A scalable approach to the control of large networks

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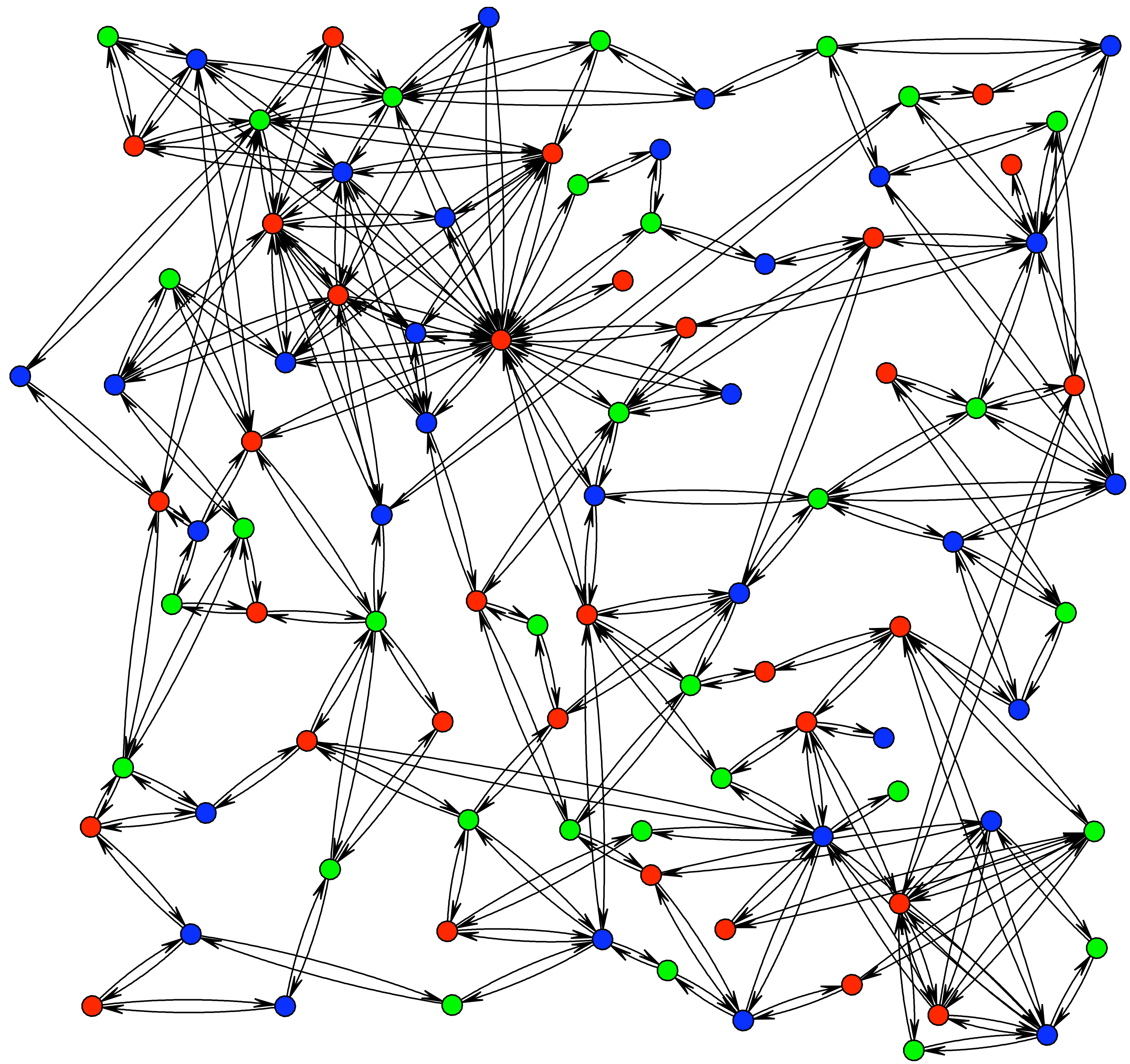
Conventional Control Problems

- Fixed architecture,
- Small, fixed number of inputs (control surfaces ...) and outputs (measurements).
- Well understood ...

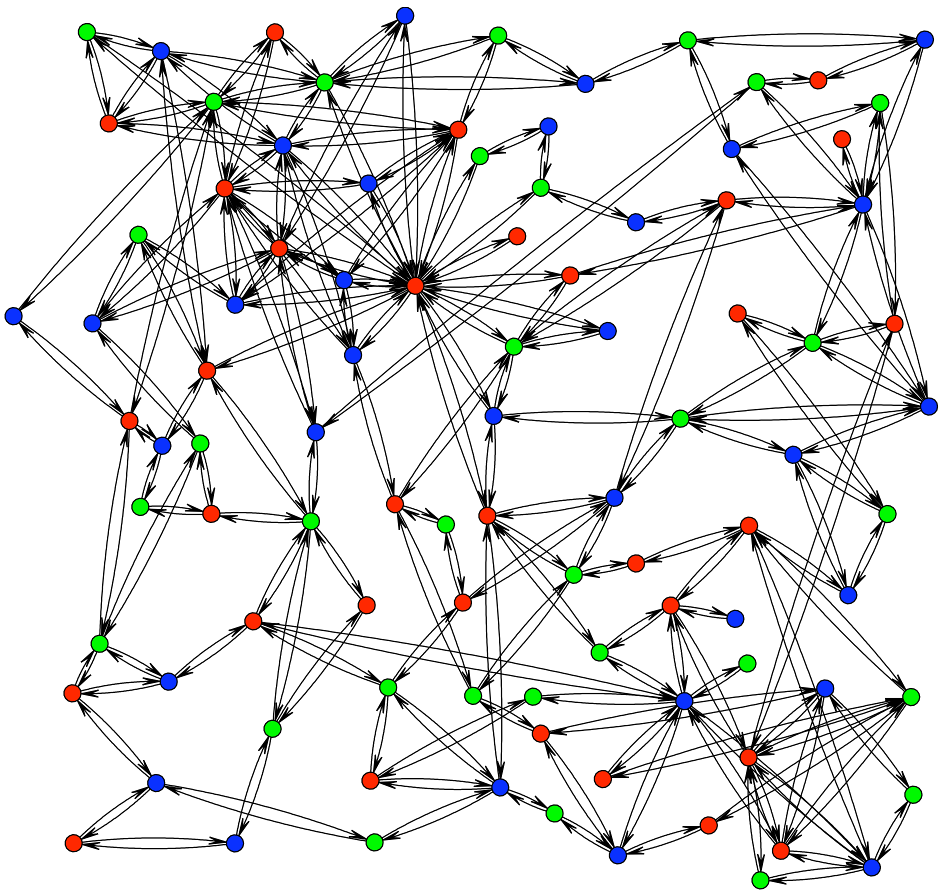
Feedback networks: A new challenge



Feedback networks: A new challenge

Examples:

- Economic networks
- Power distribution networks
- Communication networks (e.g. Internet)
- Vehicle formations (flocking phenomena, platooning etc)



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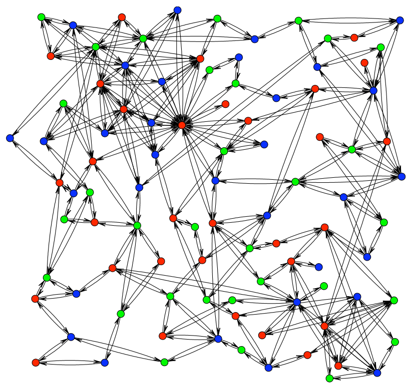

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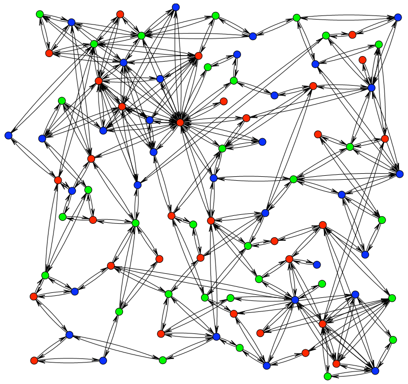

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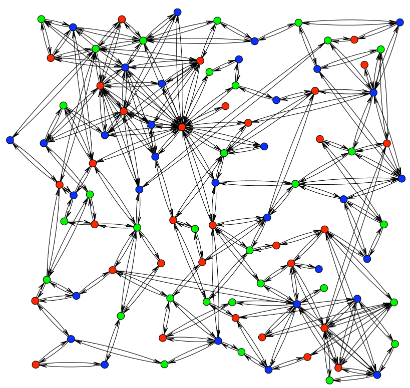

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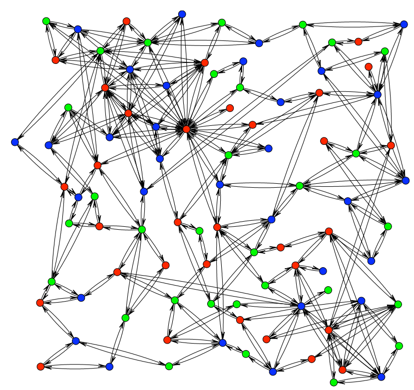
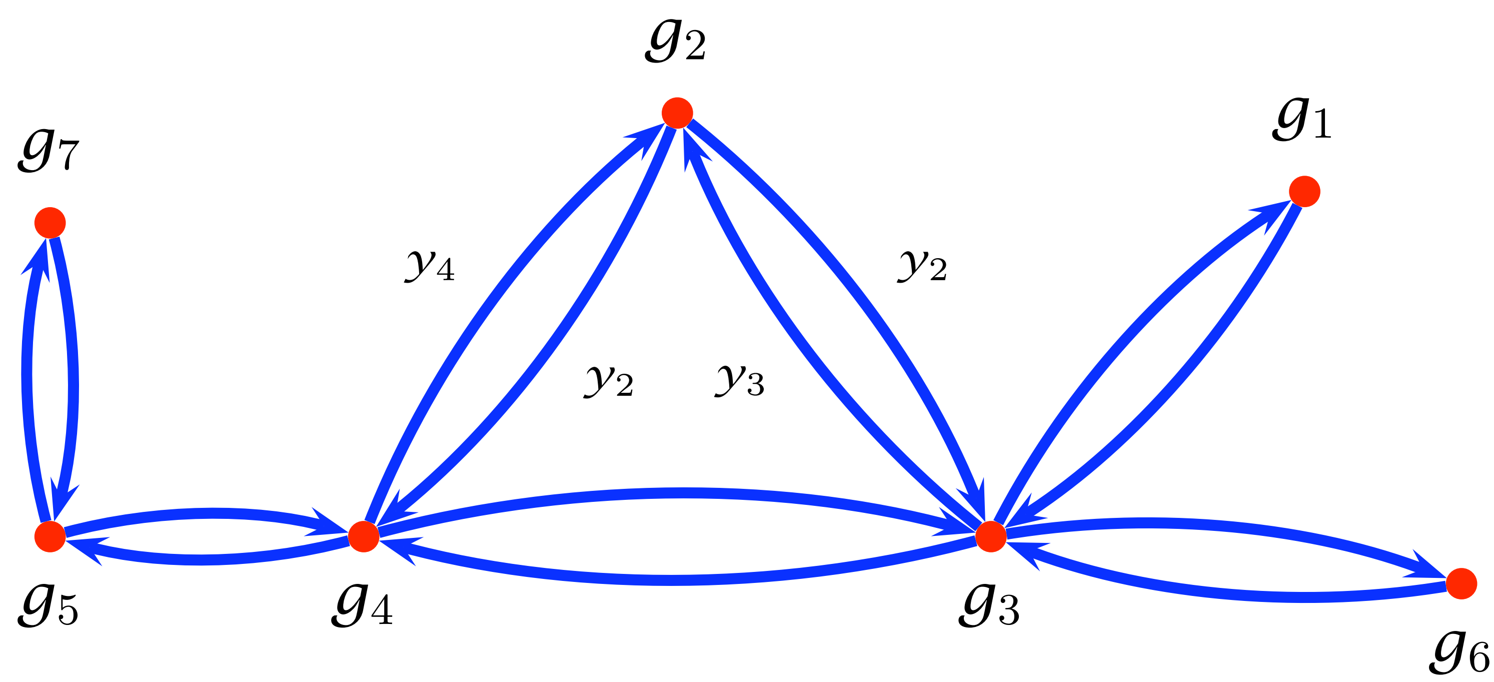


- Can we predict global properties from local interactions?
- Can we find *design rules* for the agents and their local interactions to ensure desirable global properties?
- *Note: If the agents are stable, we can always get a stable network by making the feedback gains small. We really want to know how **large** we can make them.*

Feedback networks with dynamic agents



- Each vertex: stable linear dynamical system g_i.

$$y_i(\cdot) = g_i \circ u_i(\cdot)$$

- Input: average of the signals from its neighbours.

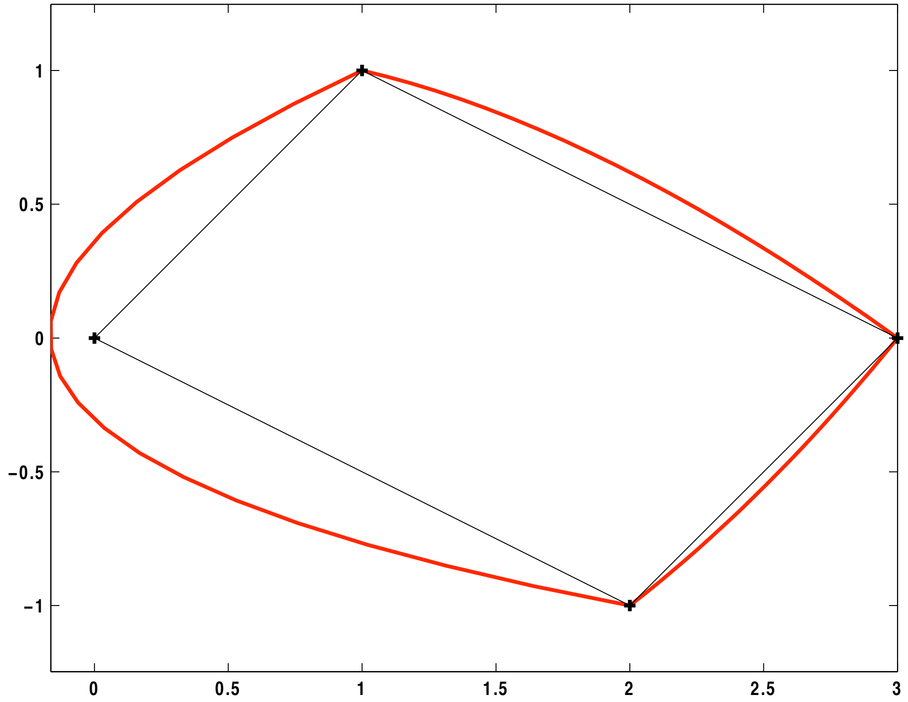
$$u_i(t) = \frac{1}{N_i} \sum_{i,k \text{ connected}} y_k(t)$$

Network Stability

Each system satisfying a convexified Nyquist-like condition with neighbours $\Longrightarrow$ Network is stable

THEOREM: Interconnection is stable if, at each ω,

$$1 \notin Co\left(\cup_i S\left(\{\bar{g}_i(j\omega)\bar{g}_k(j\omega) : k \text{ connected to } i\}\right)\right)$$



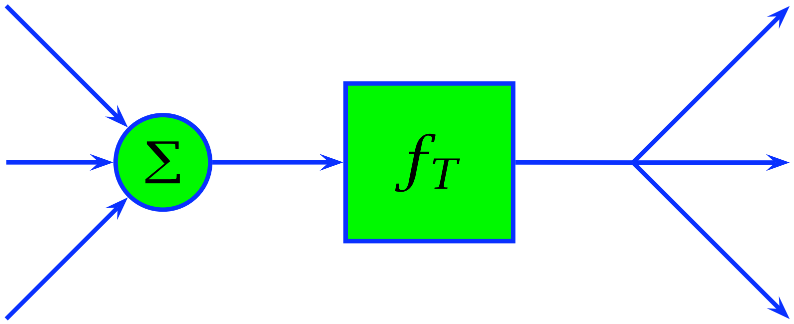
The S-hull $S(\cdot)$ is somewhat larger than the convex hull.

$S(X) = \left(\mathrm{Co}\left(\sqrt{X}\right)\right)^2$, where $\sqrt{X} := \{y : y^2 \in X\}$.

What's different?

- There are many results around, of a similar type, based on passivity/small gain and generalizations.
- These generally depend on properties if the individual systems.
- The result here depends on properties of the *loops.*

Modules and Protocols



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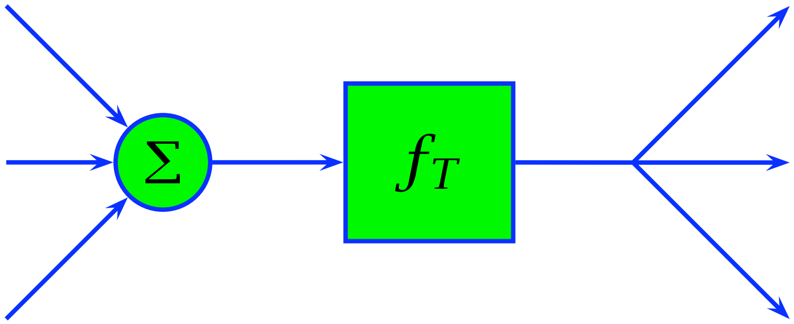

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e.g. $$\frac{dy}{dt} = \frac{k_g}{NT} \sum_{i=1}^{N} u_i(t - T)$$

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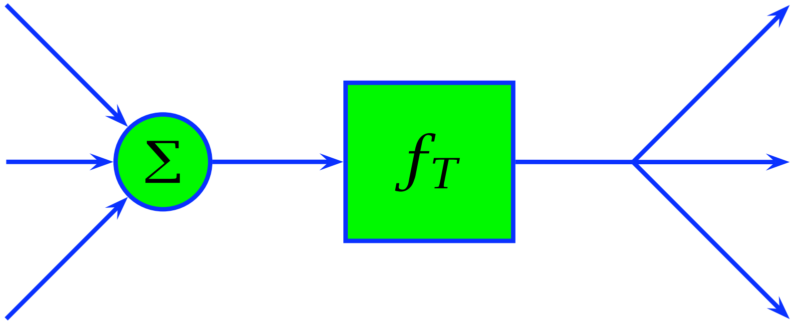

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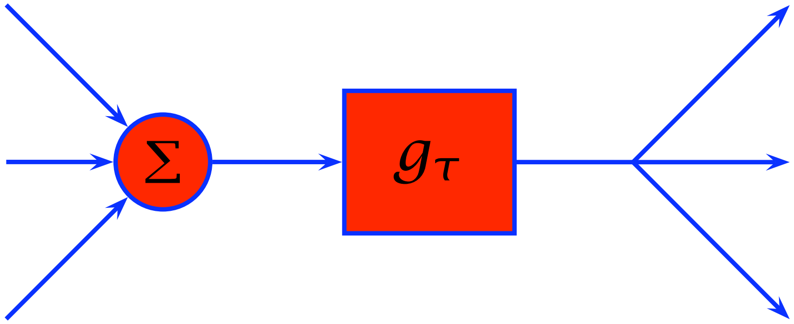

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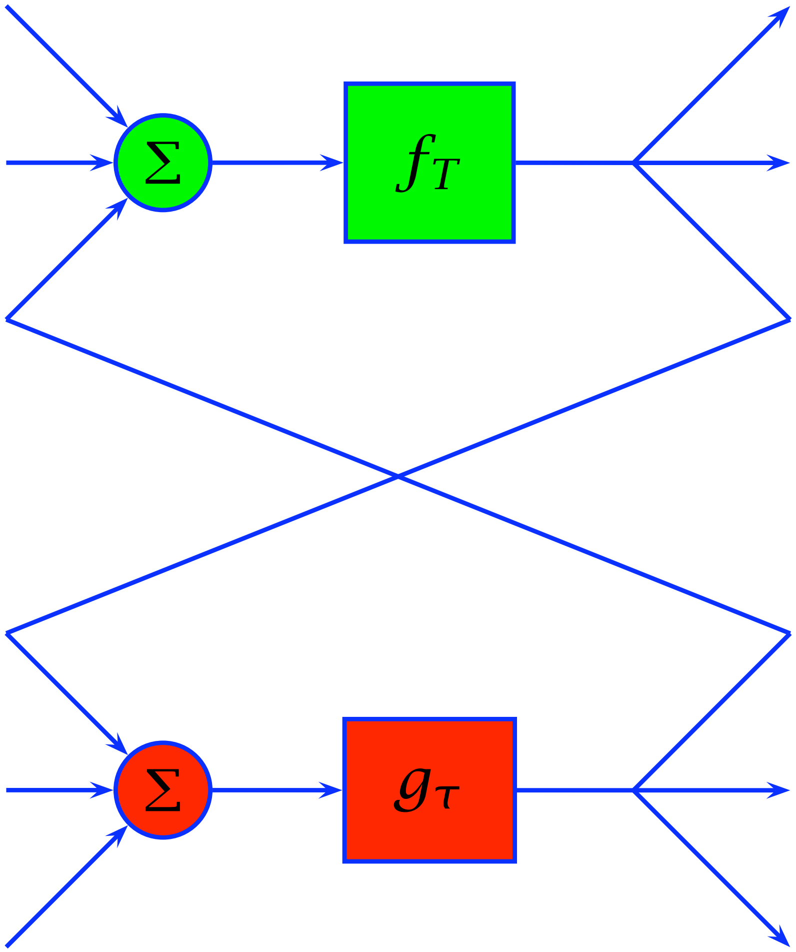

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e.g. f/b interconnection

allowed if $k_g k_r \leq 1$, $\tau < 2T$

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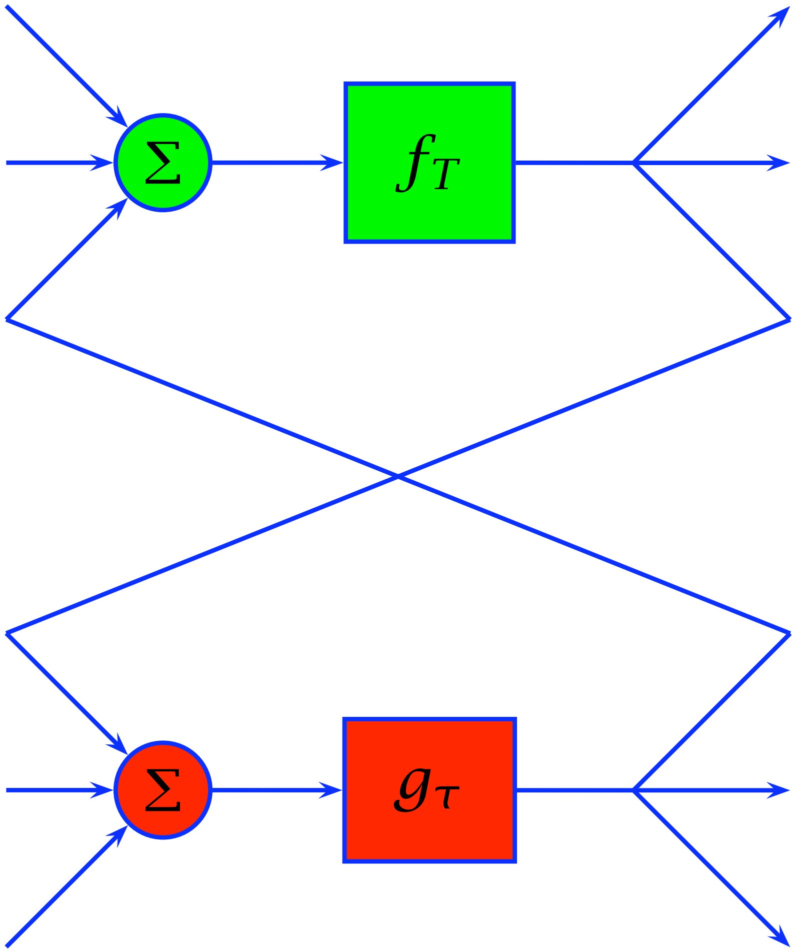

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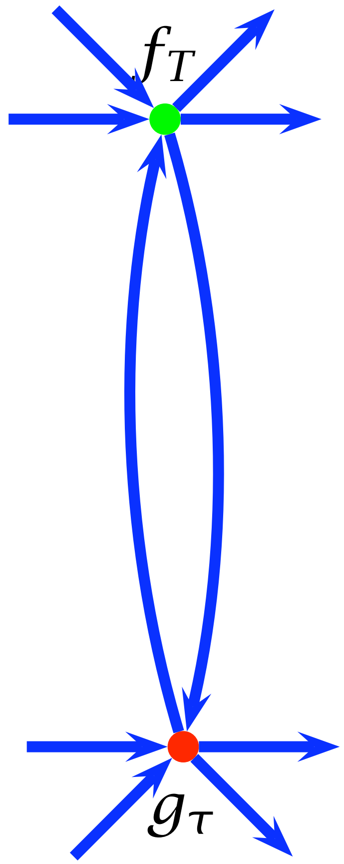

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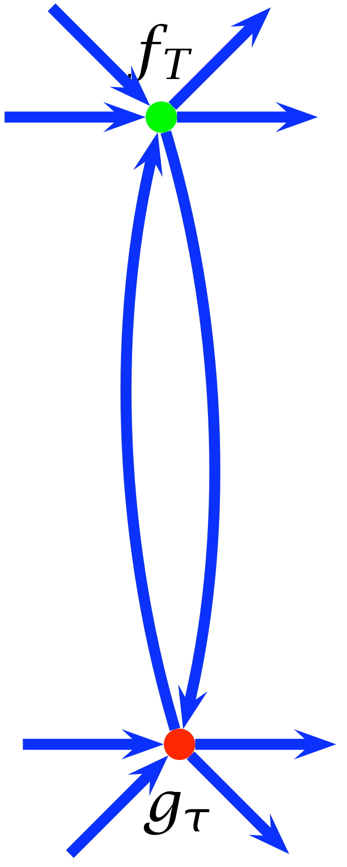

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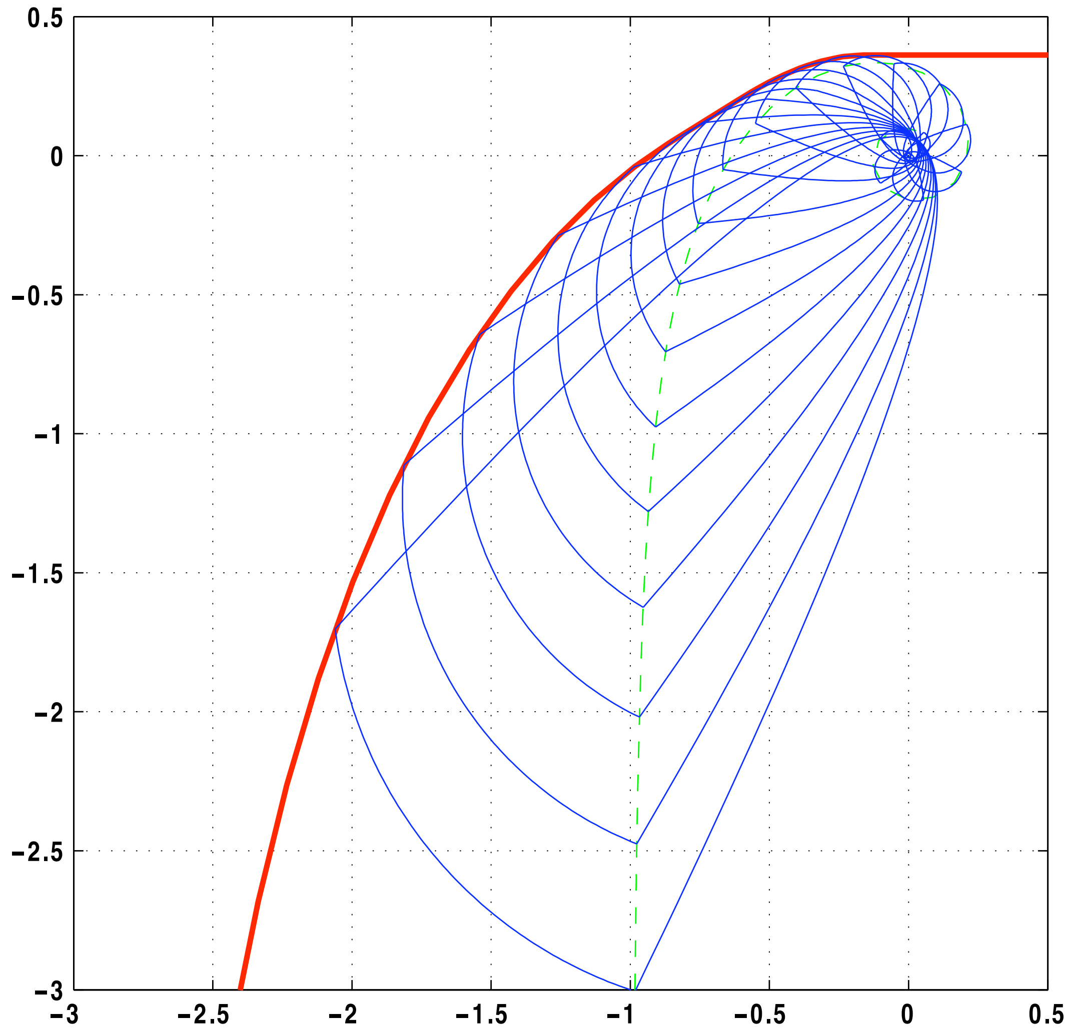
e.g. $\tau \dfrac{dy}{dt} + y = -\dfrac{k_r}{N} \sum_{i=1}^{N} u_i(t)$

- Each green has a different T, each red a different τ
- each red module can be connected to many green modules, and *vice versa*.

Stability certificate



$\Longrightarrow$ closed-loop stability.

- Note: this provides a stability certificate for all possible interconnections satisfying the *protocol*.

Details

Somewhat stronger results hold for bipartite graphs (ie two classes of systems) $f(s)$ and $g(s)$:
If

$$y_i(s) = f_i(s)\left(\sum d_{ik}u_k(s) + \eta_i\right)$$

$$u_k(s) = g_k(s)\left(\sum d'_{ki}y_k(s) + \zeta_k\right)$$

$$d_{ik} = d'_{ki} \geq 0, \quad \sum_k d_{ik} \leq 1 \,\forall k$$

then

$$1 \notin \mathrm{Co}\left(\cup_i S\left(\{f_i(j\omega)g_k(j\omega) : k \text{ connected to } i\}\right)\right)$$

suffices.

Why? because for $F = \mathrm{diag}\{f_i(j\omega)\}$ and $G = \mathrm{diag}\{g_k(j\omega)\}$

$$\left\{\cup_{\hat{R}}\mathcal{N}(G^{1/2}\hat{R}^*F\hat{R}G^{1/2}) : \rho(|\hat{R}|^T|\hat{R}|) \leq 1, \hat{R}_{ik} \neq 0 \iff d_{ik} \neq 0\right\}$$

$$= \mathrm{Co}(\cup_i S(\{f_i(j\omega)g_k(j\omega) : d_{ik} \neq 0))$$

Internet Congestion Control

- Participating Dynamics :
  - Users (generating packet flow)
  - Routers (sending congestion signals)
- Protocol : Everybody does TCP.

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- Protocol : Everybody does TCP.

- Problem: TCP performs poorly on high speed, high latency links.
- This theory suggested a modification to the way TCP parameters should scale with round trip times and bandwidth.
- This was incorporated in IETF Draft “HighSpeedTCP” (RFC 3649) (Sally Floyd, 2002)

Scalable control of power networks

- Power networks with synchronous generators are inherently only marginally stable.
- Stability provided by Power System Stabilizers (PSS) at generators and FACTS devices such as Static Var Compensators (SVC) on lines.
- The parameters of these stabilizing devices are usually set ("tuned") as part of the commissioning process for new infrastructure.
    - but the network changes ...
    - and diversification makes things worse - this approach doesn't scale.
- Renewable energy sources are often asynchronous - and offer no stabilizing effect.

Scalable control of power networks

- The scalable control theory framework applies:
  - Watts and Vars sum at connections - voltages and angles are the shared variables.
- Framework for implementation is already there:
  - “Grid Code” <--> protocol
- Lots of details ... much harder than the Internet!