Smart Grid Intro
Economic Dispatch with Battery

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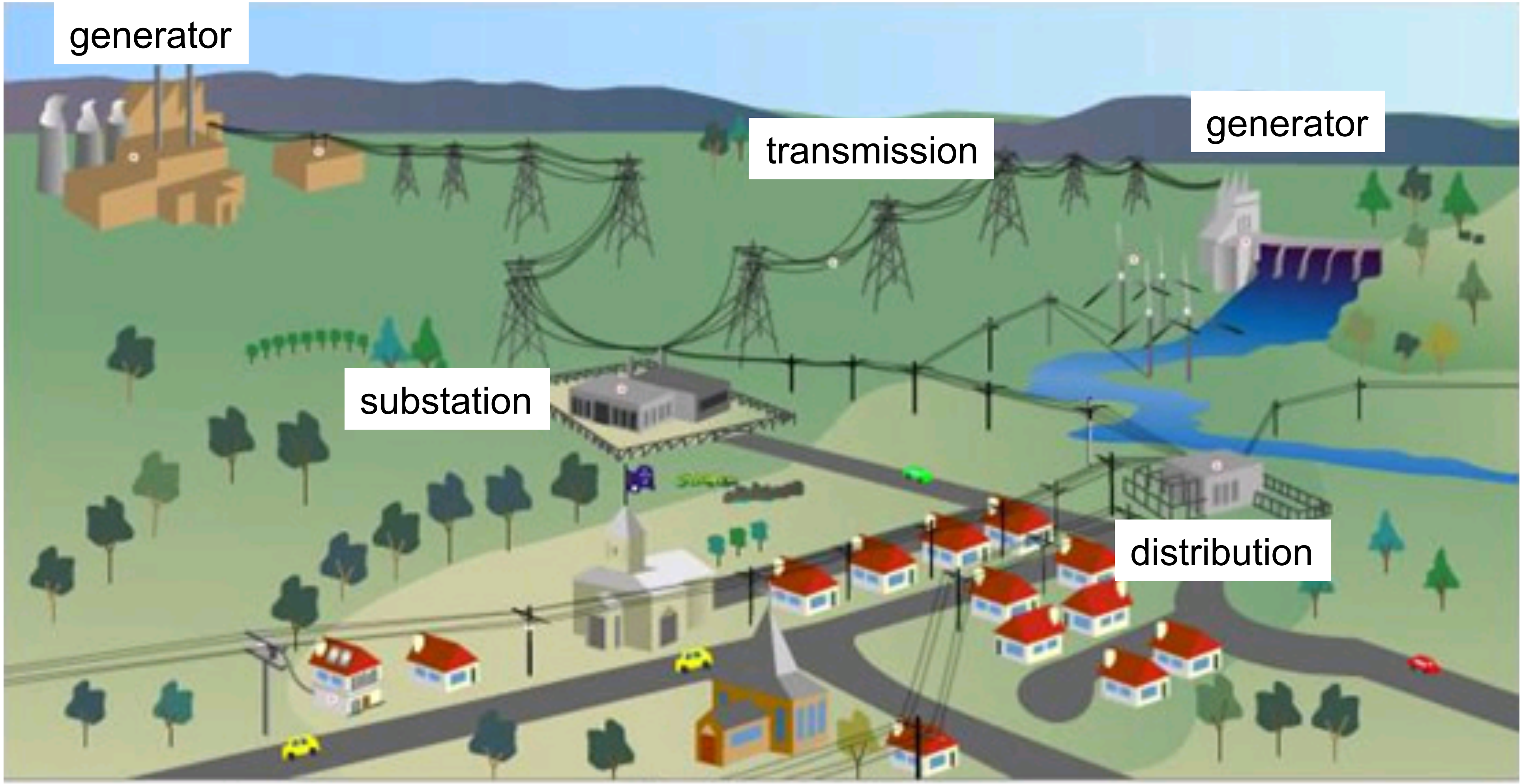
Outline

- Smart grid intro
  - Quick snapshot
  - Research issues

- Economic dispatch problem
  - Model with storage
  - Preliminary results

Where does electricity come from

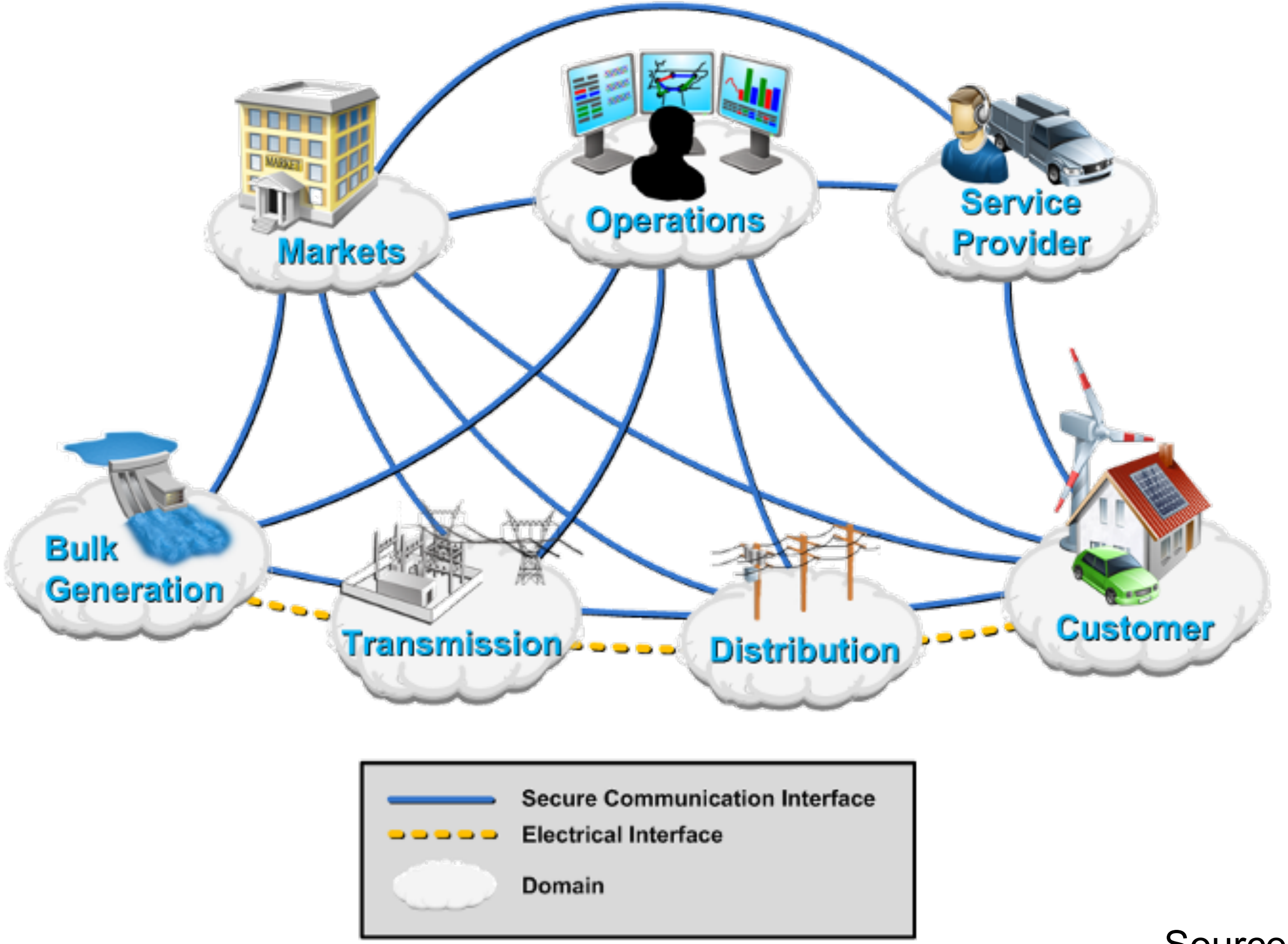


Electricity network

Transmission:	from generator to substation, long distance, high voltage
Distribution:	from substation to customer, shorter distance, lower voltage

Industry structure

Deregulation presents huge opportunities & challenges



Source: Rosa Yang

Source: EPRI Report to NIST on Smart Grid Interoperability, June 2009

US power generation & distribution

- Nation’s electricity bill: $247B
- 131M customers
- Average price: 7c/KWh
- 3,100 electric utilities
- 9,200 power plants with 1TW capacity
  - World electricity usage: 1.9 TW / 297 Wpc (2005)
  - US: 436 GW / 1460 Wpc (2005);
  - China: 326 GW / 248 Wpc (2006);
  - EU: 322 GW / 700 Wpc (2004)

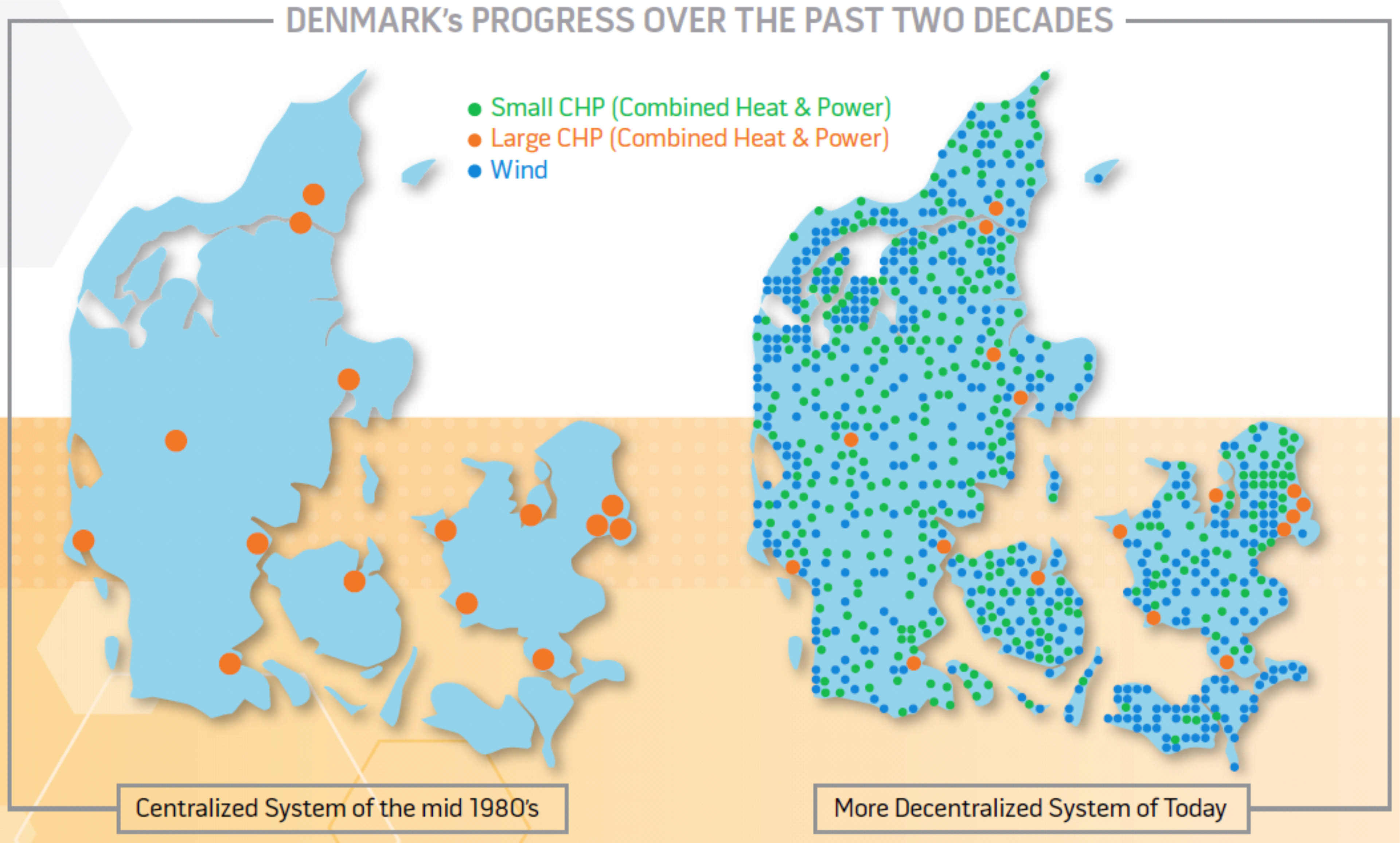
Sources:
DoE, Smart Grid Intro, 2008
DoE, GRID 2030, 2003

US power generation

- Central power plants
    - 33% thermal efficiency, unchanged since 1960s
    - Central power generation cannot recycle heat
- Distributed energy facilities
    - ~5,600 facilities, 6% of power capacity (2001)
    - 65% - 90% efficiency
    - Combine heat & power generation

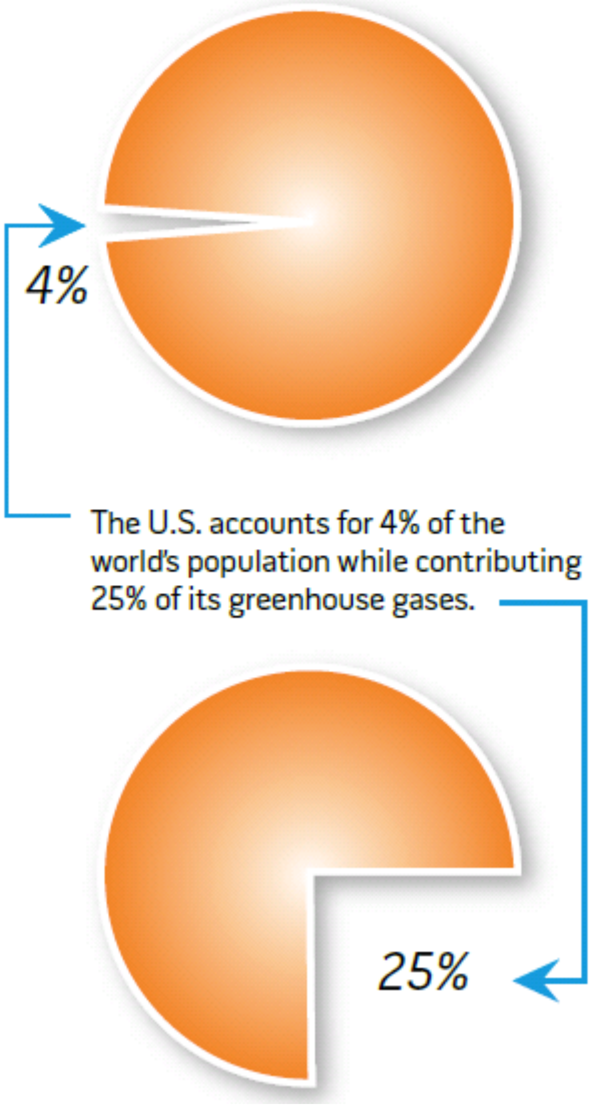
Source: DoE, GRID 2030, 2003

Future trend



Source: DoE, Smart Grid Intro, 2008

Generate more than power...



The U.S. accounts for 4% of the world's population while contributing 25% of its greenhouse gases.

US CO_2 emission
- ☐ Elect generation: 40%
- ☐ Transportation: 20%

Source: DoE, Smart Grid Intro, 2008

US power transmission

- 300,000 miles of transmission & distribution lines
  - 157K miles of high voltage (>230kV)
- Trans & distr losses: 9.5% (2001)
  - Trans & distr losses: 5% (1970)
- 99.97% reliable
- Yet, outages cost $150B/yr
  - Northeast blackout of 2003: $6B loss
  - 5 massive blackout in the last 40 yrs
  - 3 of which occurred in the last 9 yrs

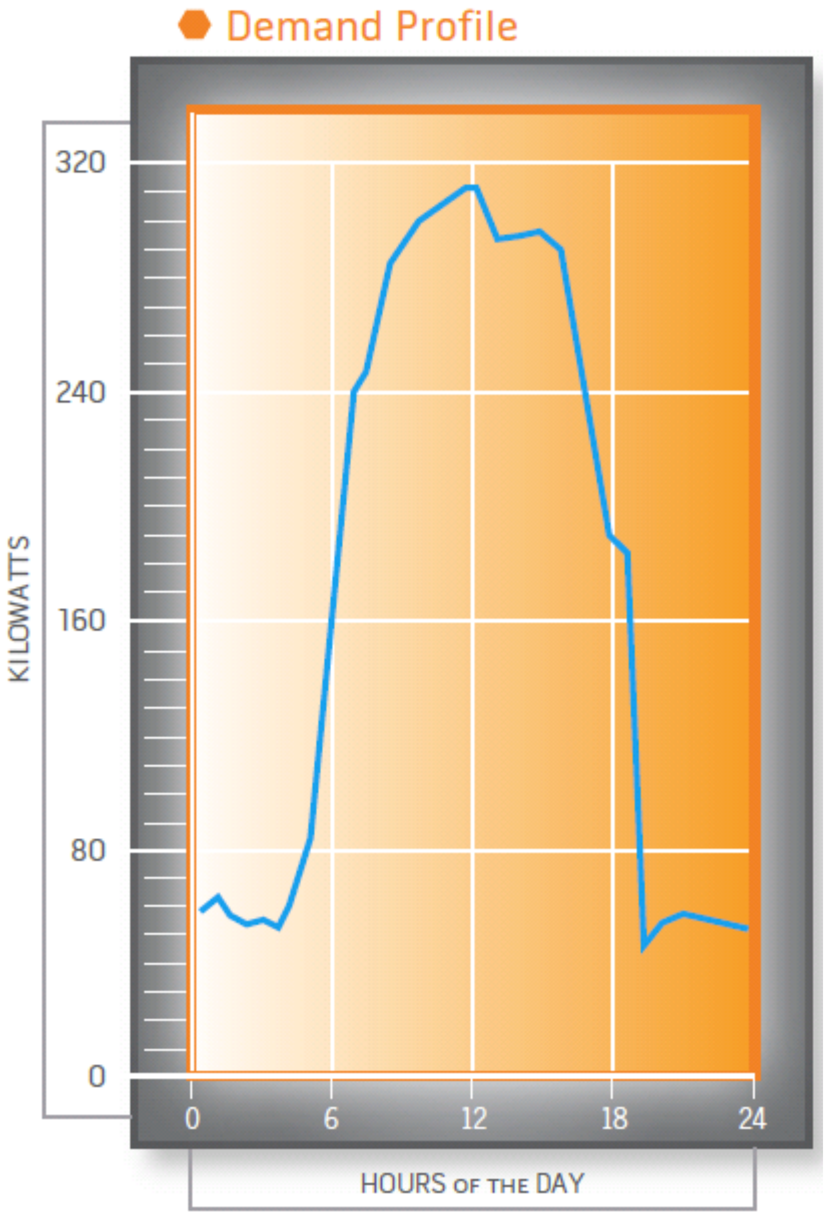
Source: DoE, Smart Grid Intro, 2008

US power transmission

- Old infrastructure
  - Average generating station built in 1960s
  - Average age of substation transformer: 42 yrs (expected life span: 40 yrs)
  - Before PCs and Internet!
- Since 1982, peak demand outgrows transmission capacity by 25%/yr
  - From 1988-98, electricity demand rose by ~30%, while transmission capacity grew by ~15%
  - Annual investment in transmission capacity has dropped 50% since 1975 [Wu et al, Proc IEEE, 2005]

Source: DoE, Smart Grid Intro, 2008

Challenges with peak



- ☐ National load factor: 55%
- ☐ 10% of generation and 25% of distribution facilities are used less than 400 hrs per year, i.e. ~5% of time
- ☐ Existing power plants can provide 73% of light vehicles
  - If they are plug-ins that recharge at night
  - Will reduce oil consumption by 6.2M barrels a day

Source: DoE, Smart Grid Intro, 2008

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Conclusions from snapshot

Grid connects generation to load

- Grid must become more efficient
  - Demand and generation will continue to outpace transmission capacity
  - 5% higher grid efficiency = 53M cars
- Grid must integrate distributed and renewable sources
  - Sustainability
- Grid must promote demand response
  - Cutting peak demand has a huge impact
  - Integrate and exploit EV

Challenges with grid control

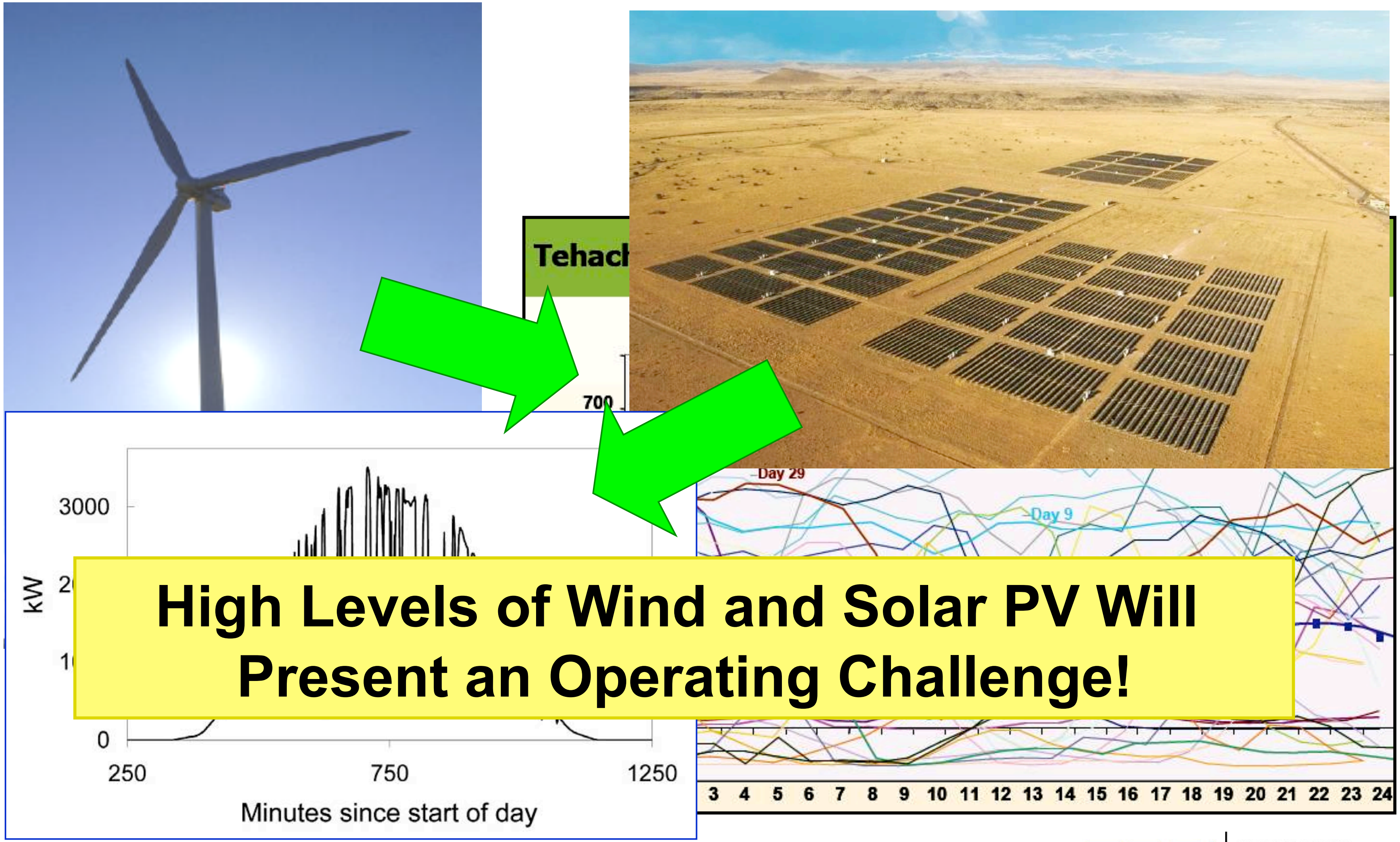
- Different timescales drive independent actions by relay system and CC
  - This gap has led to missed opportunity in preventing cascading outages
  - E.g., NA blackout & Italian blackout of 2003

- No analytical tools for emergency control
  - No dynamic system model for stability analysis after fault
  - Rely on simulation: too slow for real-time control
  - Necessitates conservative “open-loop” control

Grid + ICT

- Both issues due to ICT limitation
  - Data acquisition system (SCADA)
  - CC computational power
- … that can be drastically improved
  - More computational power in CC
  - Faster SCADA communications over WAN
  - GPS-based PMUs provide real-time and synchronized power measurements
- Future PMUs+ICT can provide system-wide synchronized data on ms timescale
  - How to control with 1000x increase in capability?

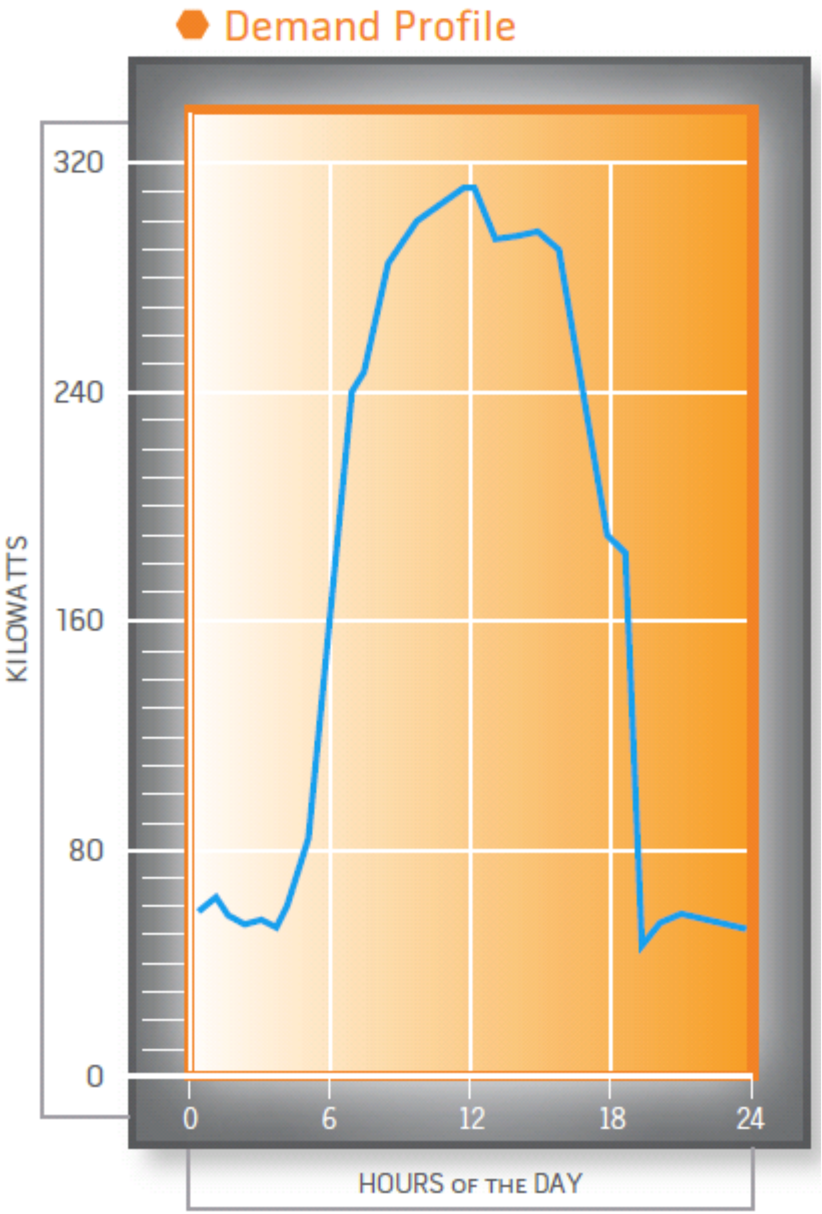
Challenges with renewables
Variability & Uncertainty



High Levels of Wind and Solar PV Will Present an Operating Challenge!

Source: Rosa Yang

Challenges with peak

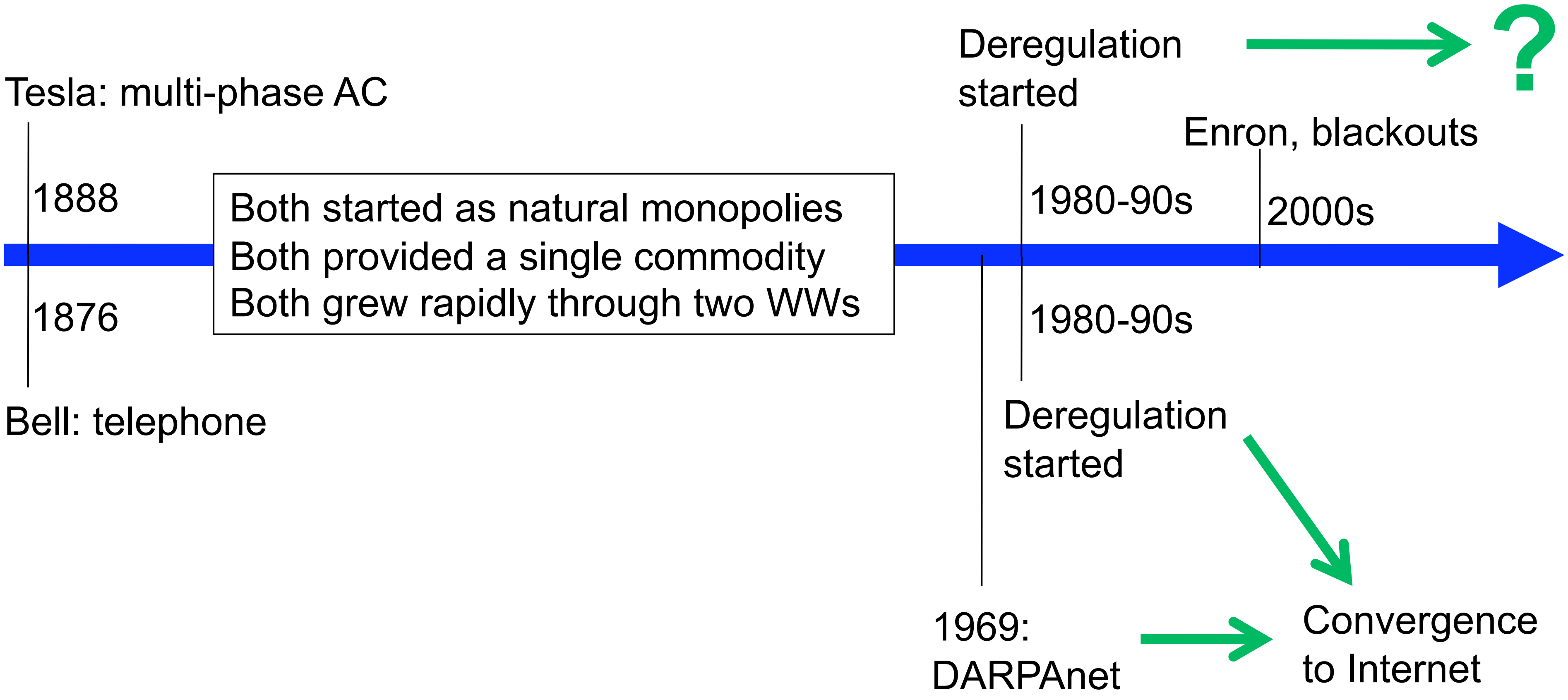


- ☐ Demand response will be deployed
- ☐ How should utility price power?
- ☐ How should users respond?
- ☐ Stability, reliability, efficiency, fairness
- ☐ IT & deployment issues

Source: DoE, Smart Grid Intro, 2008

Challenges with architecture

Power network will go through similar **architectural transformation** in the next couple decades that phone network is going through now



Tesla: multi-phase AC

1888

Both started as natural monopolies
Both provided a single commodity
Both grew rapidly through two WWs

1876

Bell: telephone

Deregulation started

1980-90s

Enron, blackouts

2000s

?

1980-90s

Deregulation started

1969: DARPAnet

Convergence to Internet

Challenges with architecture

... to become more intelligent, more distributed, more open, more autonomous, and with greater user participation

What is an architecture theory to help guide the transformation?

... while enhancing security & reliability

Enabling technologies

- AMI (Advanced Metering Infrastructure)
  - Enable demand response
  - Promote user participation
- PMU (Phasor Measurement Unit)
  - Real-time, global, synchronized measurement
  - At ms timescale, 1000x faster than now
- ICT integration with grid
  - High speed WAN allows real-time and global monitoring at control centers
  - High performance computing allows faster control decisions
- Large scale storage?

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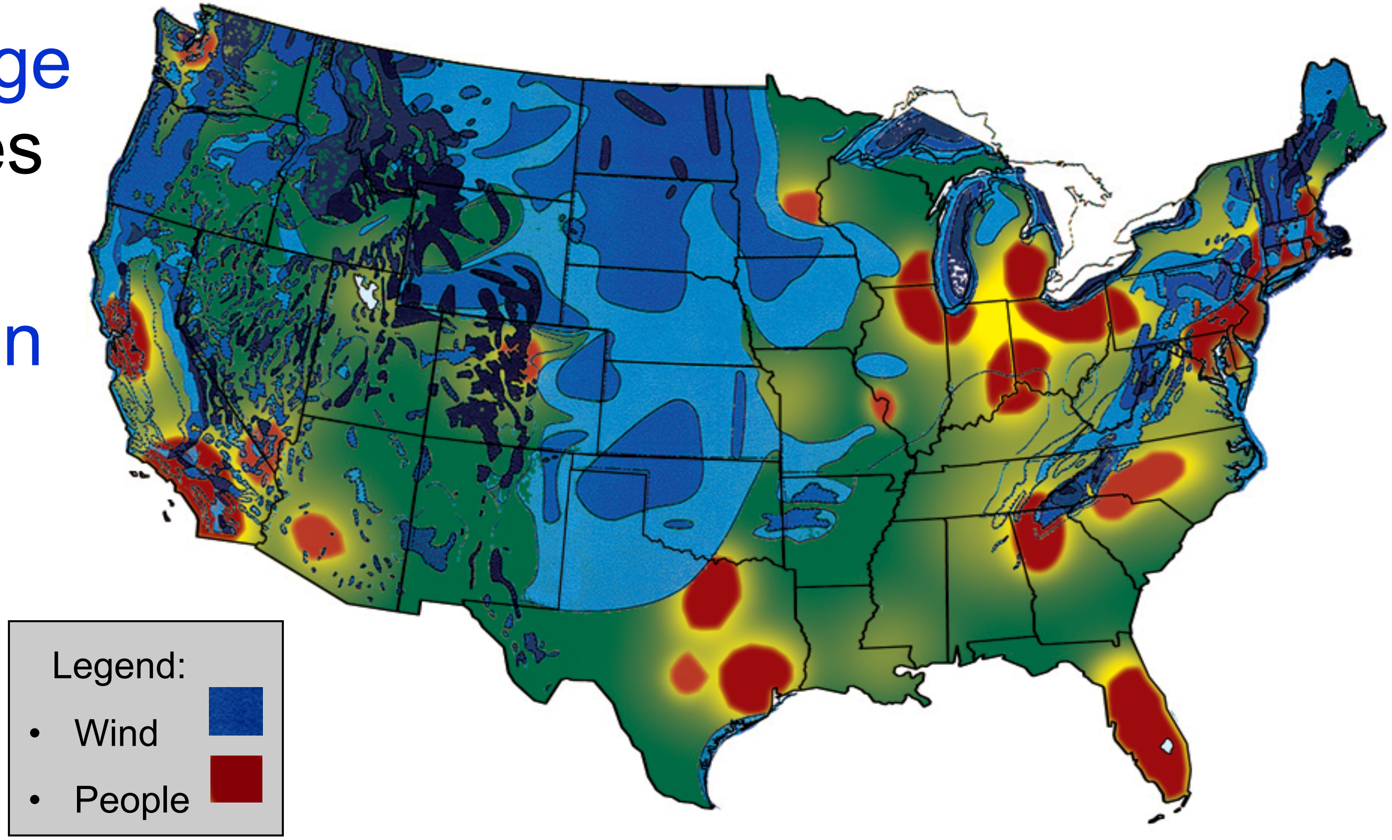
Wind & Solar are Intermittent and Far from People

- Need storage technologies
- Need transmission lines



Source: Rosa Yang

Economic dispatch problem

- Generator nodes $i \in G$
  - How much to generate $g_i(t)$
  - Operates battery $b_i(t)$
  - How much to charge/discharge battery $r_i(t)$
- Demand nodes $i \in D$
  - Given demand $d_i(t)$

- Network operator (ISO)
  - Given network admittances $Y_{ij}, i \neq j \in G \cup D$
  - Computes power angles $\theta_i(t)$ at $i \in G \cup D$
  - Set nodal and transmission prices
  - Must balance supply & demand

Network model

min $\sum_{t=1}^{T}\sum_{i\in G}\left\{c_i\left(g_i(t),t\right)+h_i\left(b_i(t),r_i(t)\right)\right\}$

over $u(t):=\left(\theta_i(t),q_i(t),i\in N;\quad g_i(t),r_i(t),b_i(t),i\in G\right)$

subject to $Y_{ij}\left(\theta_i(t)-\theta_j(t)\right)\le \bar{q}_{ij}$ $\qquad i\neq j\in N$

$q_i(t)=\sum_{j\in N}Y_{ij}\left(\theta_i(t)-\theta_j(t)\right)$ $\qquad i\in N$

$q_i(t)=g_i(t)+r_i(\mathrm{t})$ $\qquad i\in G$

$g_i(t)\geq 0$ $\qquad i\in G$

$-q_i(t)=d_i(t)$ $\qquad i\in D$

for $t=1,2,\ldots,T$

Network model

time-varying generation cost

min $$\sum_{t=1}^{T}\sum_{i\in G}\left\{c_i\left(g_i(t),t\right)+h_i\left(b_i(t),r_i(t)\right)\right\}$$

over $$u(t):=\left(\theta_i(t),q_i(t),i\in N;\quad g_i(t),r_i(t),b_i(t),i\in G\right)$$

subject to $$Y_{ij}\left(\theta_i(t)-\theta_j(t)\right)\le\bar{q}_{ij} \qquad i\neq j\in N$$

Line capacity constraint

$$q_i(t)=\sum_{j\in N}Y_{ij}\left(\theta_i(t)-\theta_j(t)\right) \qquad i\in N$$

Kirchoff Law

$$q_i(t)=g_i(t)+r_i(\mathrm{t}) \qquad i\in G$$

Power = gen + battery

$$g_i(t)\ge 0 \qquad i\in G$$

$$-q_i(t)=d_i(t) \qquad i\in D$$

Supply = demand

for $$t=1,2,\ldots,T$$

time-varying demand

Economic dispatch problem

- Initial theory developed in 1980s – 1990s
- An approximate model used in practice to compute nodal prices and transmission rights

Generation markets

- Two types of markets
  - Bilateral contracts between suppliers and consumers
  - Auction market
- Auction market
  - Generators submit bids to a centralized agent (ISO/RTO)
  - ISO/RTO determines winning bids and price
  - Markets to balance supply and demand
    - Day-ahead market
    - Real-time balancing market
    - Ancillary service market

Generation markets

- Day-ahead market
  - Forward market to calculate clearing prices for each hour of the next day
  - Based on generation & demand bids, and bilateral transaction schedules
  - Congestion management needed when reliability of transmission system is bottleneck
  - Based on LMP (locational marginal price) = differences between nodal shadow prices

Generation markets

- Real-time balancing market
  - Calculates a new set of market clearing prices (LMPs) using SCED (security constrained economic dispatch) every 5 mins
  - Based on revised generation bids and actual operating condition from state estimation
  - Any amount of generation, load, or bilateral transaction that deviates from the day-ahead schedule will pay the balancing market LMP

Network model

$$
\begin{array}{llll}
\text{min} & \displaystyle\sum_{t=1}^{T}\sum_{i\in G}\left\{c_i\left(g_i(t),t\right)+h_i\left(b_i(t),r_i(t)\right)\right\} & & \\
\text{over} & u(t):=\left(\theta_i(t),q_i(t),i\in N;\ \ g_i(t),r_i(t),b_i(t),i\in G\right) & & \\
\text{subject to} & Y_{ij}\left(\theta_i(t)-\theta_j(t)\right)\le \bar{q}_{ij} & i\ne j\in N & \mu_{ij}(t)\ge 0 \\
 & q_i(t)=\displaystyle\sum_{j\in N}Y_{ij}\left(\theta_i(t)-\theta_j(t)\right) & i\in N & p_i(t) \\
 & q_i(t)=g_i(t)+r_i(\mathrm{t}) & i\in G & \lambda_i(t) \\
 & g_i(t)\ge 0 & i\in G & \hat{\lambda}_i(t) \\
 & -q_i(t)=d_i(t) & i\in D & \delta_i(t)\ge 0 \\
\text{for} & t=1,2,\ldots,T & &
\end{array}
$$

Lagrange multipliers

Battery model

For $i \in G, t = 1,2,\ldots,T$

$$b_i(t) = \left[b_i(t-1) - r_i(t) \right]_0^{B_i}$$

$$b_i(0) \ : \ \text{given}$$

$\tilde{b}_i(t)$

$\underline{b}_i(t) \geq 0$

$\overline{b}_i(t) \geq 0$

Lagrange multipliers

Economic dispatch problem

- Without battery: optimization in each period in isolation
  - Grid allows optimization across space
  - Theory started in 1980s – 90s

- With battery: optimal control over finite horizon
  - Battery allows optimization across time

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  - ☐ Preliminary results
    - ☐ Single generator single load TI
    - ☐ General case

Simplest case

- Single generator single load (SGSL)
- Time-invariant cost and demand

$$c(g) = \frac{1}{2}\gamma g^2 \qquad h(b) = \alpha(B - b)$$

$$d(t) = d, \qquad t = 1, \ldots, T$$

- Problem reduces to constrained LQ

$$\min_{g(t) \ge 0} \quad \sum_{t=1}^{T} \{c(g(t)) + h(b(t))\}$$

$$\text{s.t.} \quad b(t) = b(t-1) - d + g(t)$$

$$0 \le b(t) \le B$$

← all the complications

Inactive battery constraint

- If battery constraint inactive

$$\begin{aligned} &\min_{g(t)\geq 0} && \sum_{t=1}^{T} \frac{1}{2}\gamma g^2(t) \ + \ \alpha(B - b(t)) \\ &\text{s. t.} && b(t) = b(t-1) - d + g(t) \end{aligned}$$

- Optimal generation decreases linearly in time

$$\bar{g}(t) = \frac{\alpha}{\gamma}\left(T + 1 - t\right) \quad \longleftarrow \quad \text{“nominal generation”}$$

- Optimality: $\underset{\substack{\uparrow \\ \text{marginal cost} \\ \text{of generation}}}{\gamma\bar{g}(t)} = \alpha\underset{\substack{\uparrow \\ \text{unit-cost-to-go} \\ \text{of storage}}}{\left(T + 1 - t\right)}$

Inactive battery constraint

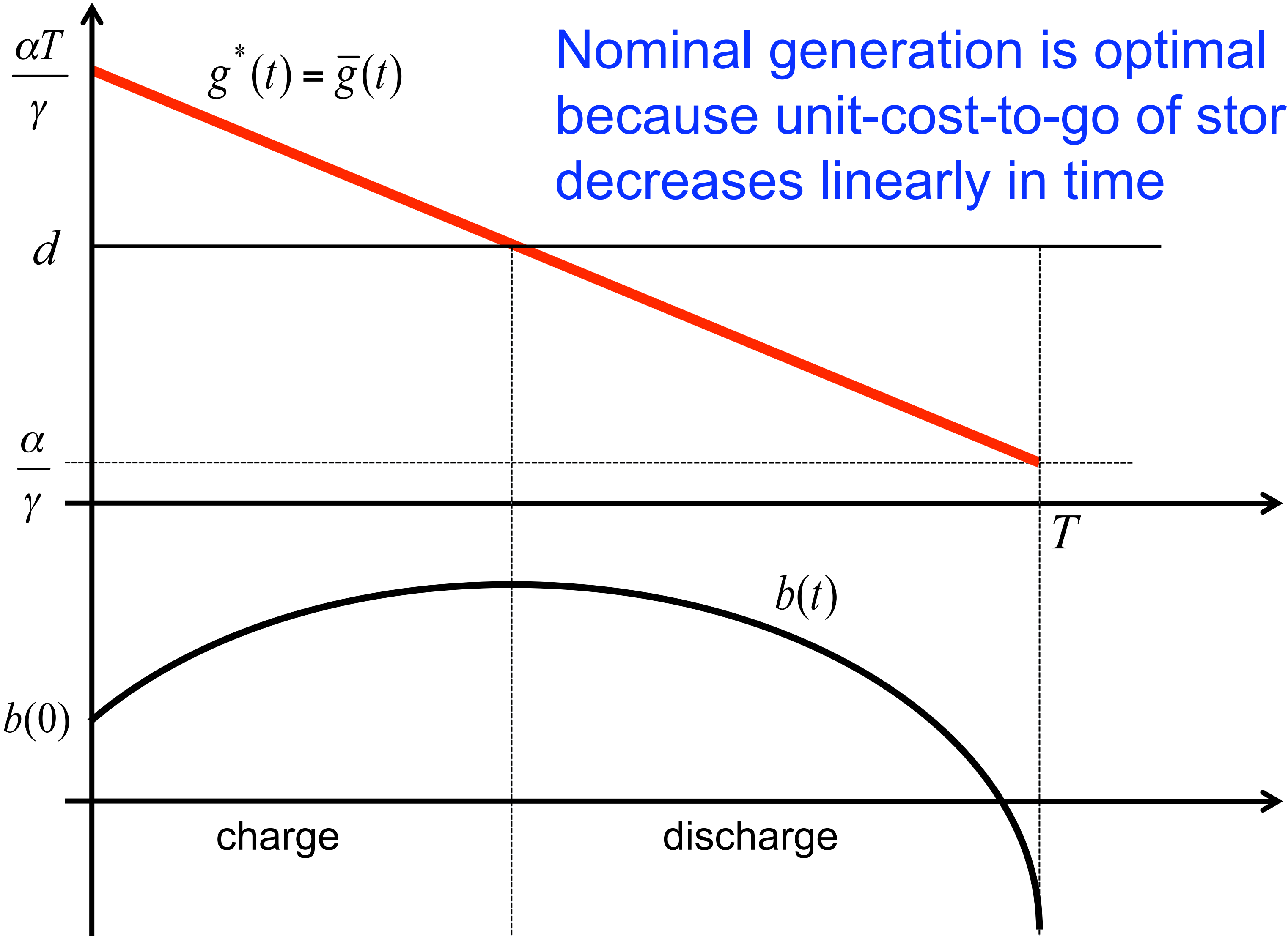
- ☐ If battery constraint inactive

$$\min_{g(t)\geq 0} \quad \sum_{t=1}^{T} \frac{1}{2}\gamma g^2(t) \; + \; \alpha(B - b(t))$$

$$\text{s. t.} \quad b(t) = b(t-1) - d + g(t)$$

- ☐ Optimal battery level varies quadratically in time
- ☐ Charge initially, then discharge
    - ☐ Unit-cost-to-go of storage decreases linearly over time

Inactive battery constraint



Nominal generation is optimal because unit-cost-to-go of storage decreases linearly in time

SGSL-TI case

- With battery constraint

$$\begin{aligned} &\min_{g(t)\geq 0} && \sum_{t=1}^{T} \frac{1}{2}\gamma g^{2}(t) \ + \ \alpha(B-b(t)) \\ &\text{s. t.} && b(t)=b(t-1)-d+g(t) \quad \in \left[0,\ B\right] \end{aligned}$$

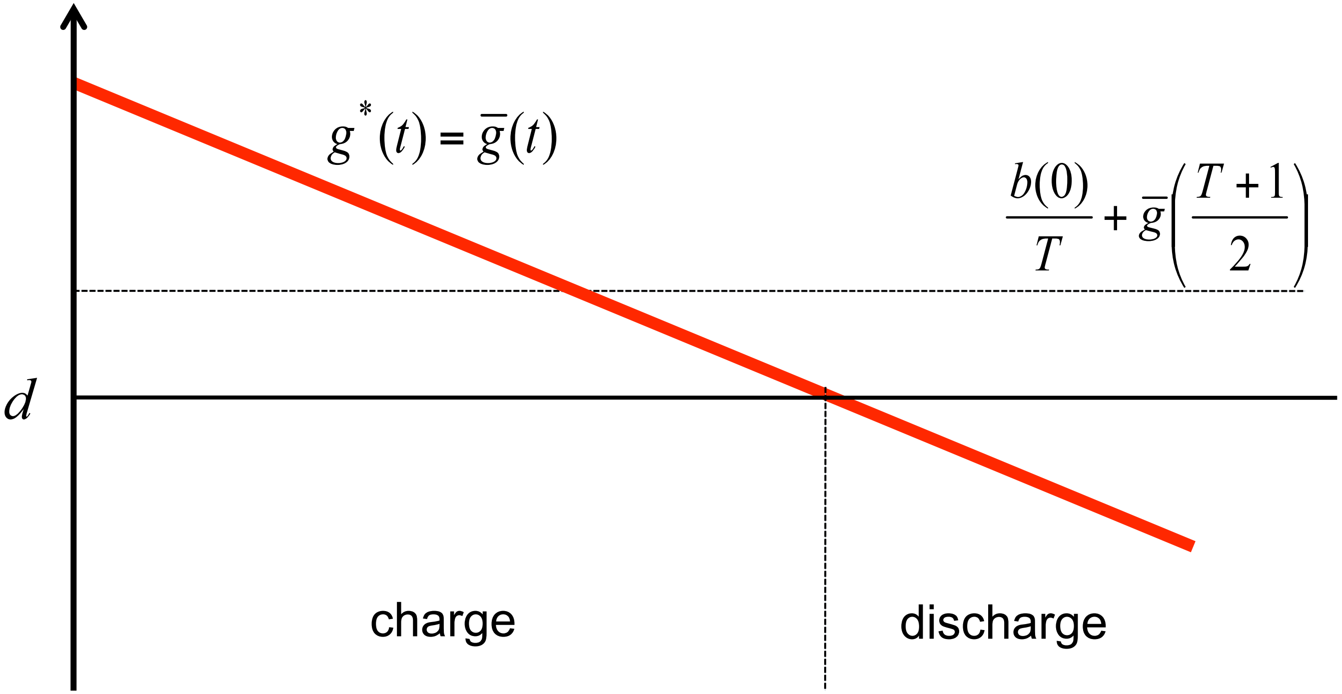
- Optimal policy anticipates future starvation and saturation
- Optimal generation has 3 phases
  - Phase 1: Charge battery, generation decreases linearly, battery increases quadratically
  - Phase 2: Generation = d (phase 2 may not exist)
  - Phase 3: Discharge battery, generation decreases linearly, battery decreases quadratically

No saturation: no phase 2

If $d \le \frac{b(0)}{T} + \bar{g}\left(\frac{T+1}{2}\right)$ then $g^*(t) = \bar{g}(t)$, $b^*(T) \ge 0$

per-period initial battery

time-averaged nominal gen



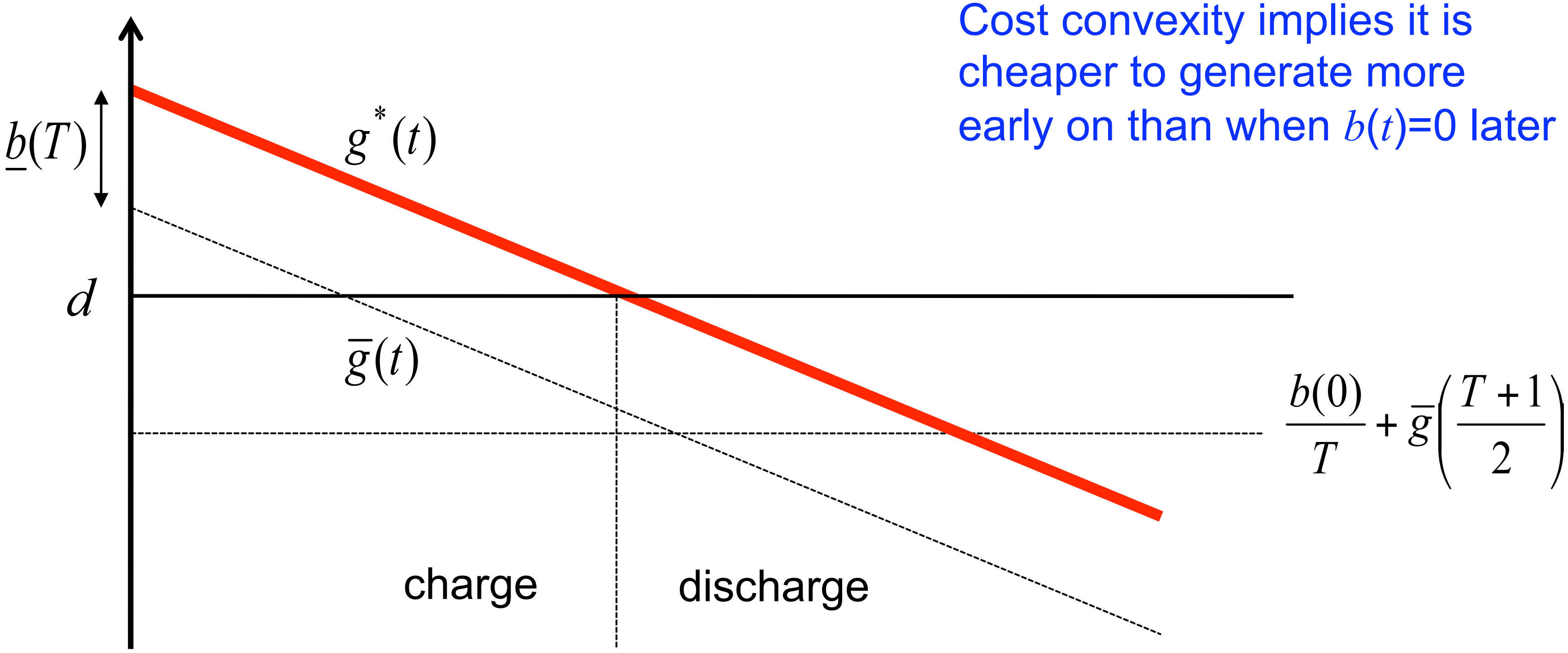
No saturation: no phase 2

If $d > \frac{b(0)}{T} + \bar{g}\left(\frac{T+1}{2}\right)$ then

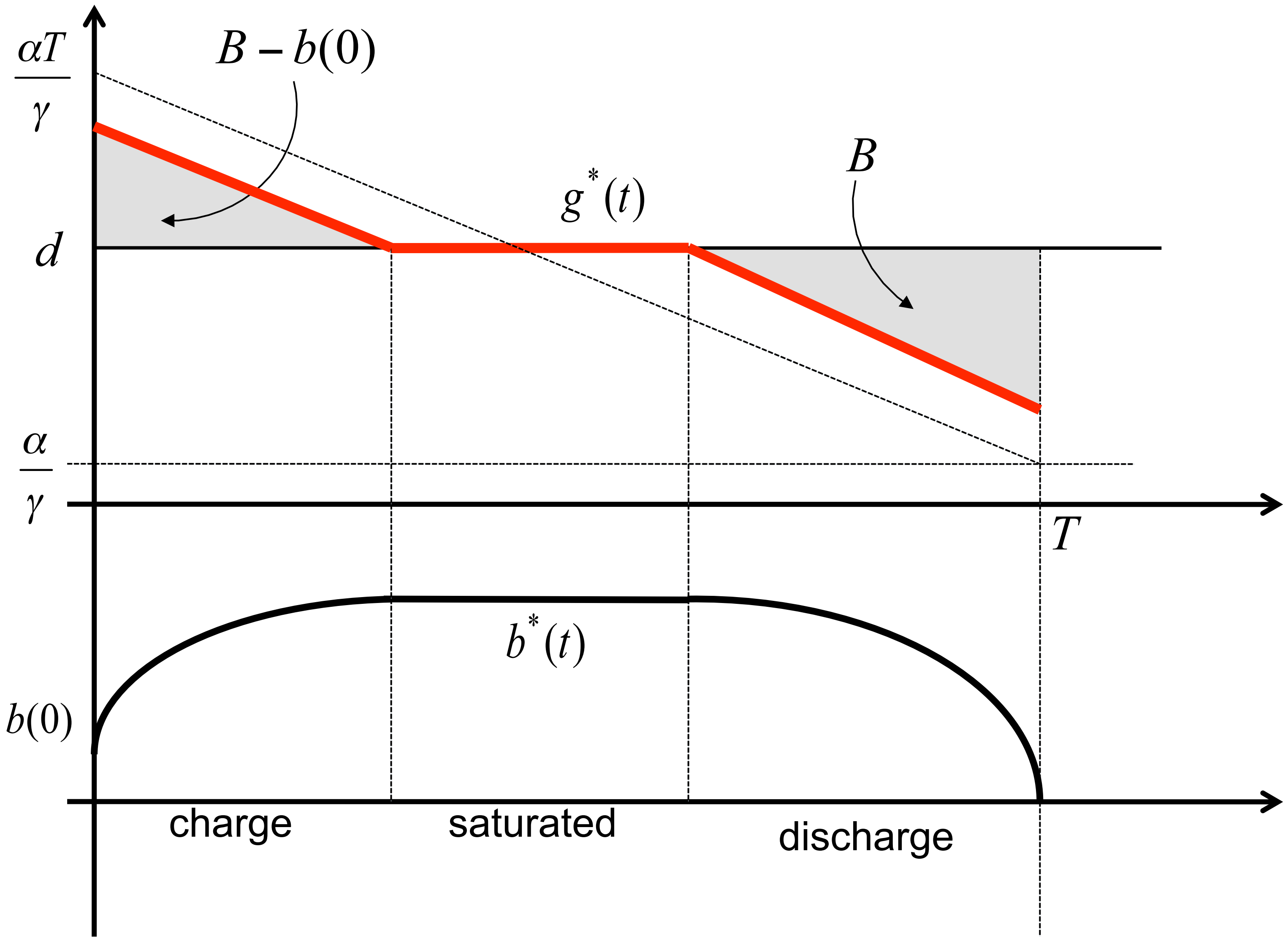
$$g^*(t) = \bar{g}(t) + \underline{b}(T)$$
$$b^*(T) = 0$$

anticipation of starvation:

Cost convexity implies it is cheaper to generate more early on than when $b(t)$=0 later



With saturation: 3 phases



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Network time-varying case

$$c_i(g_i, t) = \frac{1}{2}\gamma_i(t) g_i^{\;2}$$

$$\gamma_i(t) = \gamma_i\left(1 + \sin \omega_i t\right) + \underline{\gamma}_i$$

$$h_i(b_i) = \alpha_i\left(B_i - b_i\right)$$

Then

$$\frac{\partial c_i}{\partial g_i} = \gamma_i(t) g_i \qquad \frac{\partial h_i}{\partial b_i} = -\alpha_i$$

Marginal cost of battery level

marginal cost of
battery level

$$\tilde{b}_i(t) = \alpha_i (T + 1 - t) + B_i^*(t)$$

$$B_i^*(t) := \sum_{\tau=t}^{T} \left(\underline{b}_i(\tau)\mathbf{1}\left(b_i^*(\tau) = 0\right) - \bar{b}_i(\tau)\mathbf{1}\left(b_i^*(\tau) = B_i\right)\right)$$

cost of battery
starvation

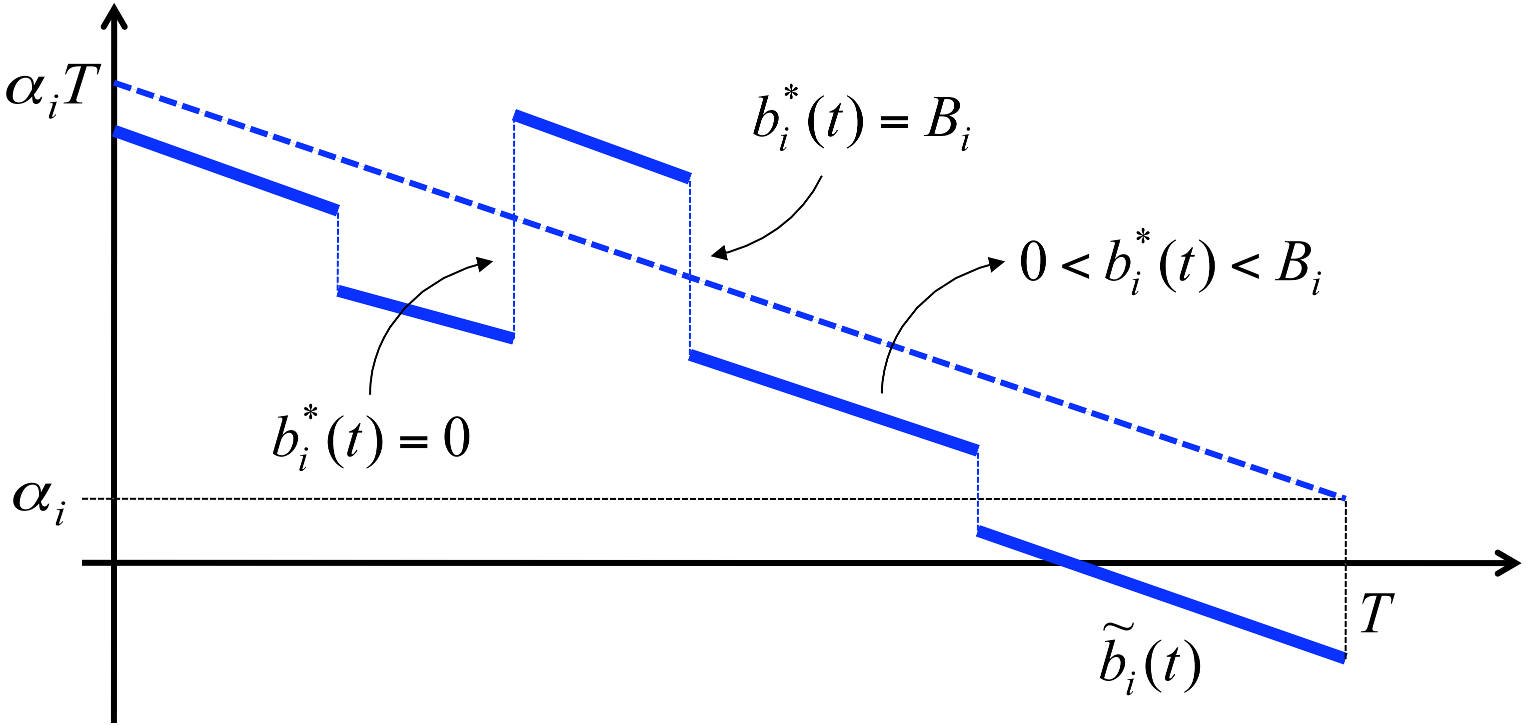
benefit of battery
saturation

cumulative cost
of battery starvation

Marginal cost of battery level

$$\tilde{b}_i(t) = \alpha_i(T + 1 - t) + B_i^*(t)$$

$$B_i^*(t) := \sum_{\tau=t}^{T} \left(\underline{b}_i(\tau)\mathbf{1}\left(b_i^*(\tau) = 0\right) - \bar{b}_i(\tau)\mathbf{1}\left(b_i^*(\tau) = B_i\right) \right)$$



Optimal generation schedule

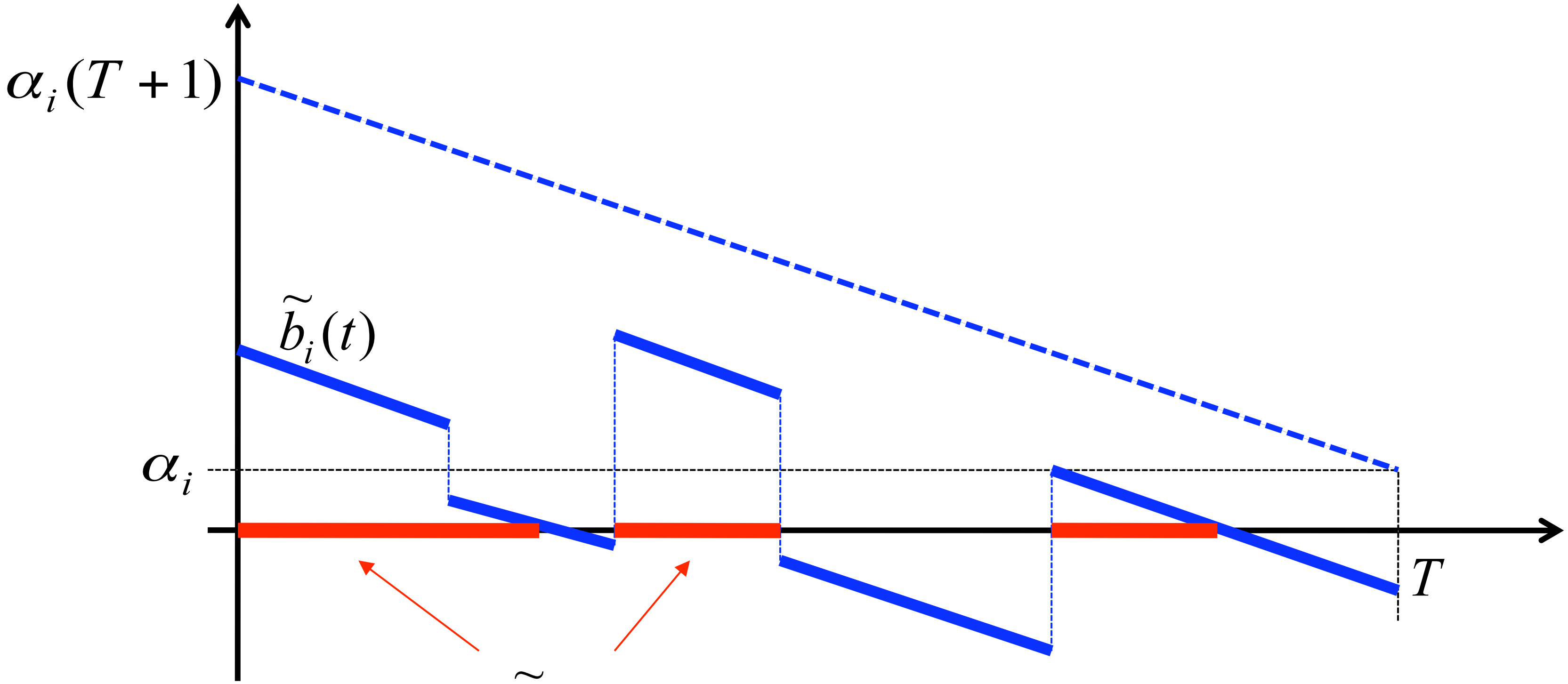
- Optimal generation schedule satisfies

$$g_i^*(t) = \left[\bar{g}_i(t) + \frac{\widetilde{B}_i^*(t)}{\gamma_i(t)} \right]^+, \qquad \bar{g}_i(t) := \frac{\alpha_i (T+1-t)}{\gamma_i(t)}$$

- Optimal generation is positive if
  - Marginal generation cost $\gamma_i(t)$ is small
  - Cumulative cost of battery starvation $\widetilde{B}_i^*(t)$ is high
- Optimal generation tends to decrease over time
  - Because marginal cost of storage tends to decrease over time

Optimal generation schedule

- Then optimal generation is positive iff
  - Marginal cost $\tilde{b}_i(t) = \lambda_i(t)$ of storage is positive



$$g^*{}_i(t) = \frac{\tilde{b}_i(t)}{\gamma_i(t)} > 0$$

Optimal generation schedule

- ☐ If $b_i^*(t) \in (0, B_i)$ and $\lambda_i(t) > 0$ then

$$g_i^*(t+1) = \left[\frac{\gamma_i(t) g_i^*(t) - \alpha_i}{\gamma_i(t+1)}\right]^+$$

  - Optimal generation tends to decrease over time
  - Optimal generation increases in the next period if and only if

$$\underbrace{\gamma_i(t) g_i^*(t) - \gamma_i(t+1) g_i^*(t+1)}_{\text{decrease in marginal cost of generation}} > \alpha_i$$

> unit cost of battery energy

Optimal generation schedule

- If $b_i^*(t) \in (0, B_i)$ and $\lambda_i(t) > 0$ then

$$g_i^*(t+1) = \left[\frac{\gamma_i(t) g_i^*(t) - \alpha_i}{\gamma_i(t+1)} \right]^+$$

$$\gamma_i(t) g_i^*(t) < \alpha_i \quad \Rightarrow \quad g_i^*(t+1) = 0$$

marginal cost of generation **<** unit cost of battery energy ⟹ charge now, and generate 0 power at t+1