Value-based Q(s, S) policy for joint replenishments

Presentation at:

LCCC Workshop: Distributed Model Predictive Control and Supply Chains

Lund, Sweden

May 19-21, 2010

Anders Thorstenson

CORAL – Centre for O.R. Applications in Logistics
Aarhus School of Business, Aarhus University, Denmark

Joint work with:

Søren Glud Johansen

Dept. of Operations Research, Aarhus University (DK)

Outline

- Objective
- Joint Replenishment Problem (JRP)
- Control policies (Literature)
- Value-based $Q(s,S)$ policy
  - Idea
  - Model
- Numerical study (Computations & Simulations)
- Conclusions & extension

Objective

Two-fold objective:

- Obtain improved solutions to the classical JRP
- Apply a dynamic decision-making procedure to improve a static decision rule

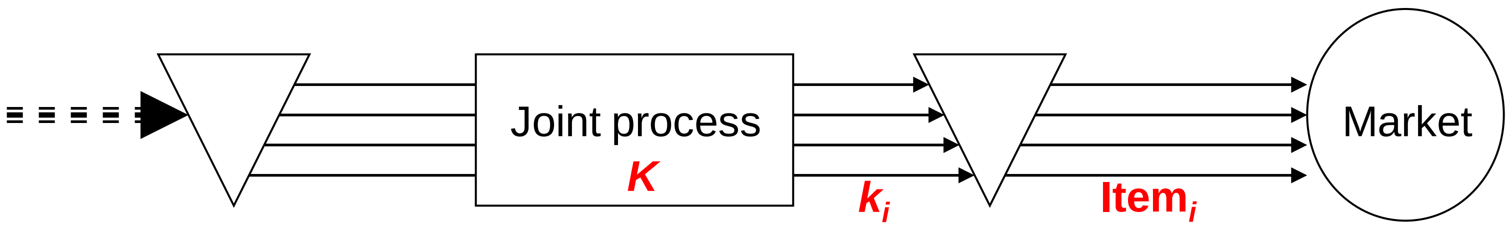
Joint Replenishment Problem (JRP)

- n items
- Setup costs
  - Joint setup cost K
  - Item-specific setup costs k_i
- Holding cost rates h_i
- Penalty costs
  - Backorder cost rate b_i
  - Shortage cost per unit π_i
- Independent (pure) Poisson demands with rates λ_i
- Deterministic lead times L_i

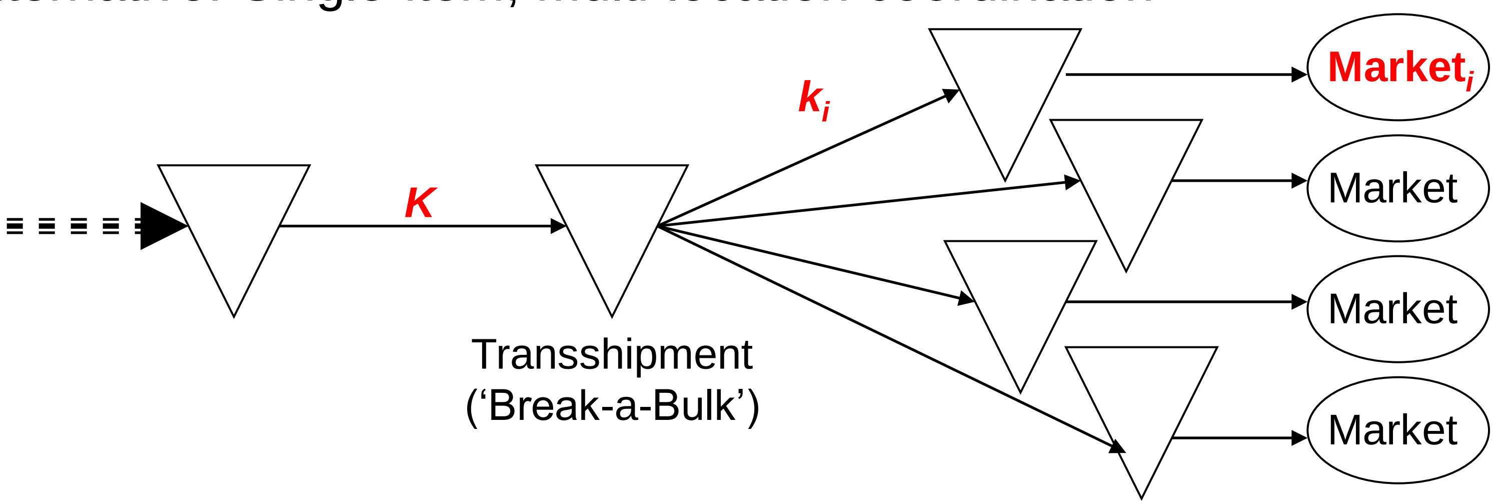
JRP (cont'd))

Interpretations:

- Conventional: Multi-item replenishment coordination



- Alternative: Single-item, multi-location coordination



Control policies (Literature)

- Can-order policies (s, c, S) — Balintfy (1964), …
  - Continuous review — Melchiors (2002)
  - Periodic review — Johansen & Melchiors (2003)
- Periodic review (R, T) policies — Atkins & Iyogun (1988)
- Periodic review P(s, S) policy — Viswanathan (1997)
- ‘Demand-reporting’ $Q(s, S)$ policies
  - QS policy — Pantumsinchai (1992)
  - Pure $Q(s, S)$ policy — Nielsen & Larsen (2005)
  - ***Value-based Q(s, S) policy*** — Johansen & Thorstenson (2006/09)
- Recent contributions — Özkaya et al. (2006), Gürbüz et al. (2007), Viswanathan (2007)

Control policies (cont'd)

(S_i, c_i, s_i) or 'Can-Order' systems

- Continuous review system (or periodic)
- S_i = *order-up-to level*, c_i = 'can'-order point, s_i = 'must'-order point

$Q(S_i)$ system

- Continuous review order-up-to system
- Q = total demand since last replenishment to trigger new order

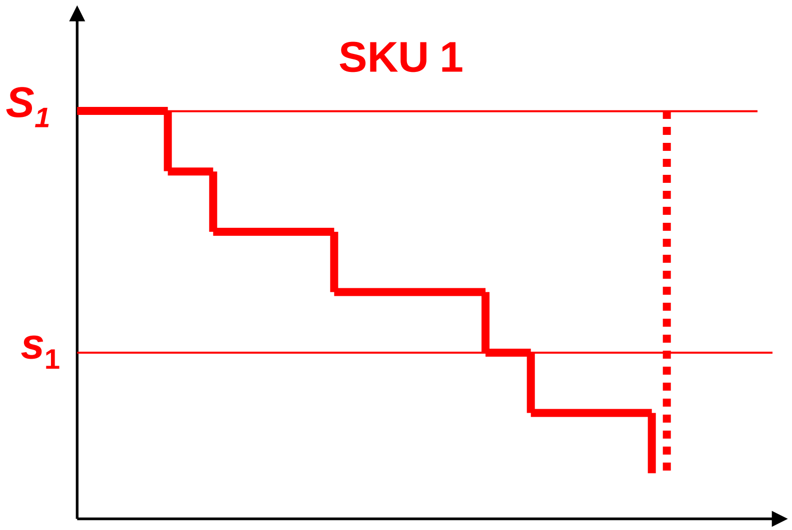
$Q(s_i, S_i)$ system

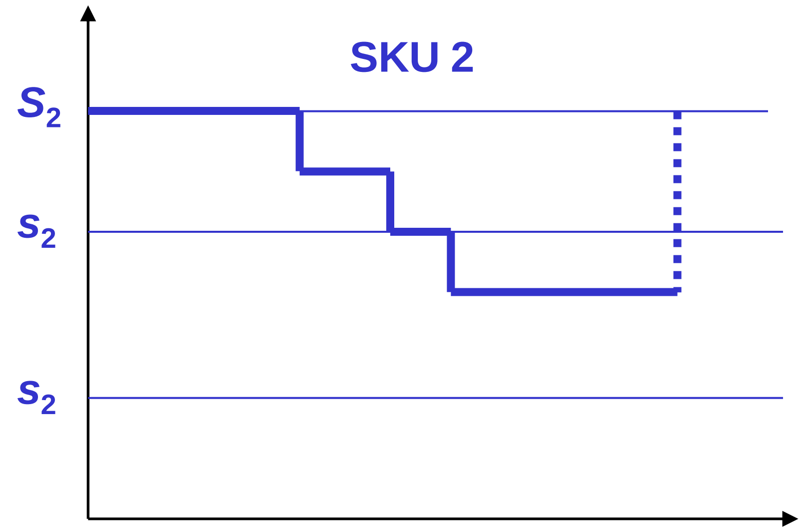
- Continuous review (s, S) system
- Q = total demand since last replenishment to trigger new order

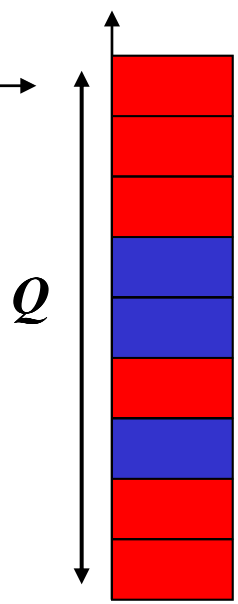
$P(s_i, S_i)$ system

- Periodic review system
- P = optimized common review period

$Q(s, S)$ policy







- **Spec. and algorithm: Nielsen & Larsen (2005)**
- **Improvement: $Q \neq Q^*$? Johansen & Thorstenson (2006/09)**

$Q(s, S)$ policy

- Intuitive
- Analytically tractable with long-run average cost:

$$C(Q) = \frac{K\mu}{Q} + \sum_{i=1}^{n} g\left(Q, s_i^*(Q), S_i^*(Q)\right)$$

- In many cases gives (very) good results
- **But:**
  Dependence on aggregate parameter Q?

Value-based $Q(s, S)$ policy

Basic ideas:

- Consider economic value of deviating from Q^* in the pure $Q(s, S)$ policy

- Approach:

  I. Estimate relative values of system's state at a decision instant

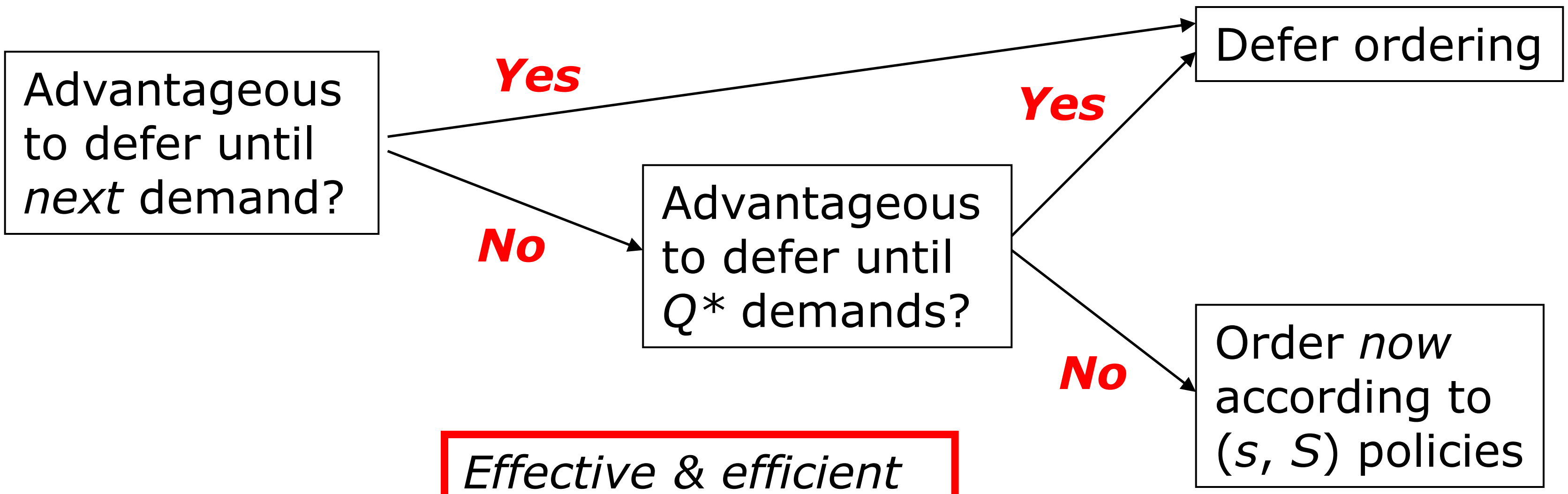
  II. Apply single policy-improvement step

Cf. Tijms (2003)
Adelman (2004)
Axsäter (2006)

Value-based $Q(s, S)$ policy

Basic ideas (cont'd):

- Value comparisons when total demand since last replenishment is 'close to' Q^* (threshold value Q_1):



Effective & efficient procedure

Research questions

- How can the value-based $Q(s, S)$ policy be computed?
- What is the effect on long-run average cost of using the value-based $Q(s, S)$ policy rather than the pure $Q(s, S)$ policy?

Value-based *Q*(*s*, *S*) policy

Model features:

- Basis in pure *Q*(*s*, *S*) policy
- Relative state values from Markov chain
- State representation: # of orders since last ordered

$$0 \le r_i \le Group(i) \quad i = 1, 2, \ldots, n$$

- Value iteration: $V_N(\mathbf{r}) = c_{\mathbf{r}} + \sum_{\mathbf{r}' \in R(\mathbf{r})} P_{\mathbf{r},\mathbf{r}'} V_{N-1}(\mathbf{r}'), \quad \mathbf{r} \in R$
- Cost comparisons: $V_{now}(\mathbf{r},\mathbf{x})\,;\, I(\mathbf{r},\mathbf{x})\,;\, V_{postpone}(\mathbf{r},\mathbf{x};q)$

Questions

- How can the value-based $Q(s, S)$ policy be computed?
- What is the effect on long-run average cost of using the value-based $Q(s, S)$ policy rather than the pure $Q(s, S)$ policy?

Numerical study

Computational tools:

- **Pure** $Q(s, S)$ policy by algorithm in VBA; *Golden section search* procedure for $Q*$
- **Value-based** $Q(s, S)$ policy by algorithm in VBA and simulation model in Arena
  - Simulation setup:
    - 95% confidence intervals
    - 10 replications
    - 10,000 time units + 100 warm-up time units
    - Initialization with $x_i = S_i^*(Q*)$

Numerical study: 12-item problems

Average costs of various policies in four cases under variations of the setup cost K in the 12-item problem introduced by Atkins and Iyogun (1988)

	Can-order		Periodic review		Demand reporting	
	Erlang[1]	Compensation[2]	(R,T)[3]	$P(s,S)$[4]	$Q(s,S)$[5]	Value-based
(d) $h=6$, $b=0$ and $\pi=30$						
$K=50$	*	*	*	*	2052.5	2035.0±1.4
$K=100$	*	*	*	*	2159.1	2140.7±1.8
$K=150$	2288.7±0.6	2313.0±0.8	2291	2267	2251.5	2233.3±1.9
$K=200$	*	*	*	*	2335.2	2319.6±1.6

Numerical study: 3-item problems

Item-specific parameter values of the three-item example in Brønmo (2005)

	item 1	item 2	item 3
demand rate	$\lambda_1 = 0.5$	$\lambda_2 = 0.5$	$\lambda_3 = 0.25$
holding cost rate	$h_1 = 2$	$h_2 = 1$	$h_3 = 2$
backorder cost rate	$b_1 = 4$	$b_2 = 2$	$b_3 = 3$
shortage cost per unit	$\pi_1 = 30$	$\pi_2 = 20$	$\pi_3 = 20$

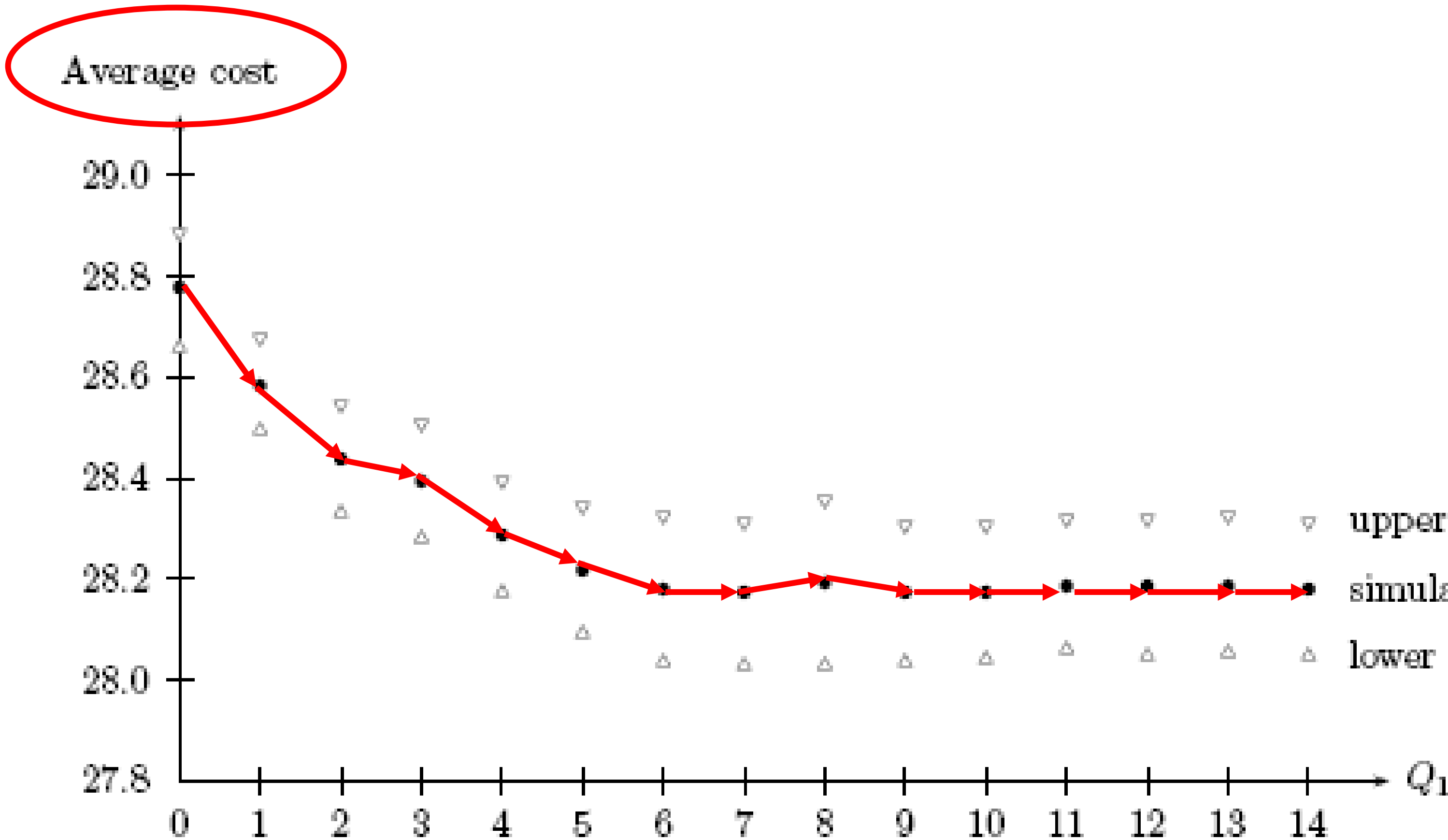
Numerical study: 3-item problems

Average costs of various policies for the examples with three items

	$Q(s,S)^\dagger$	Value-based		Optimal†
		$Q_1 = Int(\frac{Q^*}{10}) + 1$	$Q_1 = Int(\frac{Q^*}{2}) + 1$	
$k = 30$	28.88	28.44±0.13	28.19 ± 0.16	27.77

The other parameter values are $K = 30$, $L_i = 2$ and as reported in Table 2.

Numerical study: 3-item problems



Threshold value with *Q=15**

Numerical study: 3-item problems

Average costs of the pure $Q(s,S)$ policy and the value-based policy for various examples with three items

	b = (32, 16, 32)		b = (64, 32, 64)		b = (128, 64, 128)	
(c) $h_3 = 8$						
$K = 60$	45.28	$Q^*=9$	51.01	$Q^*=8$	56.64	$Q^*=8$
	40.77 +/- 0.21	10.0%	46.35 +/- 0.23	9.1%	49.69 +/- 0.26	12.3%

Pure $Q(s, S)$

Value based

Conclusions & Extension

- **Dynamic** value-based *Q*(*s*, *S*) policy
  modifies the **static** pure *Q*(*s*, *S*) policy
  by evaluating **expected cost of deviating** from it

- Value-based *Q*(*s*, *S*) policy dominates
  the pure *Q*(*s*, *S*) policy –
  in some cases cost savings of **more than 10%**

- Although state representation limits state space,
  Curse of dimensionality still may apply!

- Extension: **Value-based P(*s*, *S*) policy**
  Decomposes in items => smaller state space

Questions and …?

ath@asb.dk