Towards Parallel Implementation of Hybrid MPC

A Survey and Directions for Future Research

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Questions

- Why parallel implementation of hybrid MPC?
- How is hybrid MPC solved today?
- Are there any parallel implementations already?
- What is left to be done?

Answers

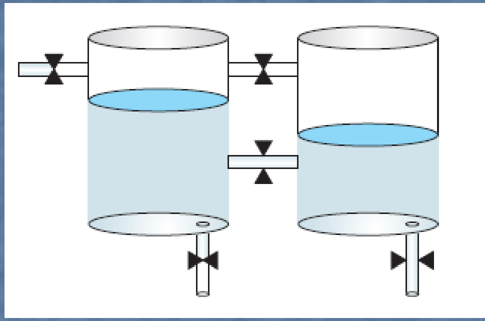
- Parallel computer architectures
- Hybrid MPC
- Distributed MPC
- Generic parallel integer programming algorithms
- Immediate possibilities
- Futuristic ideas
- Conclusions

Parallel Computer Architectures

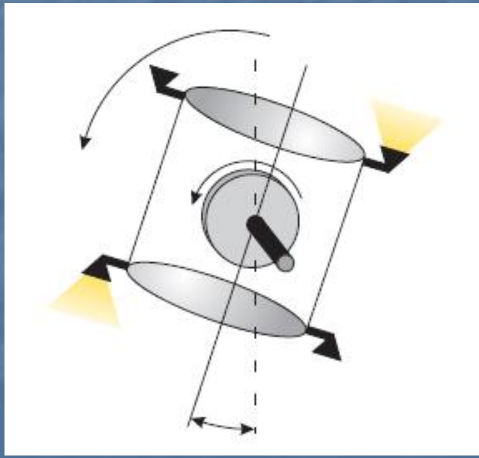
- Not possible to increase performance with higher clock frequency
- Multi-core and multi-processor computers *share memory*
- Clusters and grids are composed of computers linked via a network and usually *do not share memory*
- Parallel implementations key to efficient utilization

Hybrid Control

- Control of discrete-time hybrid systems
- Binary control signals
- Real-valued control signals
- Logical states



On/off valves



On/off thrusters

Large Scale Applications

- Transportation, logistics, and economics
- Process control
- Building control
- Airplane routing problems
- Communications

Hybrid MPC

- Modeling
- Optimal control problem
- Optimization methods

Modeling

- Mixed Logical Dynamical (MLD) models (Bemporad and Morari 1999)
- Piecewise Affine (PWA) models (Sontag 1981)
- Discrete Hybrid Automata (DHA) (Torrisi and Bemporad 2004)

Optimal Control Problem

$$\underset{u(.)}{\text{minimize}} \quad \sum_{k=0}^{N-1} l_k\left(x(k), u(k)\right)$$

subject to model of hybrid systems and constraints

Optimization Methods

- *Mixed Integer Programming* (MIP) solvers for MLD models
  - Branch and Bound (BB) (Bemporad and Morari 1999)
  - Multi-parametric programming (Bemporad et al 2000, Johansen et al 2000, Seron et al 2000, Dua et al 2002)
- *Logic-based* BB for DHA models (Bemporad and Giorgetti 2006) Based on Rodosek et al 1997, Bockmayr and Kasper 1998, Hooker 2000, Harjunkoski et al 2000, Focacci et al 2001

Parallel Implementation of Hybrid Optimal Control?

- Barth et al 2000: “Distributed Solution of Optimal Hybrid Control Problems on Networks of Workstations”
- Simulation based solution for continuous time hybrid optimal control problem

Distributed MPC

- Distributed control without collaboration (*decentralized control*)
- Distributed control with collaboration over network (*distributed control*)
- Distributed control with collaboration using coordinator (*hierarchical control for co-ordination*)

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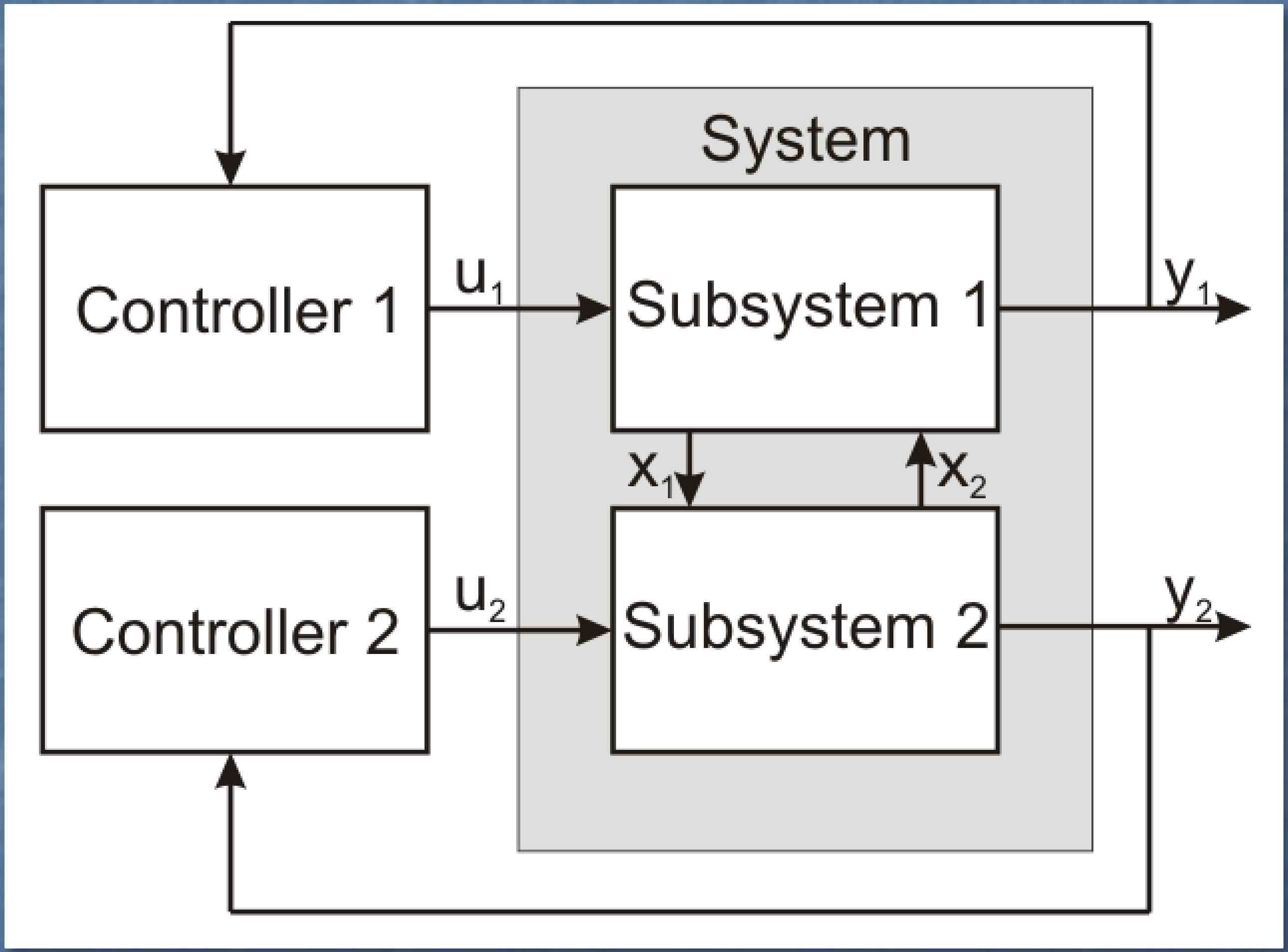
Decentralized MPC

Stability proven for decaying disturbances and for uncertainty



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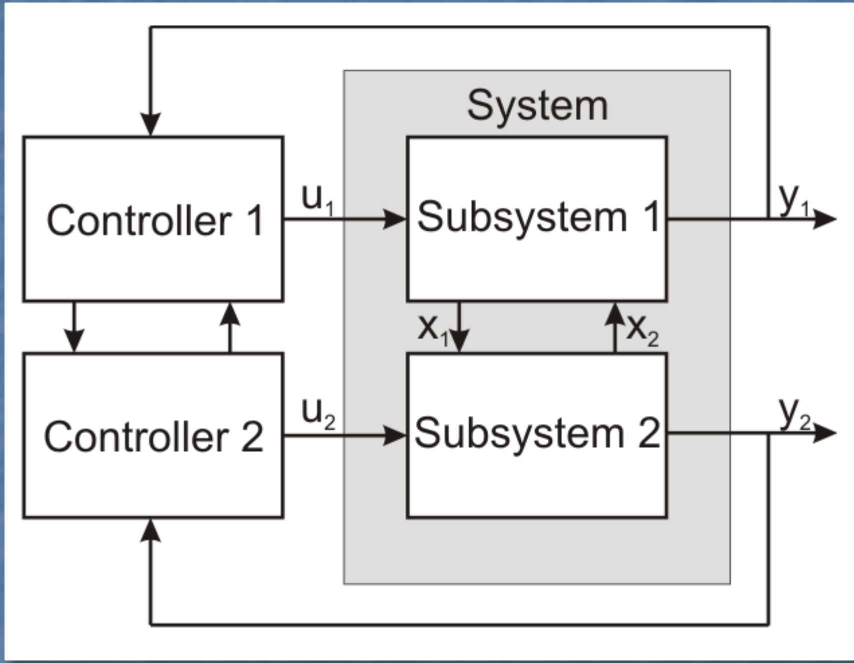
Distributed MPC

Topology: fully or partially connected

Protocol: iterative or non-iterative

Cost function: local (independent algorithm) or global (co-operating algorithm)



Distributed MPC	Independent	Co-operating
Non-iterative	Stability	-
Iterative	Nash equilibrium	Optimality of global cost

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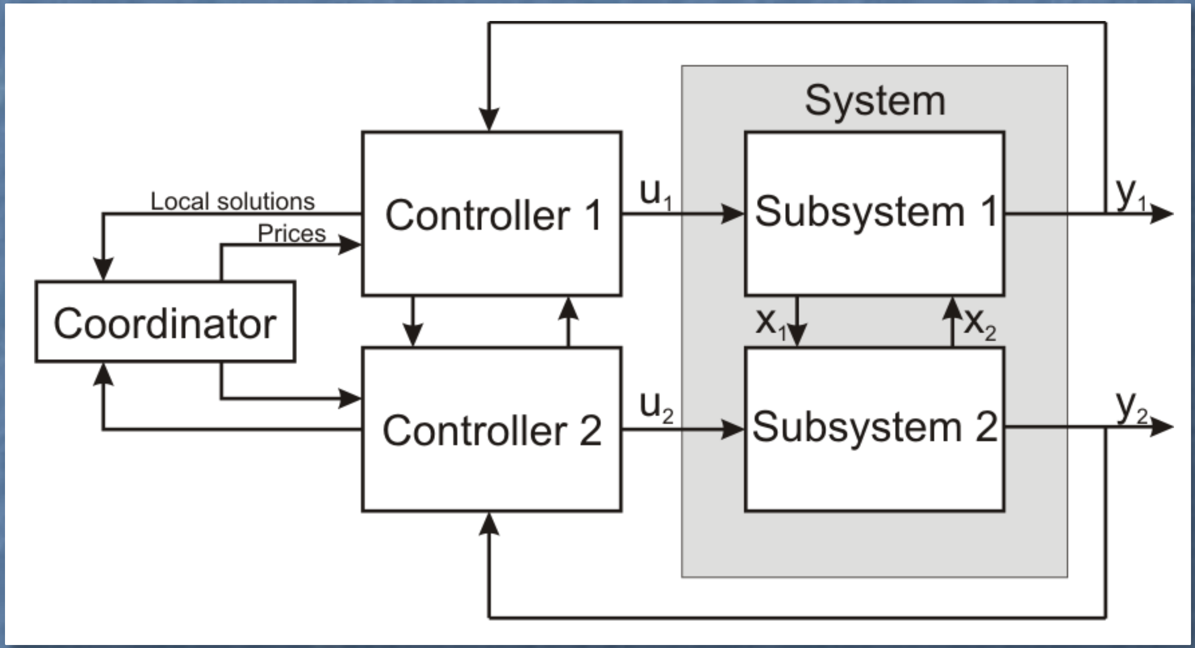
Hierarchical MPC for Coordination

Lagrange multipliers or prices used to achieve global optimality



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What about Distributed MPC for Hybrid Systems?

- Lagrange multipliers cannot be used to achieve global optimality
- Augmented Lagrangian approach shown to give feasible solutions in example (Negenborn 2007)
- Distributed hybrid MPC performs better than decentralized but worse than centralized in examples (Mestan et al 2006, Tarau et al 2009)
- *Lack of optimality results*

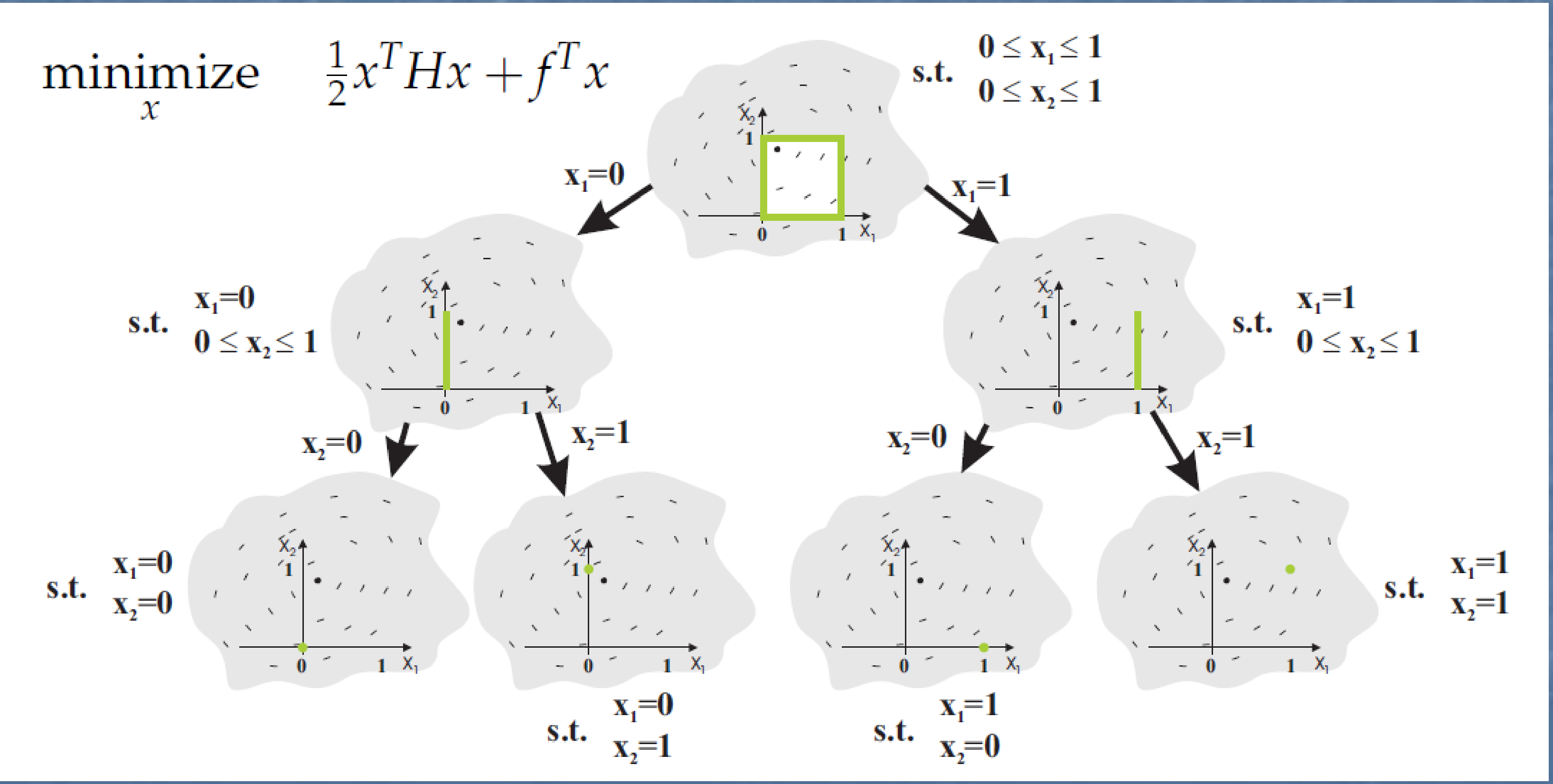
Does this mean that there is no Hope for Parallel Implementations?

The way workload is distributed between processors in parallel implementations can be quite different from how the dynamics is distributed in distributed control

Generic Parallel Integer Programming Methods

- BB type of methods
- Dynamic programming
- Constraint programming
- Genetic algorithms
- Simulated annealing
- Tabu search

Branch and Bound



Branch and Bound ctd.

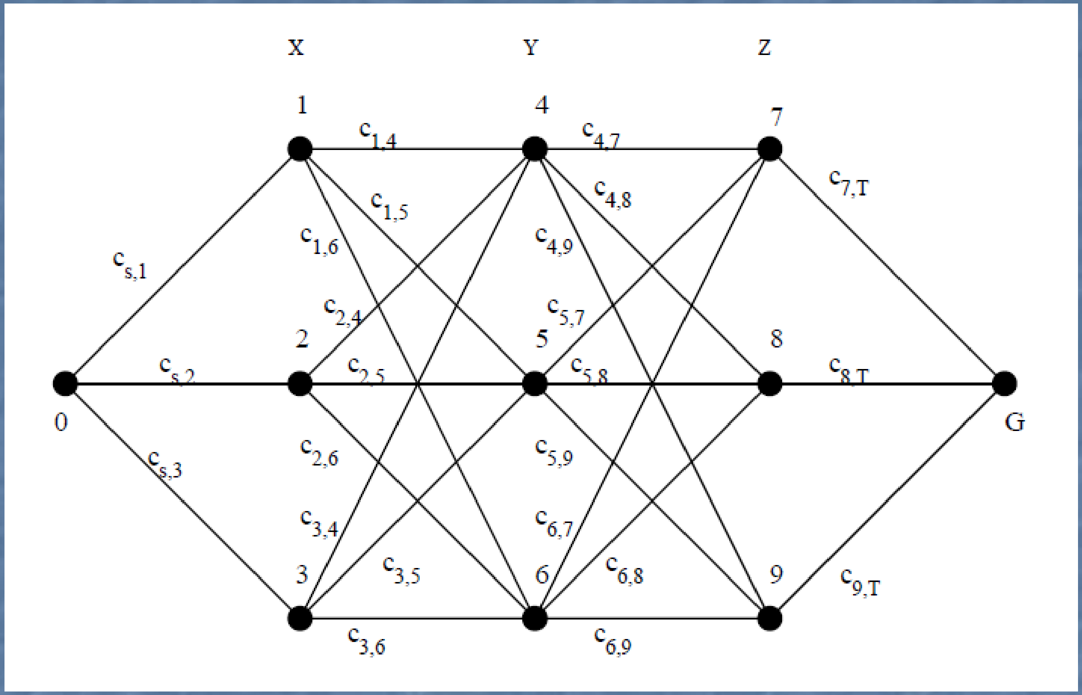
```
program depth_first_branch_and_bound
    current_best_solution = infinity;
    select initial node and place on stack;
    repeat
    begin
        select node from the top of the stack;
        if (selected node is not the solution)
        begin
            evaluate best possible solution this node can lead to;
            assign this value to node_bound;
            if (node_bound < current_best_solution)
            begin
                generate successors (if any) of selected node;
                install generated successors into the stack;
            end_if
        end_if
        else
            if (cost_of_solution < current_best_solution)
                current_best_solution = cost_of_solution;
    end_repeat
    until (stack is empty)
end_program
```

Speedup can be obtained by

- *Stack* splitting
- *Node* splitting

Grama and Kumar 1995

Dynamic Programming



Grama and Kumar 1995

- Assume n nodes at each stage
- Assign each node at each stage to one processor
- Computational cost decreases from O(n^2) to O(n)
- When dependencies between nodes are sparse, dedicated algorithms can increase efficiency further

Constraint Programming

- Adopt: Asynchronous distributed constraint optimization with quality guarantees (Modi et al 2005)
- Solving Distributed Constraint Optimization Problem Using Cooperative Mediation (Mailler and Lesser 2004)
- *Orders of magnitude in speedup*

Immediate Possibilities

- Use parallel implementation of BB to solve MIP problem for hybrid MPC based on MLD models
- Parallelize logic-based BB for DHA models using parallel implementation of BB *and/or* parallel constraint programming
- Incorporate existing tailored linear algebra (Axehill and Hansson, 2008) in a parallel BB algorithm
- Add parallel processing capabilities to some of the main operations in the gradient projection based QP solver in (Axehill and Hansson, 2008)

Futuristic Ideas

- Develop a parallel implementation of parametric BB (e.g. Acevedo and Pistikopoulos 1997) to solve hybrid MPC problems explicitly
- Processor scheduling built on recent results on off-line analysis of on-line branch and bound (Axehill and Morari 2010)

Conclusions

- Efficient parallel optimization algorithms are necessary to take advantage of today’s hardware
- Several interesting yet-to-be-explored strategies exist
- Few, if any, branches of MPC require as much computational power as hybrid MPC

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