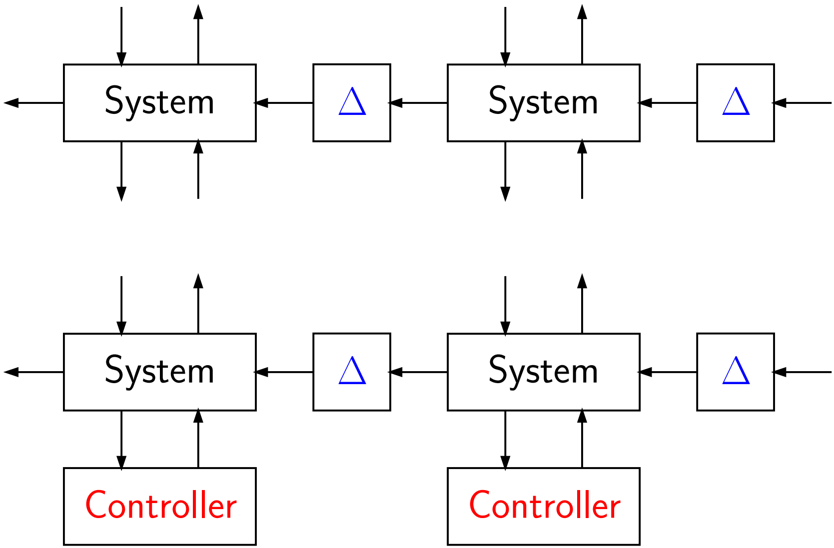
From Gain-Scheduling to Distributed Control

Carsten Scherer
Delft Center for Systems and Control
Delft University of Technology
Delft, The Netherlands

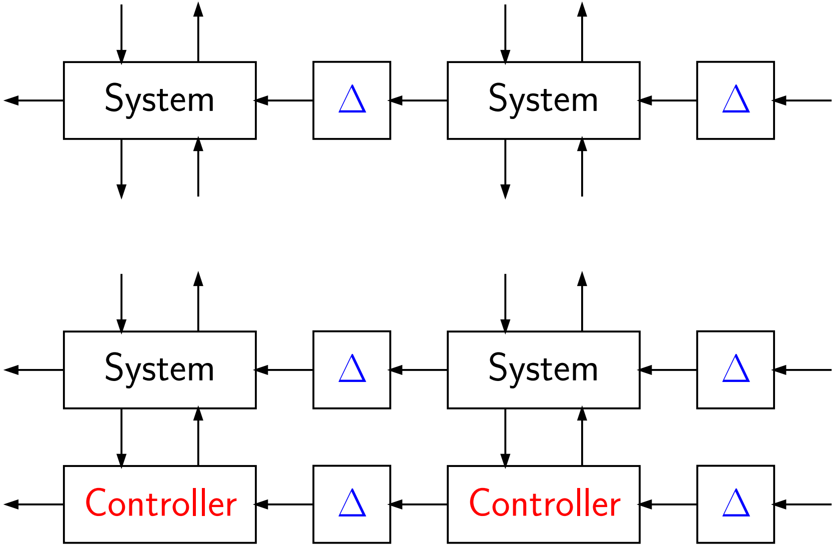
I. Emre Köse
Mechanical Engineering
Boğazici University
Istanbul, Turkey

Supported by Dutch Technology Foundation

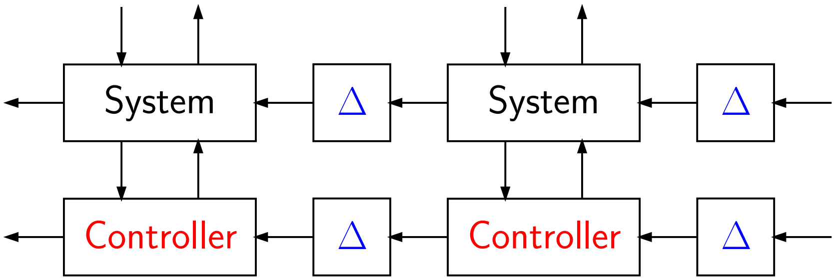
Distributed Synthesis



Distributed Synthesis



Distributed Synthesis



- Bamieh, Paganini, Daleh (2002)
- D'Andrea, Dullerud (2003)
- Langbort, D'Andrea, Chandra (2004)
- Di, Farhood, Dullerud (2006) and Fan, Antsaklis (2008)

Outline

- Analysis and Distributed Synthesis: Static IQCs
- Dynamic IQCs: Analysis
- Gain-Scheduling Synthesis with Dynamic IQCs
- Sketch of Applications and Conclusions

System Interconnection

The LTI systems

$$\begin{pmatrix} \dot{x}_i \\ \hline e_i \\ z_i \end{pmatrix} = \left(\begin{array}{c|cc} A^i & B_1^i & B_2^i \\ \hline C_1^i & D_1^i & D_{12}^i \\ C_2^i & D_{21}^i & D_2^i \end{array}\right) \begin{pmatrix} x_i \\ \hline d_i \\ w_i \end{pmatrix}, \quad i = 1, \ldots, L$$

are interconnected as

$$w_i = \sum_{j=1}^{L} \Delta_{ij}(z_j) \text{ with } \Delta_{ij} \in \boldsymbol{\Delta_{ij}} \text{ for } i, j = 1, \ldots, L.$$

Here $\boldsymbol{\Delta_{ij}}$ captures information about the

- **structure** of the interconnection (sparsity)
- **nature** of the interconnection (static, dynamic, delay)
- **uncertainties** in the interconnection (sets of dynamics)

Towards Analysis

Diagonally combine the LTI systems into

$$\begin{pmatrix} \dot{x} \\ \hline e \\ z \end{pmatrix} = \left(\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_1 & D_{12} \\ C_2 & D_{21} & D_2 \end{array}\right) \begin{pmatrix} x \\ \hline d \\ w \end{pmatrix}, \quad A = \begin{pmatrix} A^1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A^L \end{pmatrix}, \ldots$$

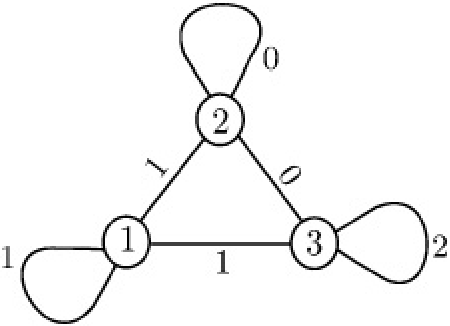
that are interconnected as $w = \Delta(z)$ with

$$\Delta \in \boldsymbol{\Delta} = \left\{ \begin{pmatrix} \Delta_{11} & \cdots & \Delta_{1L} \\ \vdots & \ddots & \vdots \\ \Delta_{L1} & \cdots & \Delta_{LL} \end{pmatrix} : \ \Delta_{ij} \in \boldsymbol{\Delta_{ij}} \ \text{ for } \ i, j = 1, \ldots, L \right\}.$$

Example: $\Delta \in \boldsymbol{\Delta}$ are matrix multiplication operators.

Structured of interconnection reflected in sparsity pattern of matrix.

Example



$$\begin{pmatrix} w_{1,1} \\ w_{1,2} \\ w_{1,3} \\ \hline w_{2,1} \\ \hline w_{3,1} \\ w_{3,2} \\ w_{3,3} \end{pmatrix} = \left(\begin{array}{ccc|c|ccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} z_{1,1} \\ z_{1,2} \\ z_{1,3} \\ \hline z_{2,1} \\ \hline z_{3,1} \\ z_{3,2} \\ z_{3,3} \end{pmatrix}$$

Langbort, D'Andrea, Chandra (2004)

Performance Analysis

$\Delta \in \boldsymbol{\Delta}$ satisfies static IQC with **multiplier** $P = P^\top$ if

$$\int_0^T \begin{pmatrix} z(t) \\ \Delta(z)(t) \end{pmatrix}^\top P \begin{pmatrix} z(t) \\ \Delta(z)(t) \end{pmatrix} dt \geq 0$$

for all $z \in \mathscr{L}_2[0,T]$ and $T \geq 0$.

Let $\boldsymbol{P}$ denote any family of multipliers

$$P = \begin{pmatrix} Q & S \\ S^\top & R \end{pmatrix}$$

for which the IQC holds for all uncertainties $\Delta \in \boldsymbol{\Delta}$.

Examples

Simplest case $\boldsymbol{\Delta} = \{\Delta_0\}$ with some matrix Δ_0 ...

... Fixed interconnection topology.

Set of **multipliers**

$$\left\{ P = P^\top = \begin{pmatrix} Q & S \\ S^\top & R \end{pmatrix} : \begin{pmatrix} I \\ \Delta_0 \end{pmatrix}^\top P \begin{pmatrix} I \\ \Delta_0 \end{pmatrix} = 0 \right\}.$$

$\boldsymbol{\Delta}$ set of time-varying matrices $\Delta(t)$ ... Time-varying topology.

Set of **multipliers**

$$\left\{ P = P^\top : \begin{pmatrix} I \\ \Delta(t) \end{pmatrix}^\top P \begin{pmatrix} I \\ \Delta(t) \end{pmatrix} \succcurlyeq 0 \text{ for all } t \geq 0,\ \Delta \in \boldsymbol{\Delta} \right\}.$$

Technical assumption: Contain at least one non-singular element.

Main Analysis Result

Interconnection well-posed, stable and $\mathscr{L}_2$-gain of $d \to e$ bounded by γ

if there exists $X \succ 0$ and a multiplier $\begin{pmatrix} Q & S \\ S^\top & R \end{pmatrix} \in \boldsymbol{P}$ with

$$\begin{pmatrix} A & B_1 & B_2 \\ I & 0 & 0 \\ \hline C_1 & D_1 & D_{12} \\ 0 & I & 0 \\ \hline C_2 & D_{21} & D_2 \\ 0 & 0 & I \end{pmatrix}^\top \left(\begin{array}{cc|cc|cc} 0 & X & 0 & 0 & 0 & 0 \\ X & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma^2 I & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & Q & S \\ 0 & 0 & 0 & 0 & S^\top & R \end{array}\right) \begin{pmatrix} A & B_1 & B_2 \\ I & 0 & 0 \\ \hline C_1 & D_1 & D_{12} \\ 0 & I & 0 \\ \hline C_2 & D_{21} & D_2 \\ 0 & 0 & I \end{pmatrix} \prec 0.$$

Very closely related to classical stability/dissipation theory.

Popov, Yakubovich, Zames, Willems, Hill, Moylan, Desoer, Vidyasagar, ...

Idea of Proof: Performance Bound

LMI implies along any interconnection trajectory that

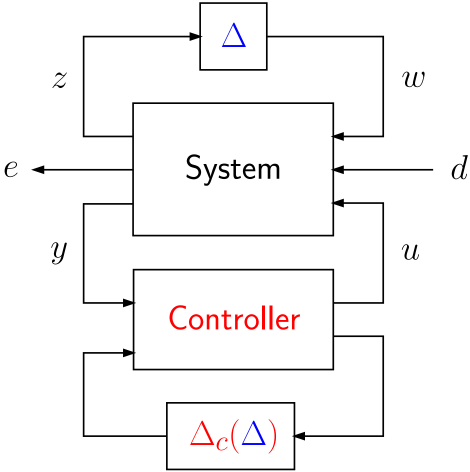
$$\int_0^T \frac{d}{dt} x(t)^\top X x(t) - \gamma^2 \|d(t)\|^2 + \|e(t)\|^2 \, dt +$$

$$+ \int_0^T \begin{pmatrix} z(t) \\ w(t) \end{pmatrix}^\top P \begin{pmatrix} z(t) \\ w(t) \end{pmatrix} dt \leq 0.$$

Since $w(t) = \Delta(z)(t)$ the last term is non-negative. With $X \succ 0$ get

$$\int_0^T \|e(t)\|^2 \, dt \leq \gamma^2 \int_0^T \|d(t)\|^2 \, dt + x(0)^\top X x(0).$$

Distributed Controller Synthesis



Synthesis of controller and scheduling function for robust stability/performance

↓

Convex Optimization!

Packard (94)
Apkarian, Gahinet (94)
Helmersson (95)
Scorletti & El-Ghaoui (98)
Scherer (01)

Our work allows for **general static multipliers**.

Example

Fixed interconnection topology $\boldsymbol{\Delta} = \{\Delta_0\}$

In the class of multipliers

$$\boldsymbol{P} = \left\{ P = P^\top = \begin{pmatrix} Q & S \\ S^\top & R \end{pmatrix} : \begin{pmatrix} I \\ \Delta_0 \end{pmatrix}^\top P \begin{pmatrix} I \\ \Delta_0 \end{pmatrix} = 0 \right\}$$

let Q, S, R share their block-diagonal structure with system matrices.

Key Observation

Exists $X \succ 0$ with

$$\left(\begin{array}{cc} A^1 & 0 \\ 0 & A^2 \\ \hline I & 0 \\ 0 & I \end{array}\right)^\top \left(\begin{array}{cc|cc} 0 & 0 & X_1 & X_{12} \\ 0 & 0 & X_{21} & X_2 \\ \hline X_1 & X_{12} & 0 & 0 \\ X_{21} & X_2 & 0 & 0 \end{array}\right) \left(\begin{array}{cc} A^1 & 0 \\ 0 & A^2 \\ \hline I & 0 \\ 0 & I \end{array}\right) \prec 0$$

iff exist $X_1 \succ 0$, $X_2 \succ 0$ with

$$\begin{pmatrix} I \\ A^1 \end{pmatrix}^\top \begin{pmatrix} 0 & X_1 \\ X_1 & 0 \end{pmatrix} \begin{pmatrix} I \\ A^1 \end{pmatrix} \prec 0, \quad \begin{pmatrix} I \\ A^2 \end{pmatrix}^\top \begin{pmatrix} 0 & X_2 \\ X_2 & 0 \end{pmatrix} \begin{pmatrix} I \\ A^2 \end{pmatrix} \prec 0.$$

Can work with **diagonally structured** X without loss of generality.

Example

Fixed interconnection topology $\boldsymbol{\Delta} = \{\Delta_0\}$

In the class of multipliers

$$\boldsymbol{P} = \left\{ P = P^\top = \begin{pmatrix} Q & S \\ S^\top & R \end{pmatrix} : \begin{pmatrix} I \\ \Delta_0 \end{pmatrix}^\top P \begin{pmatrix} I \\ \Delta_0 \end{pmatrix} = 0 \right\}$$

let Q, S, R share their block-diagonal structure with system matrices.

- Synthesis conditions: L LMIs and multiplier equation constraints
- Controller shares interconnection structure Δ_0 with system.
- Less conservative than what's known.

Reduction of conservatism by adapting **structure** of multipliers.

Outline

- Analysis and Distributed Synthesis: Static IQCs
- Dynamic IQCs: Analysis
- Gain-Scheduling Synthesis with Dynamic IQCs
- Sketch of Applications and Conclusions

Dynamic Multipliers

Recall the IQC

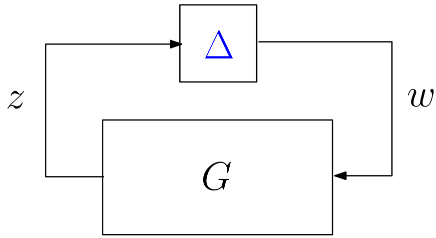
$$\int_0^T \begin{pmatrix} z(t) \\ \Delta(z)(t) \end{pmatrix}^\top P \begin{pmatrix} z(t) \\ \Delta(z)(t) \end{pmatrix} dt \geq 0 \quad \text{for all} \quad T \geq 0.$$

Static multipliers P are conservative.

Use **dynamic** multipliers. IQC then reads in the frequency domain as

$$\int_{-\infty}^{\infty} \begin{pmatrix} \hat{z}(i\omega) \\ \widehat{\Delta(z)}(i\omega) \end{pmatrix}^* \Pi(i\omega) \begin{pmatrix} \hat{z}(i\omega) \\ \widehat{\Delta(z)}(i\omega) \end{pmatrix} dt \geq 0.$$

Robust Stability Analysis



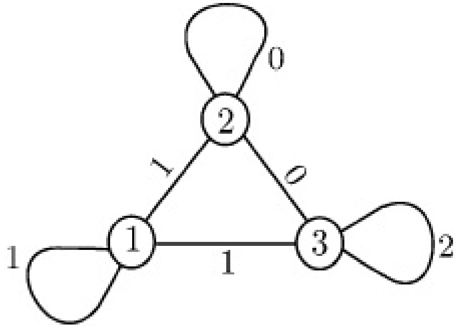
Interconnection

$$z = Gw \quad \text{and} \quad w = \Delta(z)$$

remains robustly stable if

$$\begin{pmatrix} G(i\omega) \\ I \end{pmatrix}^* \Pi(i\omega) \begin{pmatrix} G(i\omega) \\ I \end{pmatrix} \prec 0 \quad \text{for all} \quad \omega \in \mathbb{R} \cup \{\infty\}.$$

Example



$$\begin{pmatrix} w_{1,1} \\ w_{1,2} \\ w_{1,3} \\ \hline w_{2,1} \\ \hline w_{3,1} \\ w_{3,2} \\ w_{3,3} \end{pmatrix} = \left(\begin{array}{ccc|c|ccc} \delta_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_3 & 0 & 0 \\ \hline 0 & \delta_4 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & \delta_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_6 \end{array}\right) \begin{pmatrix} z_{1,1} \\ z_{1,2} \\ z_{1,3} \\ \hline z_{2,1} \\ \hline z_{3,1} \\ z_{3,2} \\ z_{3,3} \end{pmatrix}, \quad \|\delta_j\|_{\mathcal{H}_\infty} = 1$$

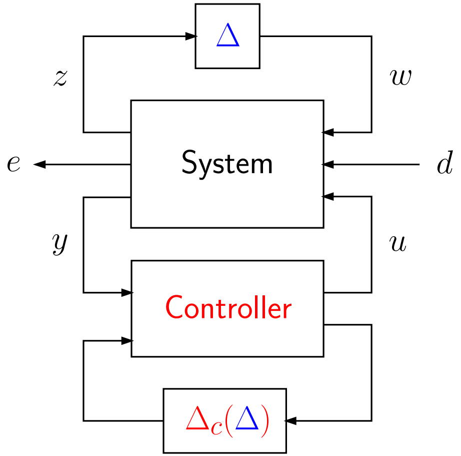
Example

Suitable class of multipliers $\Pi(i\omega)$ is

$$\left(\begin{array}{cccccc|cccccc}
q_1(i\omega) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & q_2(i\omega) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & q_3(i\omega) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & q_4(i\omega) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & q_5(i\omega) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & Q_6(i\omega) & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & -q_1(i\omega) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_4(i\omega) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_5(i\omega) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_2(i\omega) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_3(i\omega) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q_6(i\omega)
\end{array}\right)$$

Corresponding static multipliers used by **D'Andrea, Dullerud (2003)**

Distributed Synthesis



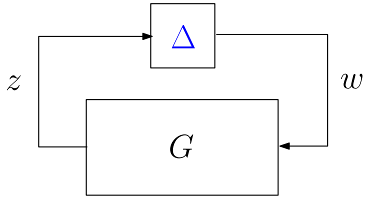
Synthesis with **dynamic** multipliers was completely open.

A More Classical Case

Consider structured uncertainty

$$\Delta = \begin{pmatrix} \delta_1 I & 0 \\ 0 & \delta_2 I \end{pmatrix}$$

with linear time-invariant SISO systems δ_1, δ_2 whose gains are bounded by 1.



With frequency-dependent multiplier

$$Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix} \quad \text{satisfying} \quad \Delta Q = Q \Delta,$$

robust stability guaranteed by

$$\begin{pmatrix} G \\ I \end{pmatrix}^* \begin{pmatrix} Q & 0 \\ 0 & -Q \end{pmatrix} \begin{pmatrix} G \\ I \end{pmatrix} \prec 0 \quad \text{and} \quad Q \succ 0 \quad \text{on} \quad \mathbb{C}^0.$$

Computations

For pole $p > 0$ choose **basis** that has dense span in RH_∞:

$$\psi(s) = \begin{pmatrix} I \\ \left(\frac{s-p}{s+p}\right) I \\ \vdots \\ \left(\frac{s-p}{s+p}\right)^l I \end{pmatrix}, \quad l = 0, 1, 2, \ldots$$

Parameterize structured scalings with structured M as

$$Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix} = \begin{pmatrix} \psi & 0 \\ 0 & \psi \end{pmatrix}^* \underbrace{\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}}_{M} \underbrace{\begin{pmatrix} \psi & 0 \\ 0 & \psi \end{pmatrix}}_{\Psi} = \Psi^* M \Psi$$

Towards State-Space

Parametrization $Q = \Psi^* M \Psi$ leads to FDIs

$$\begin{pmatrix} \Psi G \\ \Psi \end{pmatrix}^* \begin{pmatrix} M & 0 \\ 0 & -M \end{pmatrix} \begin{pmatrix} \Psi G \\ \Psi \end{pmatrix} \prec 0 \quad \text{and} \quad \Psi^* M \Psi \succ 0 \quad \text{on} \quad \mathbb{C}^0.$$

Choose realizations

$$\Psi = \left[\begin{array}{c|c} A_\Psi & B_\Psi \\ \hline C_\Psi & D_\Psi \end{array}\right] \quad \text{and} \quad G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

and thus

$$\begin{pmatrix} \Psi G \\ \Psi \end{pmatrix} = \left[\begin{array}{ccc|c} A_\Psi & 0 & B_\Psi C & B_\Psi D \\ 0 & A_\Psi & 0 & B_\Psi \\ 0 & 0 & A & B \\ \hline C_\Psi & 0 & D_\Psi C & D_\Psi D \\ 0 & C_\Psi & 0 & D_\Psi \end{array}\right] =: \left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & D_p \end{array}\right].$$

Towards LMIs

The two FDIs translate into feasibility of LMIs

$$\begin{pmatrix} I & 0 \\ A_p & B_p \\ C_p & D_p \end{pmatrix}^T \begin{pmatrix} 0 & X & 0 \\ X & 0 & 0 \\ 0 & 0 & \mathrm{diag}(M, -M) \end{pmatrix} \begin{pmatrix} I & 0 \\ A_p & B_p \\ C_p & D_p \end{pmatrix} \prec 0$$

$$\begin{pmatrix} I & 0 \\ A_\Psi & B_\Psi \\ C_\Psi & D_\Psi \end{pmatrix}^T \begin{pmatrix} 0 & \hat{X} & 0 \\ \hat{X} & 0 & 0 \\ 0 & 0 & M \end{pmatrix} \begin{pmatrix} I & 0 \\ A_\Psi & B_\Psi \\ C_\Psi & D_\Psi \end{pmatrix} \succ 0.$$

How to characterize **nominal stability** of A?

Towards LMIs

A is **stable** and the FDIs hold iff the following LMIs are feasible:

$$\begin{pmatrix} I & 0 \\ A_p & B_p \\ C_p & D_p \end{pmatrix}^T \begin{pmatrix} 0 & X & 0 \\ X & 0 & 0 \\ 0 & 0 & \mathrm{diag}(M, -M) \end{pmatrix} \begin{pmatrix} I & 0 \\ A_p & B_p \\ C_p & D_p \end{pmatrix} \prec 0,$$

$$\begin{pmatrix} I & 0 \\ A_\Psi & B_\Psi \\ C_\Psi & D_\Psi \end{pmatrix}^T \begin{pmatrix} 0 & \hat{X} & 0 \\ \hat{X} & 0 & 0 \\ 0 & 0 & M \end{pmatrix} \begin{pmatrix} I & 0 \\ A_\Psi & B_\Psi \\ C_\Psi & D_\Psi \end{pmatrix} \succ 0,$$

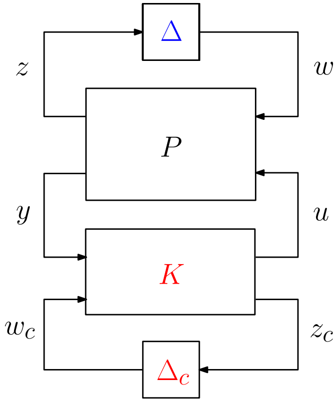
$$\begin{pmatrix} X_{11} - \hat{X} & X_{12} & X_{13} \\ X_{21} & X_{22} + \hat{X} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix} \succ 0.$$

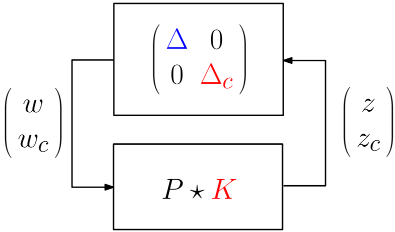
LMIs for nominal and robust stability analysis.

Outline

- Analysis and Distributed Synthesis: Static IQCs
- Dynamic IQCs: Analysis
- Gain-Scheduling Synthesis with Dynamic IQCs
- Sketch of Applications and Conclusions

Gain-Scheduling Controller Synthesis





Scalings for extended uncertainty: For Q, Q_{12}, Q_{22} as above have

$$\begin{pmatrix} \Delta & 0 \\ 0 & \Delta_c \end{pmatrix} \begin{pmatrix} Q & Q_{12} \\ Q_{12}^* & Q_{22} \end{pmatrix} = \underbrace{\begin{pmatrix} Q & Q_{12} \\ Q_{12}^* & Q_{22} \end{pmatrix}}_{\succ 0} \begin{pmatrix} \Delta & 0 \\ 0 & \Delta_c \end{pmatrix}$$

Gain-Scheduling Controller Synthesis

Note that $P \star K$ is given by

$$\begin{pmatrix} z \\ z_c \\ \hline y \\ w_c \end{pmatrix} = \left(\begin{array}{cc|cc} P_{11} & 0 & P_{12} & 0 \\ 0 & 0 & 0 & I \\ \hline P_{21} & 0 & P_{22} & 0 \\ 0 & I & 0 & 0 \end{array}\right) \begin{pmatrix} w \\ w_c \\ \hline u \\ z_c \end{pmatrix}, \quad \begin{pmatrix} u \\ z_c \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} y \\ w_c \end{pmatrix}$$

With abbreviation

$$L = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \left(I - \begin{pmatrix} P_{22} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \right)^{-1}$$

we have

$$P \star K = \begin{pmatrix} P_{11} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} P_{12} & 0 \\ 0 & I \end{pmatrix} L \begin{pmatrix} P_{21} & 0 \\ 0 & I \end{pmatrix}.$$

Gain-Scheduling Controller Synthesis

Analysis FDI

$$(\star)^* \left(\begin{array}{cc|cc} Q & Q_{12} & 0 & 0 \\ Q_{12}^* & Q_{22} & 0 & 0 \\ \hline 0 & 0 & -Q & -Q_{12} \\ 0 & 0 & -Q_{12}^* & -Q_{22} \end{array}\right) \left(\begin{array}{c} \begin{pmatrix} P_{11} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} P_{12} & 0 \\ 0 & I \end{pmatrix} L \begin{pmatrix} P_{21} & 0 \\ 0 & I \end{pmatrix} \\ \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \end{array} \right) \prec 0$$

Apply the elimination lemma to get rid of L. Note that the inverse

$$\begin{pmatrix} \tilde{Q} & \tilde{Q}_{12} \\ \tilde{Q}_{12}^* & \tilde{Q}_{22} \end{pmatrix} = \begin{pmatrix} Q & Q_{12} \\ Q_{12}^* & Q_{22} \end{pmatrix}^{-1}$$

shares its structure with the original scaling.

Gain-Scheduling Controller Synthesis

Elimination of L leads to

$$(P_{21})_\perp^* \begin{pmatrix} P_{11} \\ I \end{pmatrix}^* \begin{pmatrix} Q & 0 \\ 0 & -Q \end{pmatrix} \begin{pmatrix} P_{11} \\ I \end{pmatrix} (P_{21})_\perp \prec 0$$

and

$$(P_{12}^*)_\perp^* \begin{pmatrix} I \\ -P_{11}^* \end{pmatrix}^* \begin{pmatrix} \tilde{Q} & 0 \\ 0 & -\tilde{Q} \end{pmatrix} \begin{pmatrix} I \\ -P_{11}^* \end{pmatrix} (P_{12}^*)_\perp \succ 0$$

and

$$\begin{pmatrix} Q & I \\ I & \tilde{Q} \end{pmatrix} \succ 0.$$

Obtain **convex constraints** on Q and $\tilde{Q}$!

Problem: We neglected that controller has to be internally stabilizing!

Synthesis LMIs: Dynamic Scalings

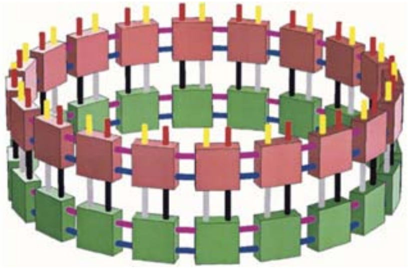
$$U^T \begin{pmatrix} I & 0 \\ A_p & B_p \\ C_p & D_p \end{pmatrix}^T \begin{pmatrix} 0 & X & 0 \\ X & 0 & 0 \\ 0 & 0 & \mathrm{diag}(M, -M) \end{pmatrix} \begin{pmatrix} I & 0 \\ A_p & B_p \\ C_p & D_p \end{pmatrix} U \prec 0$$

$$V^T \begin{pmatrix} -A_d^T & -C_d^T \\ I & 0 \\ B_d^T & D_d^T \end{pmatrix}^T \begin{pmatrix} 0 & Y & 0 \\ Y & 0 & 0 \\ 0 & 0 & \mathrm{diag}(N, -N) \end{pmatrix} \begin{pmatrix} -A_d^T & -C_d^T \\ I & 0 \\ B_d^T & D_d^T \end{pmatrix} V \succ 0$$

$$\left(\begin{array}{ccc|ccc} X_{11} - \hat{X} & X_{12} & X_{13} & -\hat{Z} & 0 & 0 \\ X_{21} & X_{22} + \hat{X} & X_{23} & 0 & -\hat{Z} & 0 \\ X_{31} & X_{32} & X_{33} & 0 & 0 & I \\ \hline -\hat{Z}^T & 0 & 0 & Y_{11} - \hat{Y} & Y_{12} & Y_{13} \\ 0 & -\hat{Z}^T & 0 & Y_{21} & Y_{22} + \hat{Y} & Y_{23} \\ 0 & 0 & I & Y_{31} & Y_{32} & Y_{33} \end{array}\right) \succ 0.$$

Range of Applications

- Reduction of conservatism by dynamics in scalings
- Allows scheduling on dynamic changes in plant
- **Lossless** gain-scheduling synthesis for slowly time-varying dynamic uncertainties
- Graceful mixing of scheduled and robust synthesis
- Distributed synthesis



D'Andrea, Dullerud (03)

Conclusions

Have seen:

- Relation of gain-scheduling and distributed synthesis
- Recap of technique with static multipliers
- Sketch of complete solution for dynamic D-scalings

Next steps:

- Numerical implementations and experimentation
- Precise understanding: Interconnection and multiplier structures
- Extension to general IQC multipliers (expected to be tough)