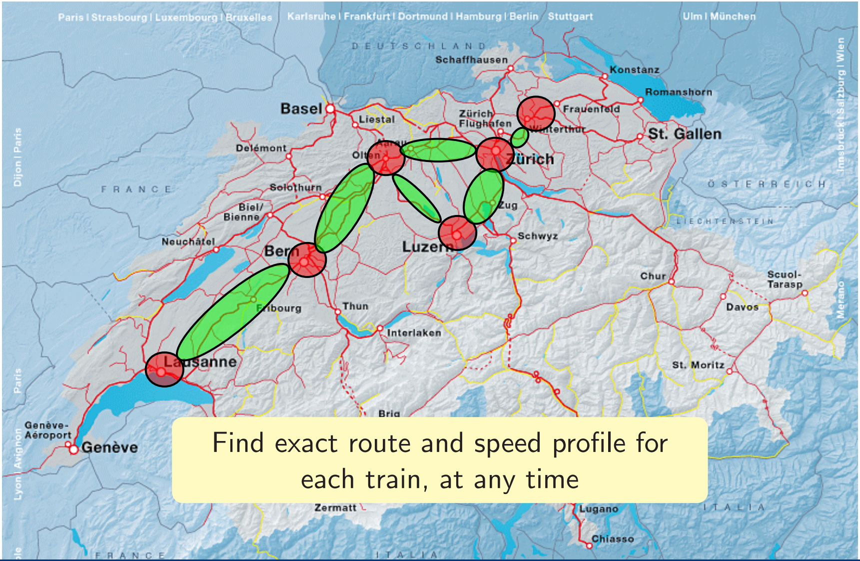
Problem



Find exact route and speed profile for each train, at any time

Problem and geographical subdivision



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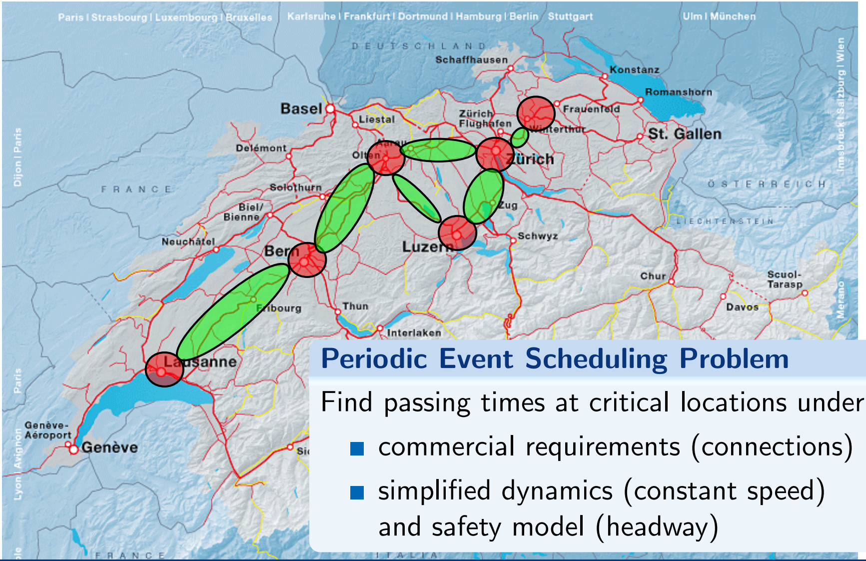
Macroscopic model



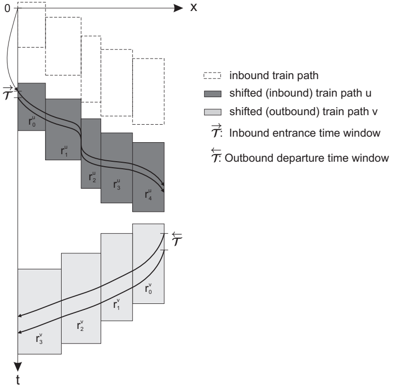
Periodic Event Scheduling Problem

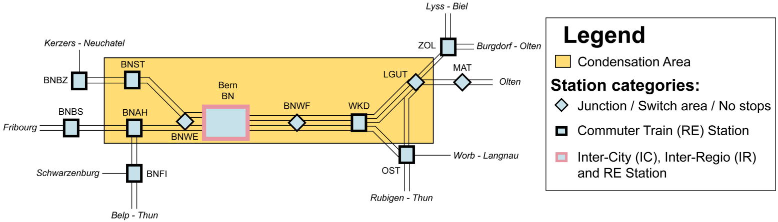
Find passing times at critical locations under

- commercial requirements (connections)
- simplified dynamics (constant speed) and safety model (headway)

Microscopic model

Junction / Switch area





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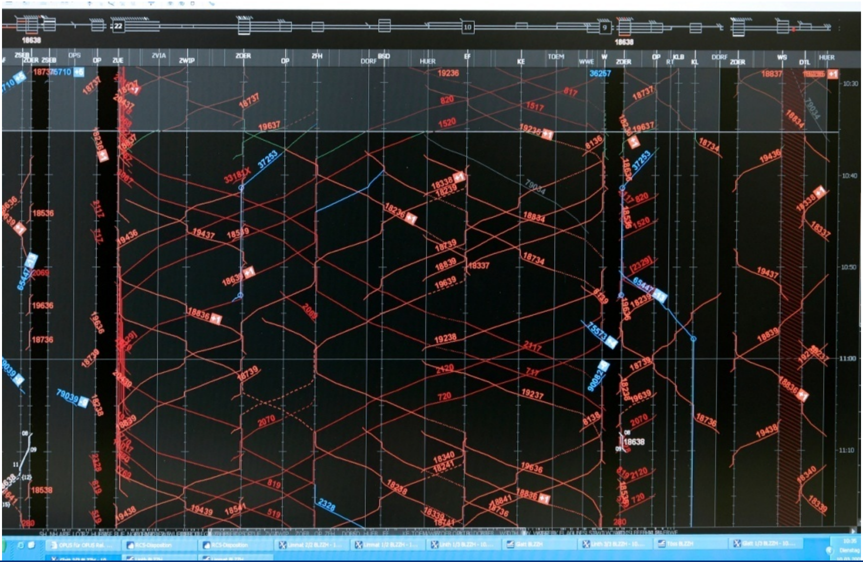
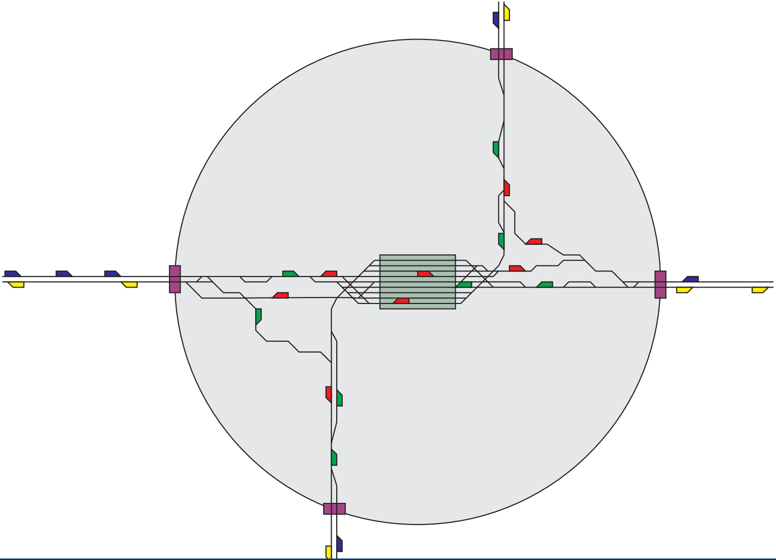
Receding horizon “control” – current practise



MPC concept for station region



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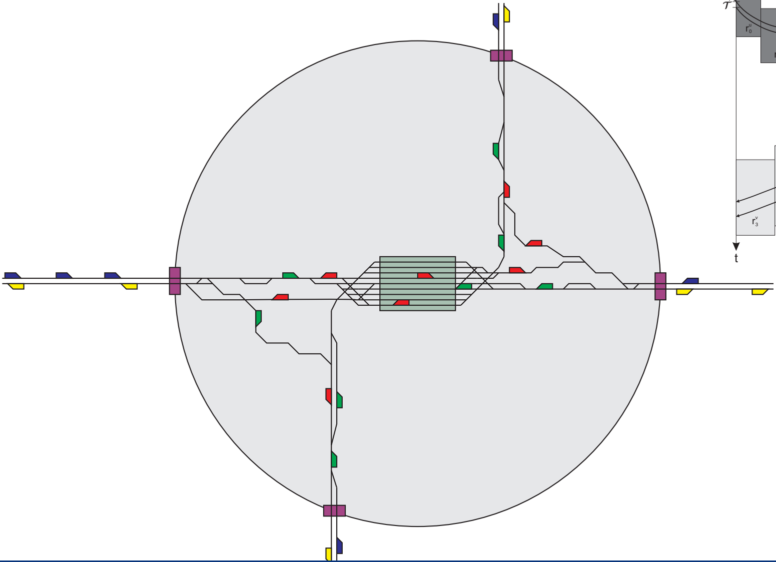

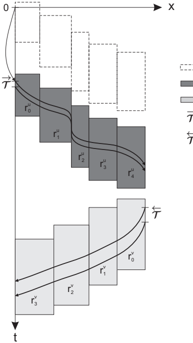

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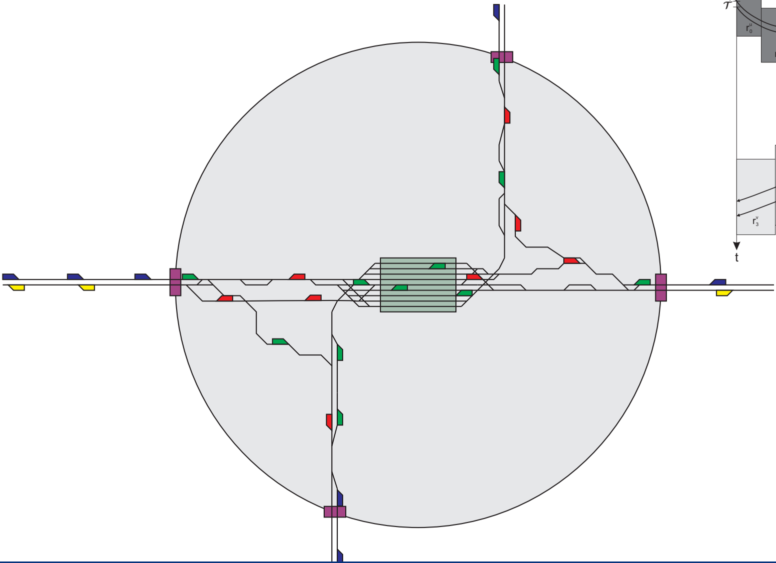

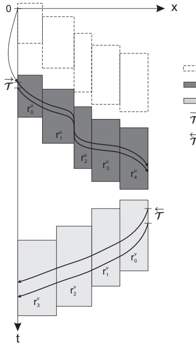

MPC – IP formulation

$$\text{Maximize} \quad - \sum_{\substack{p \in \overrightarrow{P}(z) \\ z \in Z}} x_p \cdot \overbrace{f(\mathcal{A}^{*z} - A(p))}^{\text{arrival time difference}} + \sum_{\substack{p \in \overleftarrow{P}(z) \\ z \in Z}} x_p \cdot \overbrace{g(\mathcal{D}^{*z} - D(p))}^{\text{departure time difference}}$$

$$+ \sum_{(z_i, z_j) \text{ weakly connected}} I^c_{z_i, z_j} \cdot y^c_{z_i, z_j} \qquad \text{(connections kept)}$$

$$+ \sum_{(z_i, z_j) \text{ weakly sequenced}} I^s_{z_i, z_j} \cdot y^s_{z_i, z_j} \qquad \text{(sequences kept)}$$

$$- \sum_{\substack{\mathcal{F}^{*z} \neq F(p) \\ p \in P(z), z \in Z}} h(\mathcal{F}^{*z}, F(p)) \cdot x_p \qquad \text{(platform changes)}$$

$$\text{subject to} \quad \sum_{p \in P^{\overrightarrow{m}}, \overrightarrow{m} \in \overrightarrow{M}^z} x_p = 1, \quad \forall z \in Z$$

Conclusions

- Offline train scheduling problem already intractable
  - $\longrightarrow$ decomposition and simplification necessary
- MPC approach for online train control of a single station area
- Coordination an open problem, options:
  - local coordination beween neighboring nodes
  - global coordination via macroscopic layer