Distributed Model Predictive Control for Water Infrastructures

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LCCC Workshop on Multi-agent Coordination and Estimation

Delft Center for Systems and Control

Delft University of Technology

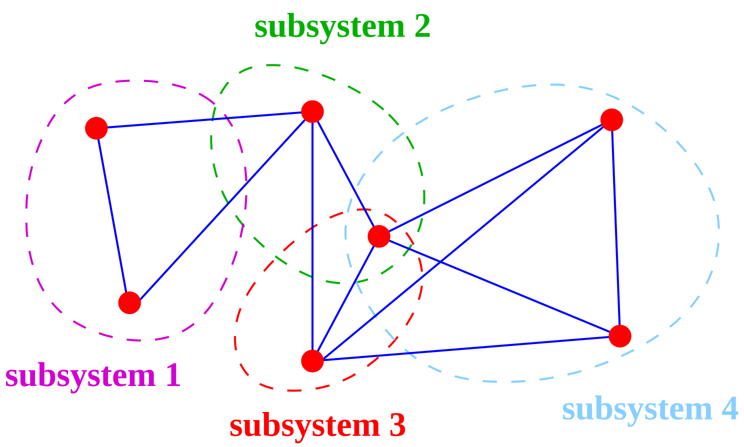
Overview

- Hierarchical and distributed MPC (HD-MPC)
- Distributed model predictive control
- Applications in water control for
  - Irrigation canals
  - Dutch river system
  - Water supply and sewer networks
- A multi-scale approach for HD-MPC
- Concluding remarks

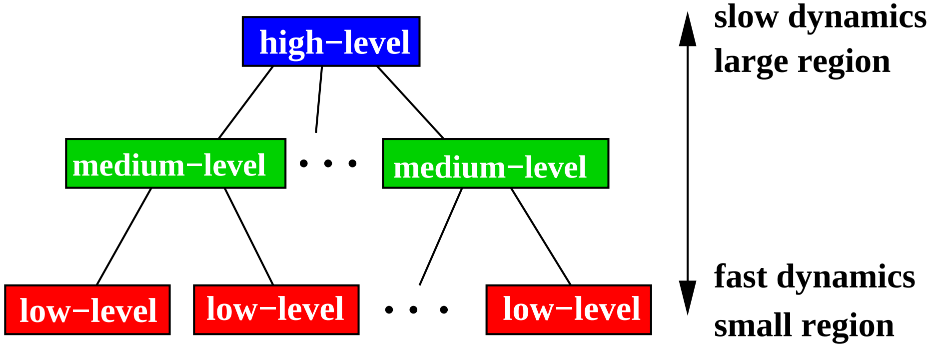
1. HD-MPC

Hierarchical and Distributed Model Predictive Control

Objective: Development of new methods for distributed and hierarchical model-based predictive control of large-scale systems



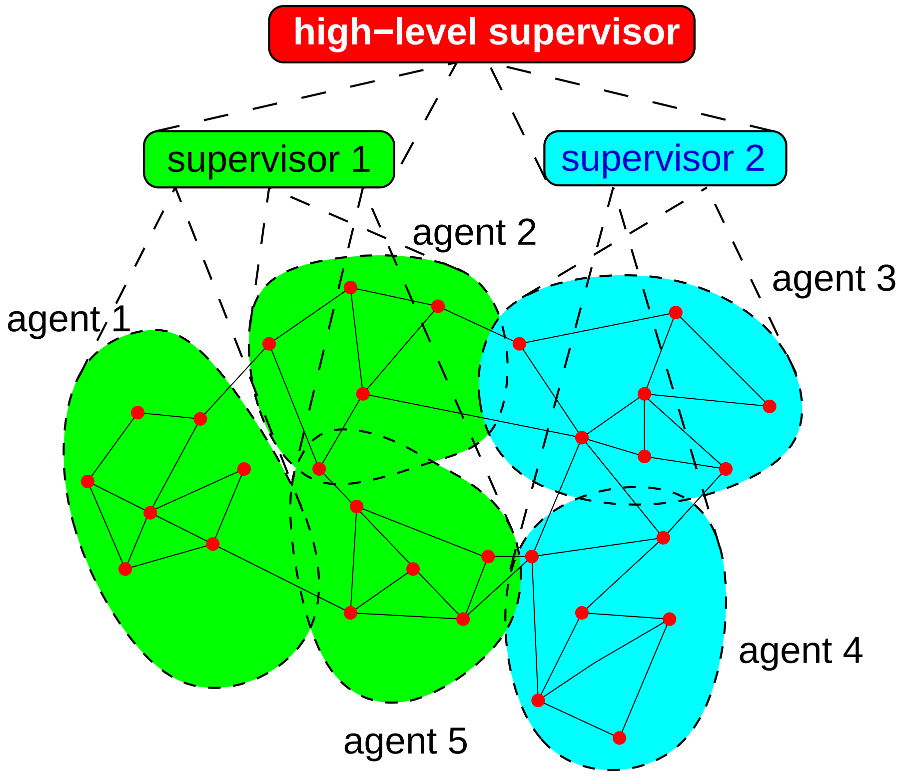
Partners: TUD, EDF, K.U.Leuven, POLIMI, RWTH Aachen, US, UNAL, Supelec, Inocsa, UWM



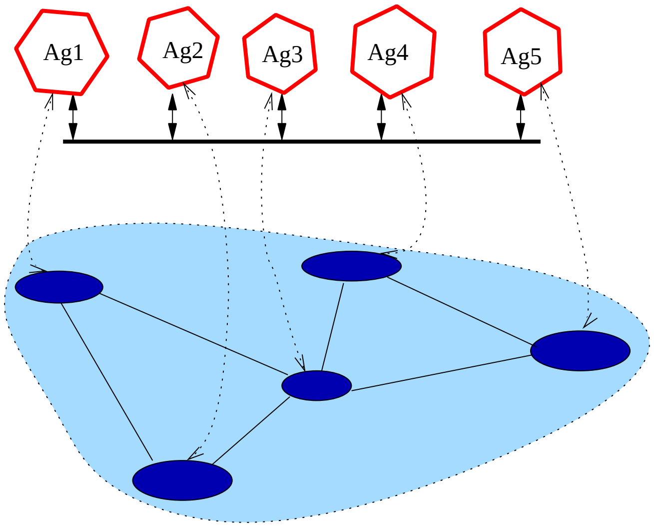
http://www.ict-hd-mpc.eu/

1. HD-MPC

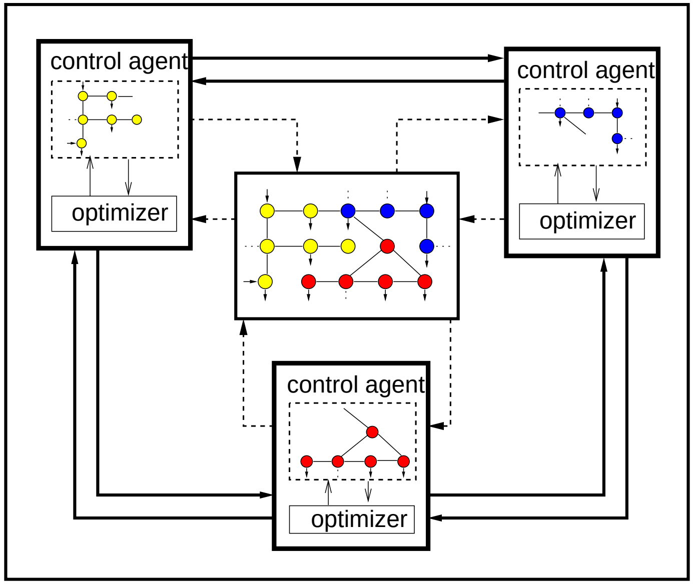
- Challenges in control of large-scale networks:
  - large-scale networks
  - distributed vs centralized control
  - optimality ↔ computational efficiency/tractability
  - global ↔ local
  - scalability
  - robustness
  - → multi-level multi-agent approach



2. Distributed MPC



- subnetworks instead of overall network
- single agent/controller for each subnetwork
  - limited action capabilities
  - limited information gathering

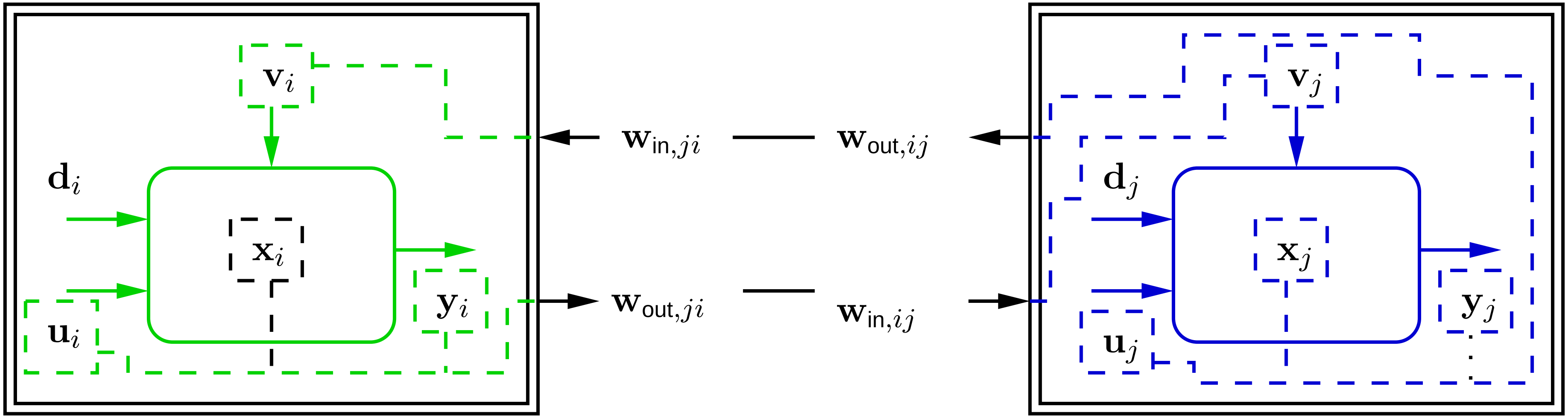


Challenge

agents should choose local inputs that are globally optimal

2. Distributed MPC

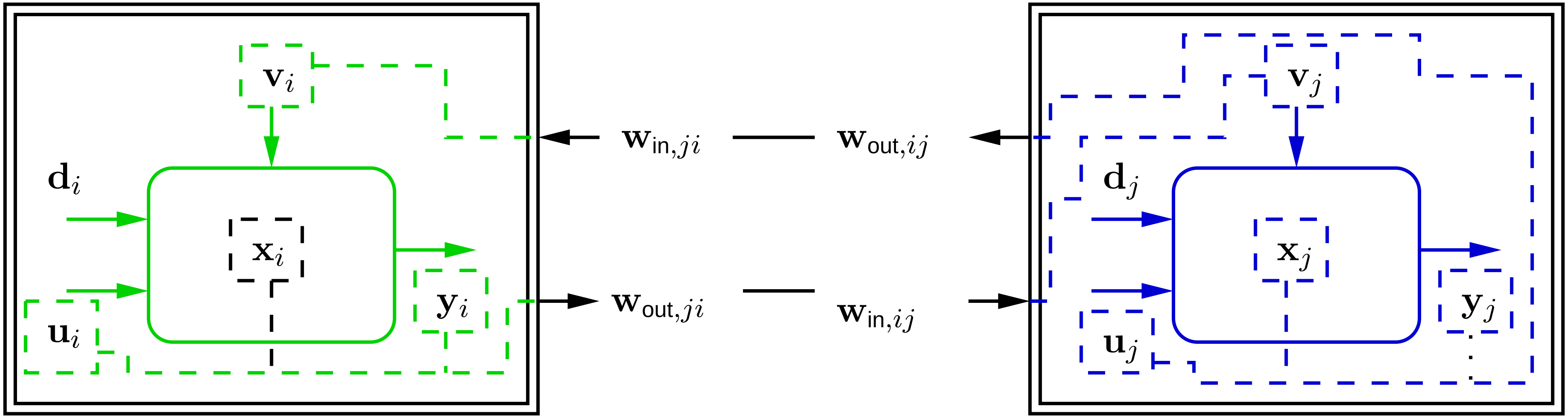
Interconnection between between control agents



$$\mathbf{x}_i(k+1) = \mathbf{f}_i(\mathbf{x}_i(k), \mathbf{u}_i(k), \mathbf{d}_i(k), \mathbf{v}_i(k))$$

2. Distributed MPC

Interconnection between between control agents



$$\mathbf{x}_i(k+1) = \mathbf{f}_i(\mathbf{x}_i(k), \mathbf{u}_i(k), \mathbf{d}_i(k), w_{\text{in},j_1 i}(k), \ldots, w_{\text{in},j_{m_i} i}(k))$$

$$\mathbf{w}_{\text{out},ji}(k+1) = \mathbf{h}^{ji}_{\text{out}}(\mathbf{u}^i_k, \mathbf{y}^i_k, \mathbf{x}^i_{k+1})$$ for each neighbor j of i

2. Distributed MPC

Local MPC control problem of agent i at decision step k

$$\min_{\tilde{\mathbf{u}}_i(k),\tilde{\mathbf{x}}_i(k+1)} J_{\text{local},i}(\tilde{\mathbf{u}}_i(k),\tilde{\mathbf{x}}_i(k+1))$$

subject to

- subnetwork dynamics: prediction model

$$\mathbf{x}_i(k+1) = \mathbf{f}_i(\mathbf{x}_i(k),\mathbf{u}_i(k),\mathbf{d}_i(k),\ldots)$$

$$\vdots$$

$$\mathbf{x}_i(k+N) = \mathbf{f}_i(\mathbf{x}_i(k+N-1),\mathbf{u}_i(k+N-1),\mathbf{d}_i(k+N-1),\ldots)$$

- initial local state, disturbances, and additional constraints

2. Distributed MPC

Local MPC control problem of agent i at decision step k

$$\min_{\tilde{\mathbf{u}}_i(k), \tilde{\mathbf{x}}_i(k+1)} J_{\text{local},i}(\tilde{\mathbf{u}}_i(k), \tilde{\mathbf{x}}_i(k+1))$$

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$$\mathbf{x}_i(k+1) = \mathbf{f}_i(\mathbf{x}_i(k), \mathbf{u}_i(k), \mathbf{d}_i(k), \mathbf{w}_{\text{in},j_1 i}(k), \ldots, \mathbf{w}_{\text{in},j_{m_i} i}(k))$$

$$\mathbf{w}_{\text{out},ji}(k+1) = \mathbf{h}_{\text{out},ji}(\mathbf{u}_i(k), \mathbf{y}_i(k), \mathbf{x}_i(k+1)) \quad \text{for each neighbor } j \text{ of } i$$

$$\vdots$$

$$\mathbf{x}_i(k+N) = \mathbf{f}_i(\mathbf{x}_i(k+N-1), \mathbf{u}_i(k+N-1), \mathbf{d}_i(k+N-1), \mathbf{w}_{\text{in},j_1 i}(k+N-1), \ldots, \mathbf{w}_{\text{in},j_{m_i} i}(k+N-1))$$

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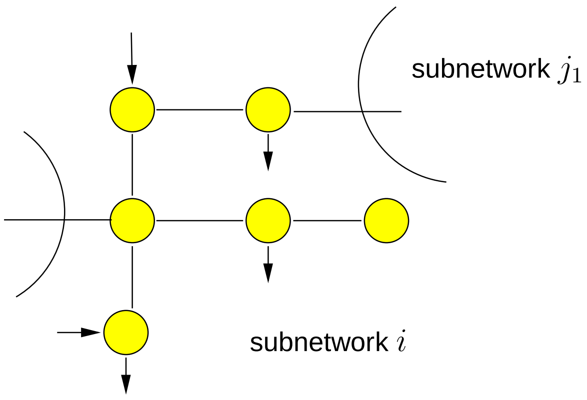
- initial local state, disturbances and additional constraints

2. Distributed MPC

Interconnecting constraints

- constraints on interconnecting variables
- imposed by dynamics of overall network
- *what goes in into i equals what goes out from j*
- satisfaction necessary for accurate predictions



$$\begin{aligned}
\mathbf{w}_{\text{in},ji}(k) &= \mathbf{w}_{\text{out},ij}(k) \\
\mathbf{w}_{\text{out},ji}(k) &= \mathbf{w}_{\text{in},ij}(k) \\
\vdots \quad & \quad \vdots \\
\mathbf{w}_{\text{in},ji}(k+N-1) &= \mathbf{w}_{\text{out},ij}(k+N-1) \\
\mathbf{w}_{\text{out},ji}(k+N-1) &= \mathbf{w}_{\text{in},ij}(k+N-1)
\end{aligned}$$

For agent controlling subnetwork i

- $\mathbf{w}_{\text{in},ij}$ and $\mathbf{w}_{\text{out},ij}$ of neighbor j unknown
- how make accurate predictions?
  $\rightarrow$ via negotiations

2. Distributed MPC

A multiple-iterations scheme

- agree on values of interconnecting variables
- each agent
  - computes optimal local *and* interconnecting variables
  - communicates interconnecting variables to neighbors
  - updates parameters $\tilde{\lambda}_{\text{in}}^{ji}, \tilde{\lambda}_{\text{out}}^{ji}$ of additional cost term J_{inter}^{i}
- iterations until stopping criterion satisfied
- scheme converges to overall optimal solution under convexity assumptions

$$\min_{\tilde{\mathbf{u}}_i(k),\tilde{\mathbf{x}}_i(k+1),\tilde{\mathbf{w}}_{\text{in},li}(k),\tilde{\mathbf{w}}_{\text{out},li}(k)} J_{\text{local},i}(\tilde{\mathbf{u}}_i(k),\tilde{\mathbf{x}}_i(k+1)) + \sum_{j\in\text{neighbors}_i} J_{\text{inter},i}(\tilde{\mathbf{w}}_{\text{in},ji}(k),\tilde{\mathbf{w}}_{\text{out},ji}(k))$$

subject to

- dynamics of subnetwork i over the horizon
- initial local state, disturbances, additional constraints

2. Distributed MPC

- Scheme based on augmented Lagrangian and block coordinate descent + serial implementation
  Alternative: auxiliary problem principle with parallel implementation

- Additional objective function $J^{(s)}_{\text{inter},i}(\tilde{\mathbf{w}}_{\text{in},ji}(k), \tilde{\mathbf{w}}_{\text{out},ji}(k)) =$

$$\begin{bmatrix} \tilde{\boldsymbol{\lambda}}^{(s)}_{\text{in},ji}(k) \\ -\tilde{\boldsymbol{\lambda}}^{(s)}_{\text{out},ij}(k) \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} \tilde{\mathbf{w}}_{\text{in},ji}(k) \\ \tilde{\mathbf{w}}_{\text{out},ji}(k) \end{bmatrix} + \frac{\gamma}{2} \left\| \begin{bmatrix} \tilde{\mathbf{w}}_{\text{in,prev},ij}(k) - \tilde{\mathbf{w}}_{\text{out},ji}(k) \\ \tilde{\mathbf{w}}_{\text{out,prev},ij}(k) - \tilde{\mathbf{w}}_{\text{in},ji}(k) \end{bmatrix} \right\|_2^2,$$

  where for each j that is a neighbor that solved its problem before i in iteration s:

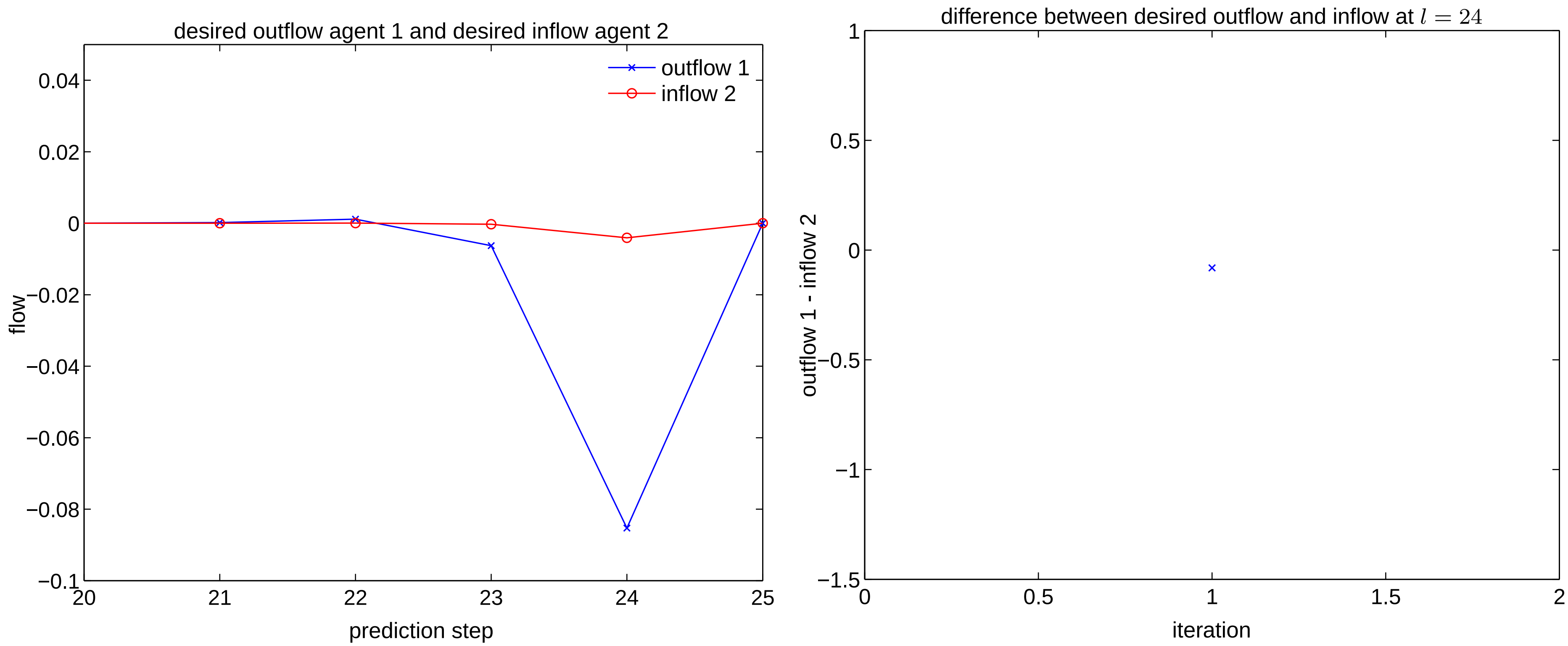
$$\tilde{\mathbf{w}}_{\text{in,prev},ij}(k) = \tilde{\mathbf{w}}^{(s)}_{\text{in},ij} \quad \text{and} \quad \tilde{\mathbf{w}}_{\text{out,prev},ij}(k) = \tilde{\mathbf{w}}^{(s)}_{\text{out},ij},$$

  and where for each j that has not solved its problem in iteration s yet:

$$\tilde{\mathbf{w}}_{\text{in,prev},ij}(k) = \tilde{\mathbf{w}}^{(s-1)}_{\text{in},ij} \quad \text{and} \quad \tilde{\mathbf{w}}_{\text{out,prev},ij}(k) = \tilde{\mathbf{w}}^{(s-1)}_{\text{out},ij}$$

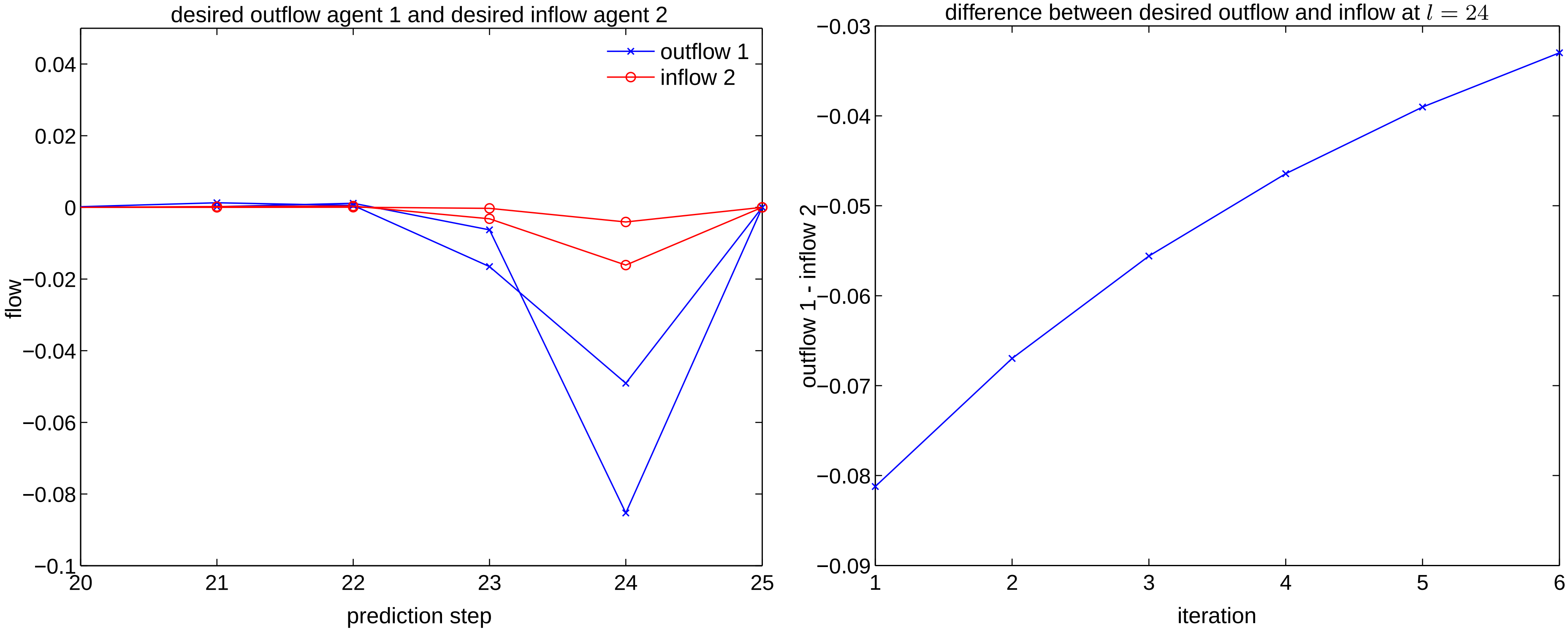
- Update of $\tilde{\boldsymbol{\lambda}}_{\text{in},ji}$: $\tilde{\boldsymbol{\lambda}}^{(s+1)}_{\text{in},ji}(k) = \tilde{\boldsymbol{\lambda}}^{(s)}_{\text{in},ji} + \gamma \left(\tilde{\mathbf{w}}^{(s)}_{\text{in},ji}(k) - \tilde{\mathbf{w}}^{(s)}_{\text{out},ij}(k) \right)$

2. Distributed MPC



Obtaining agreement on flows between two subnetworks.

2. Distributed MPC



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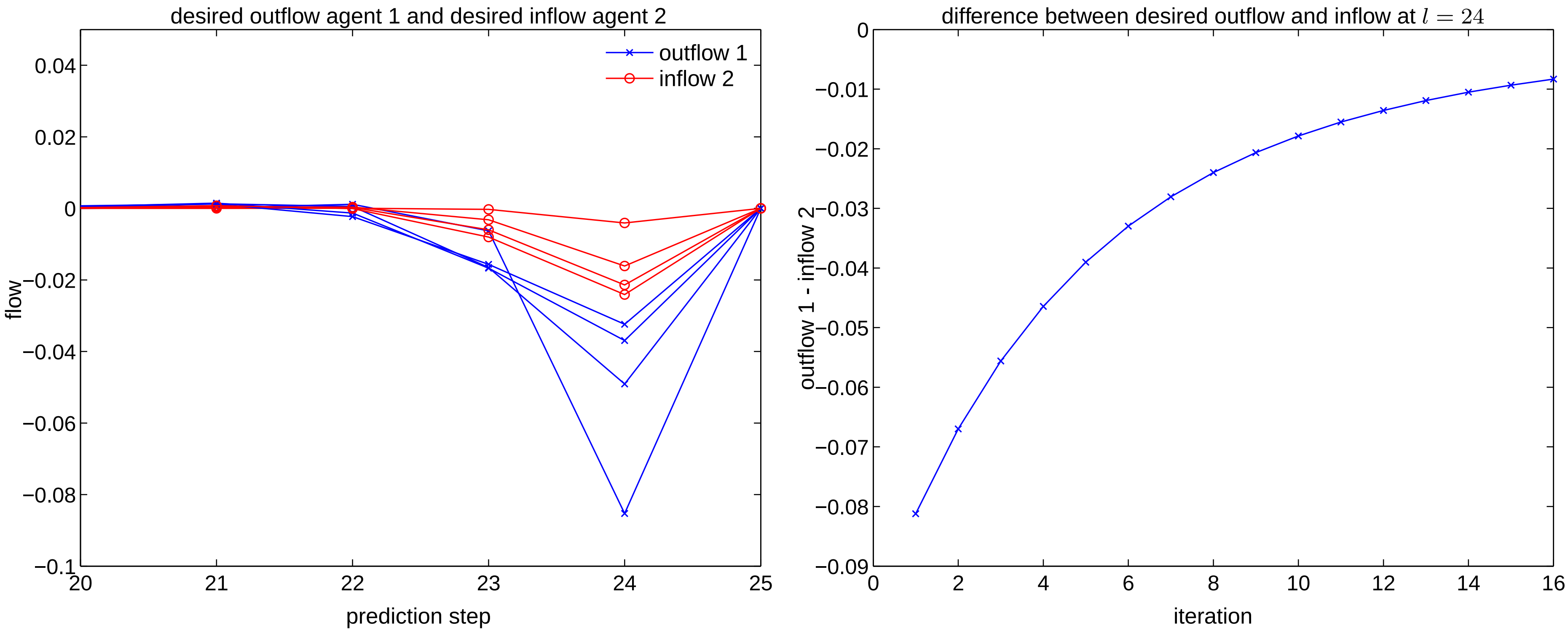

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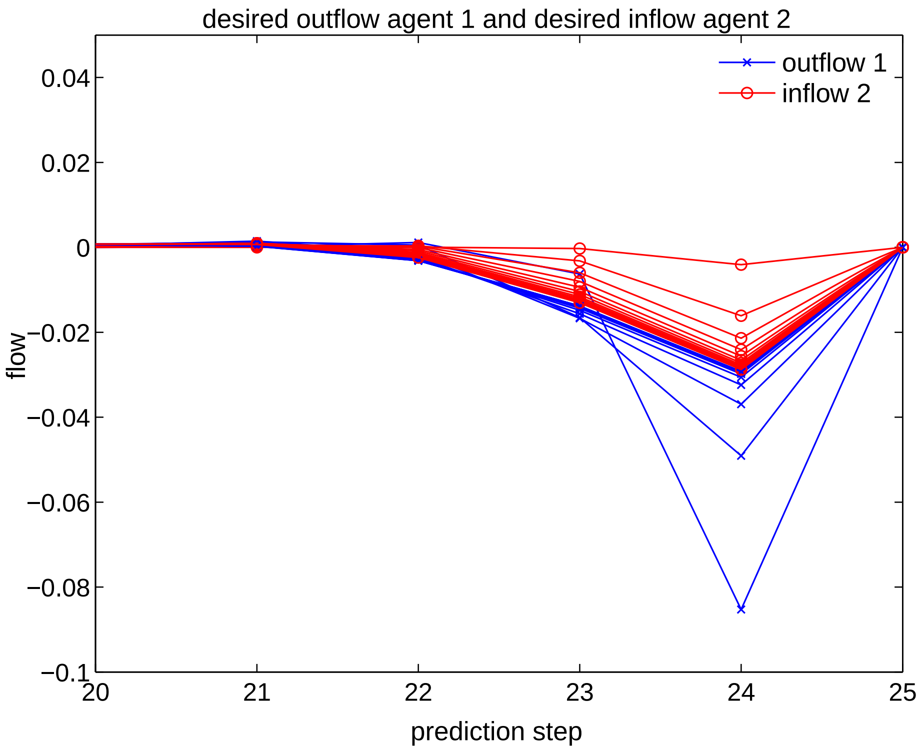

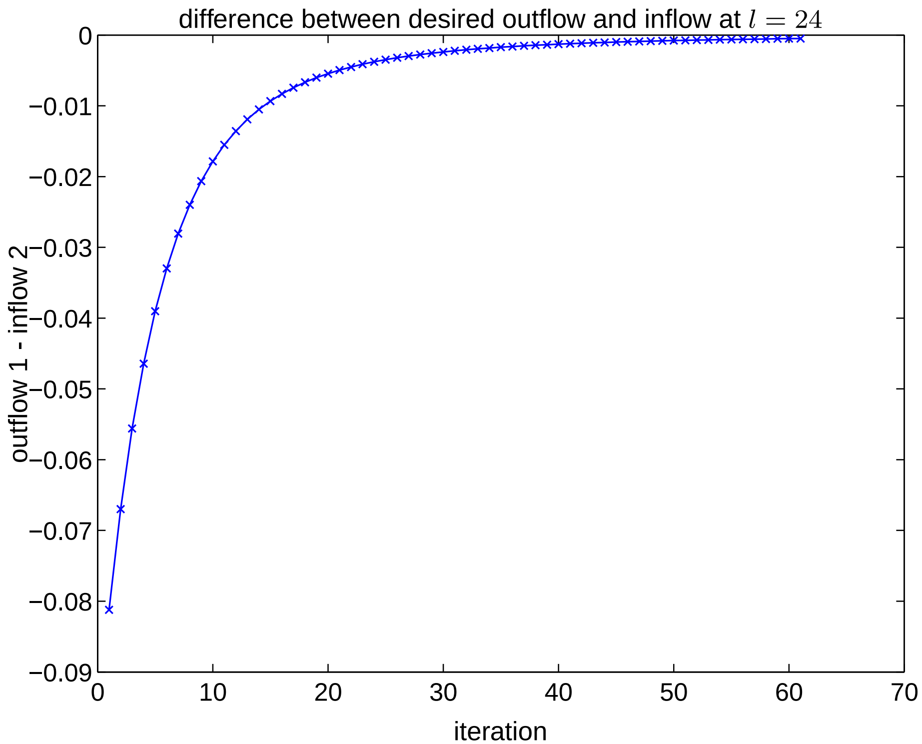

Obtaining agreement on flows between two subnetworks.

3. Cooperative water control

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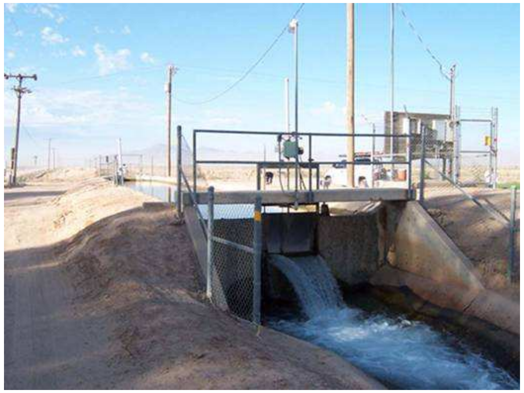
Cooperation to
improve performance

3.1. Irrigation canals



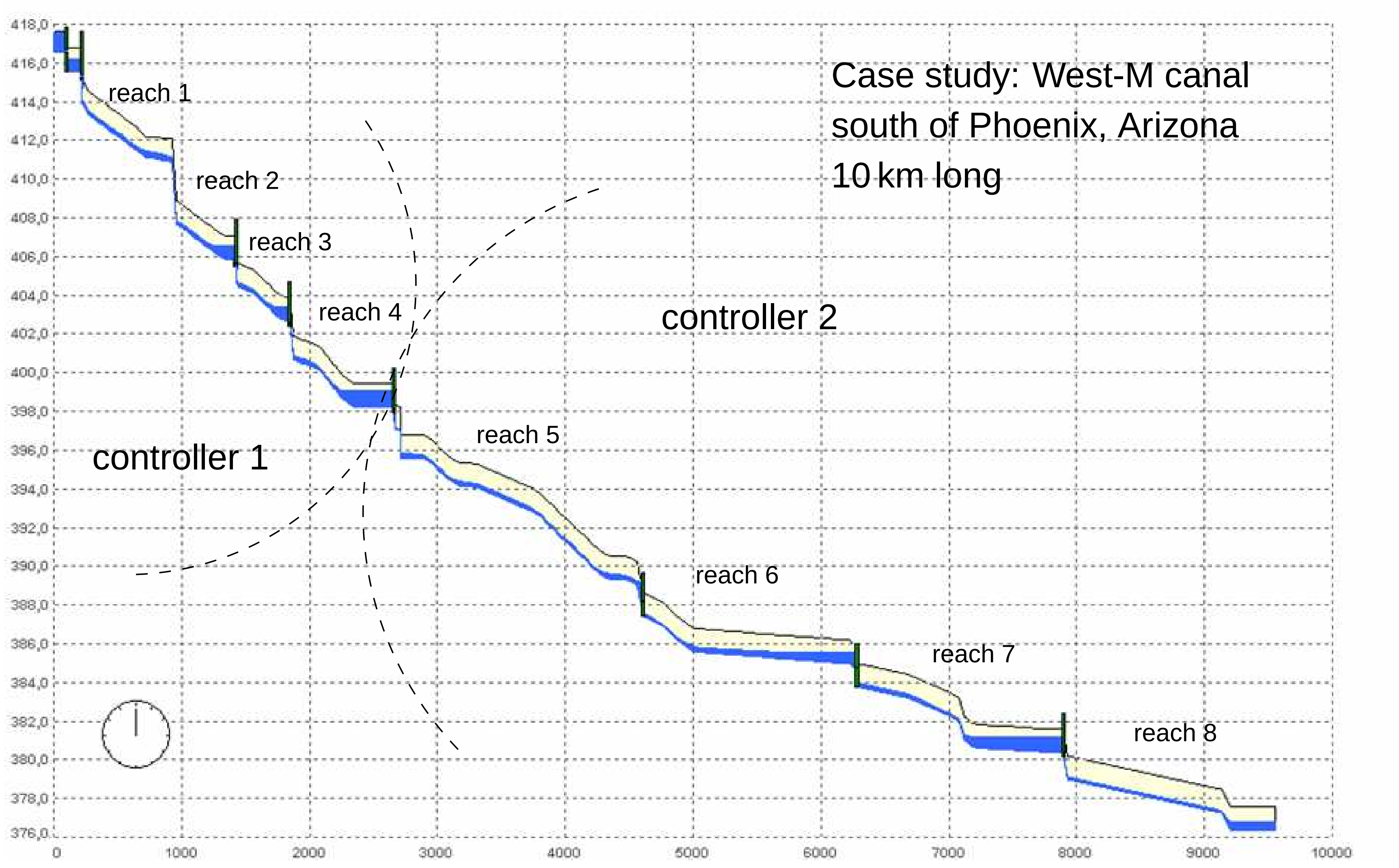
Irrigation accounts for about 70% of global fresh water usage

Irrigation canals should deliver water at the right time to the right location

Components:

- control structures
- off-takes
- canal reaches
- water users

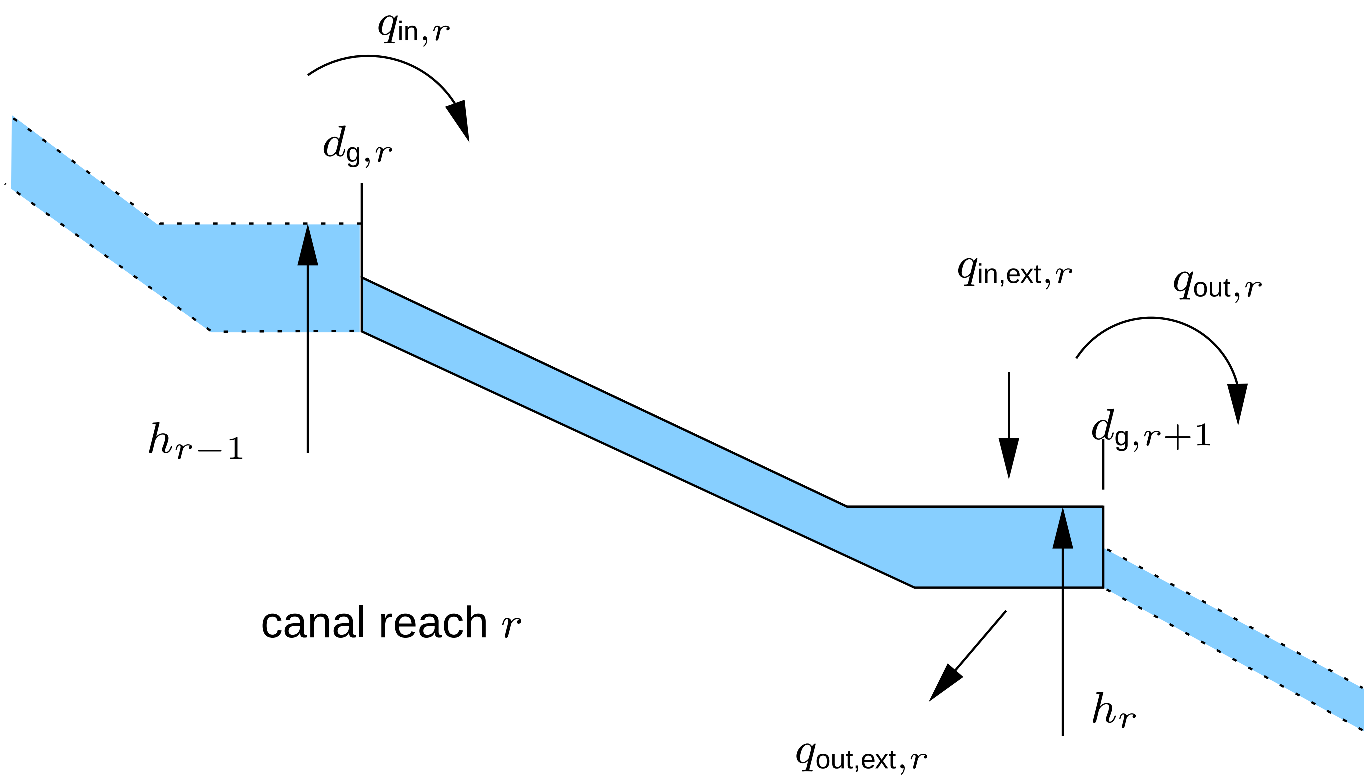
3.1. Irrigation canals



Adjust gates to maintain water levels, while satisfying demand and actuator constraints.

3.1. Irrigation canals

Dynamics of a canal reach



3.1. Irrigation canals

Dynamics of a canal reach

$$h_r(k+1) = h_r(k) + \frac{T_\mathrm{c}}{c_r} q_{\mathrm{in},r}(k - k_{\mathrm{d},r}) - \frac{T_\mathrm{c}}{c_r} q_{\mathrm{out},r}(k) + \frac{T_\mathrm{c}}{c_r} q_{\mathrm{ext,in},r}(k) - \frac{T_\mathrm{c}}{c_r} q_{\mathrm{ext,out},r}(k)$$

$$q_{\mathrm{in},r}(k) = q_{\mathrm{in},r}(k-1) + C_{e,r} \Delta h_{r-1}(k) + C_{u,r} \Delta d_{\mathrm{g},r}(k)$$

$$q_{\mathrm{out},r}(k) = q_{\mathrm{out},r}(k-1) + C_{e,r+1} \Delta h_r(k) + C_{u,r+1} \Delta d_{\mathrm{g},r+1}(k)$$

with constant

$$C_{e,r} = \frac{g c_{\mathrm{w},r} W_{\mathrm{s},r} \mu_r \underline{d}_{\mathrm{g},r}}{\sqrt{2g(\underline{h}_{r-1} - (z_{\mathrm{s},r} + \mu_r \underline{d}_{\mathrm{g},r}))}}$$

$$C_{u,r} = c_{\mathrm{w},r} W_{\mathrm{s},r} \mu_r \sqrt{2g(\underline{h}_{r-1} - (z_{\mathrm{s},r} + \mu_r \underline{d}_{\mathrm{g},r}))} - \frac{g c_{\mathrm{w},r} W_{\mathrm{s},r} \mu_r^2 \underline{d}_{\mathrm{g},r}}{\sqrt{2g(\underline{h}_{r-1} - (z_{\mathrm{s},r} + \mu_r \underline{d}_{\mathrm{g},r}))}},$$

where $\underline{h}, \underline{d}$ are given linearization points

3.1. Irrigation canals

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where $\underline{h}, \underline{d}$ are given linearization points

3.1. Irrigation canals

Control objectives

- Minimize deviations of water levels from set-points
- Minimize changes in gate positions

$$J_{\text{local},i} = \sum_{l=0}^{N-1} \sum_{r \in \mathcal{R}_i} \Big(\alpha_r \left(h_r(k+1+l) - h_{r,\text{ref}}\right)^2 + \beta_r \left(d_{\text{g},r}(k+l) - d_{\text{g},r}(k+l-1)\right)^2 \Big)$$

Constraints

- maximum on the change in the gate position, both upwards and downwards
- gate position should always be positive
- gate should not be lifted out of the water

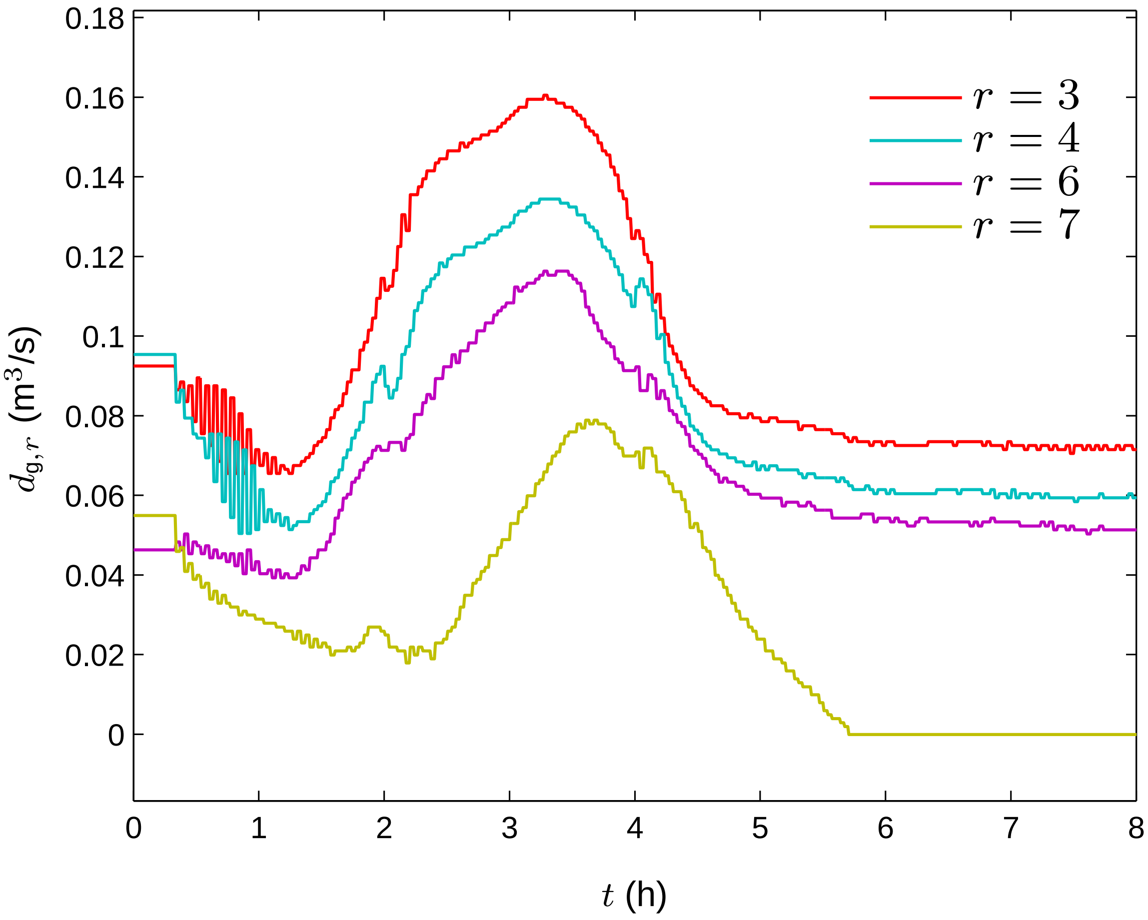
3.1. Irrigation canals

Setup

- Implementation
  - Nonlinear, validated model of the canal implemented in SOBEK
  - MPC controllers with linearized models implemented in Matlab
  - Optimization using CPLEX v10.0 through Tomlab 5.7 interface
- Parameters
  - $T_{\mathrm{c}} = 120\,\mathrm{s}$, $N = 30$ steps
  - Distributed MPC scheme parameters: $\gamma = 1000$, $\varepsilon = 1.10^{-4}$
  - Cost coefficients: $\alpha_r = 0.15$, $\beta_r = 0.0075$
- Scenario
  - 8 hour simulation
  - at $t = 2$: increase of 0.1 m^3/s in offtake of reach 3
  - at $t = 4$: decrease of 0.1 m^3/s in offtake of reach 3

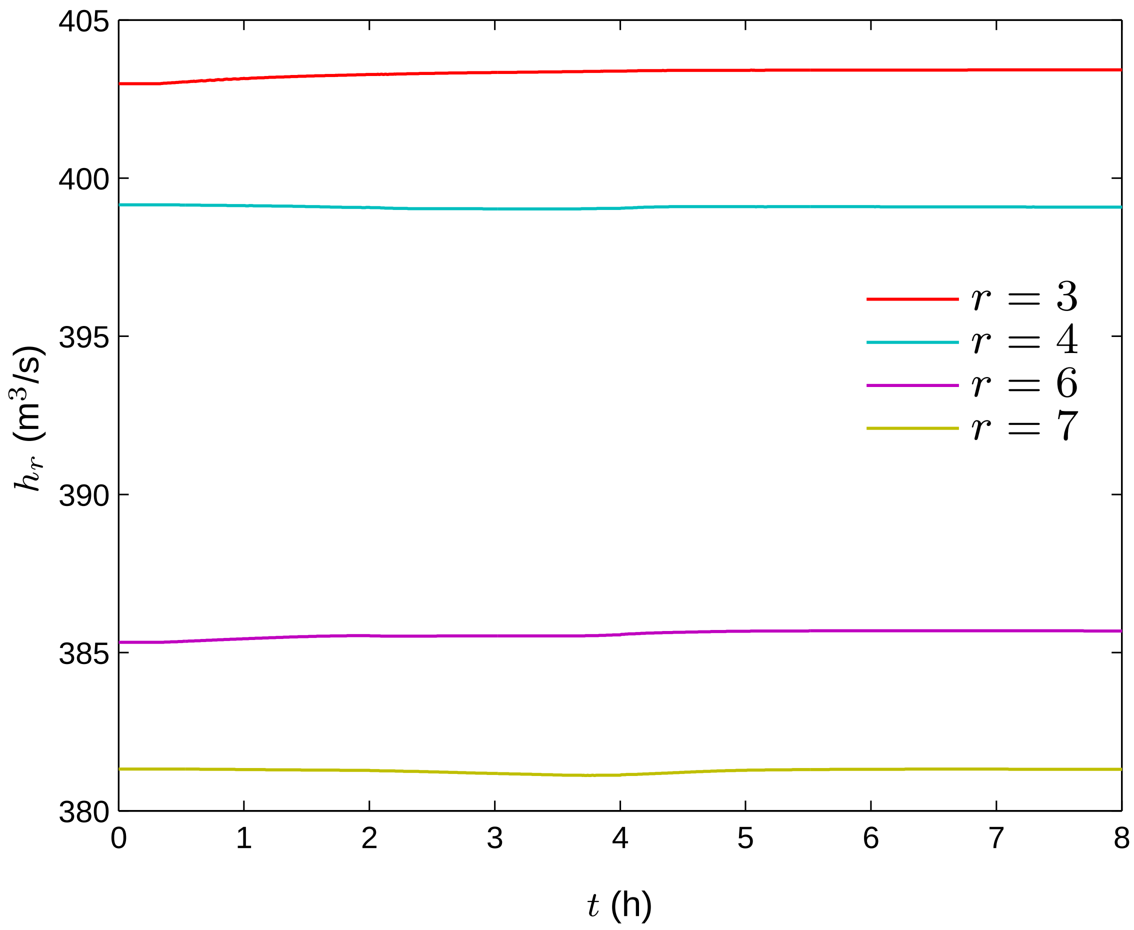
3.1. Irrigation canals

Evolution of actions over the full simulation



3.1. Irrigation canals

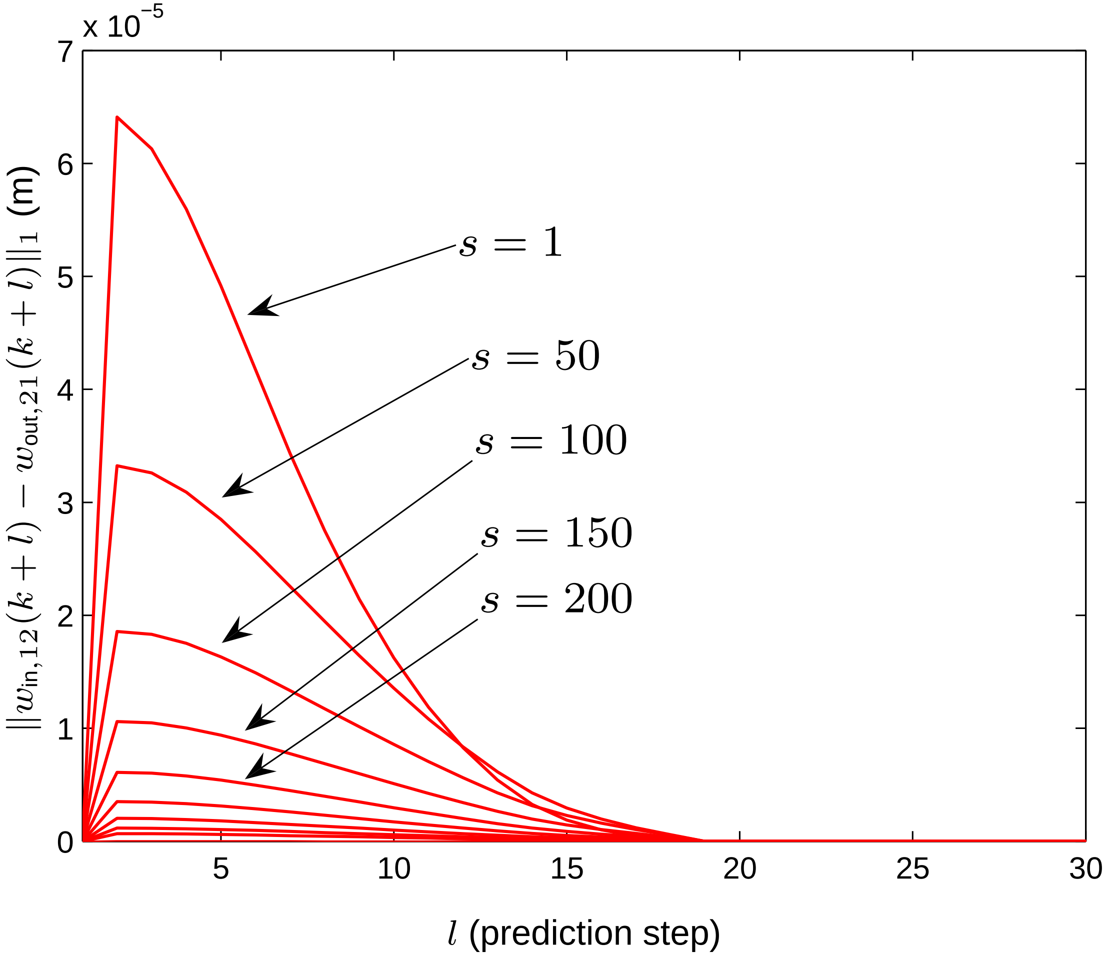
Evolution of water levels over the full simulation



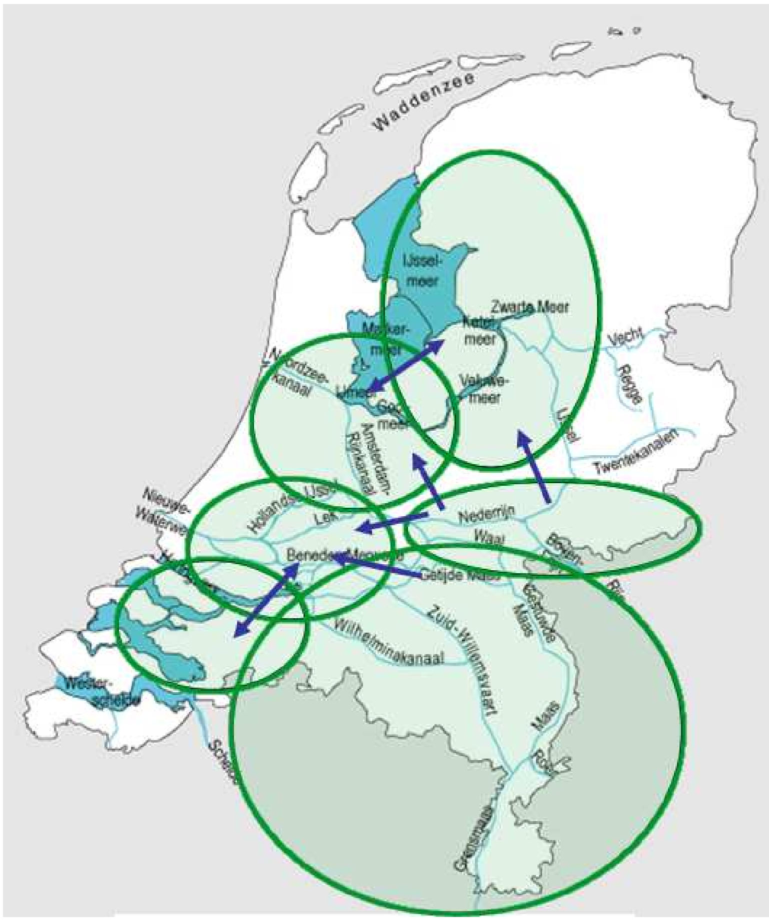
performance within
10% of centralized

3.1. Irrigation canals

Evolution of absolute error over the iterations at $t = 2.23$

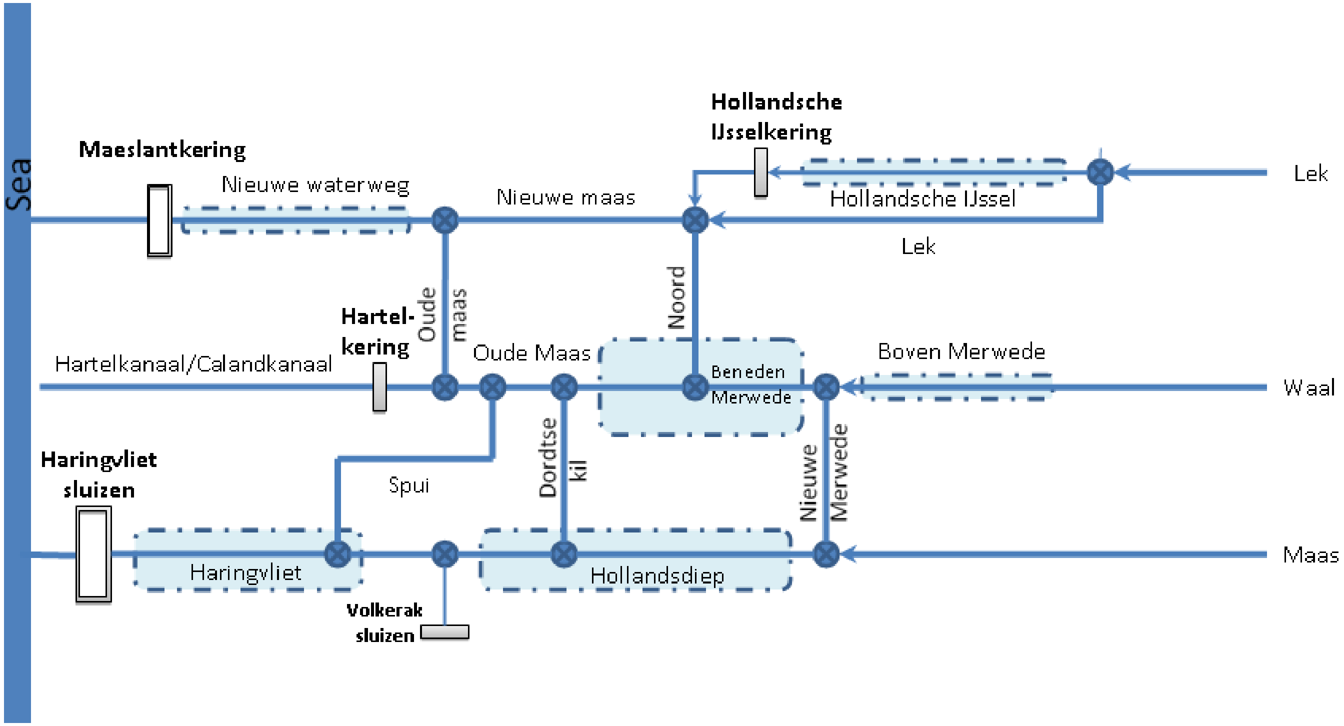


3.2. Dutch river system



3.2. Dutch river system

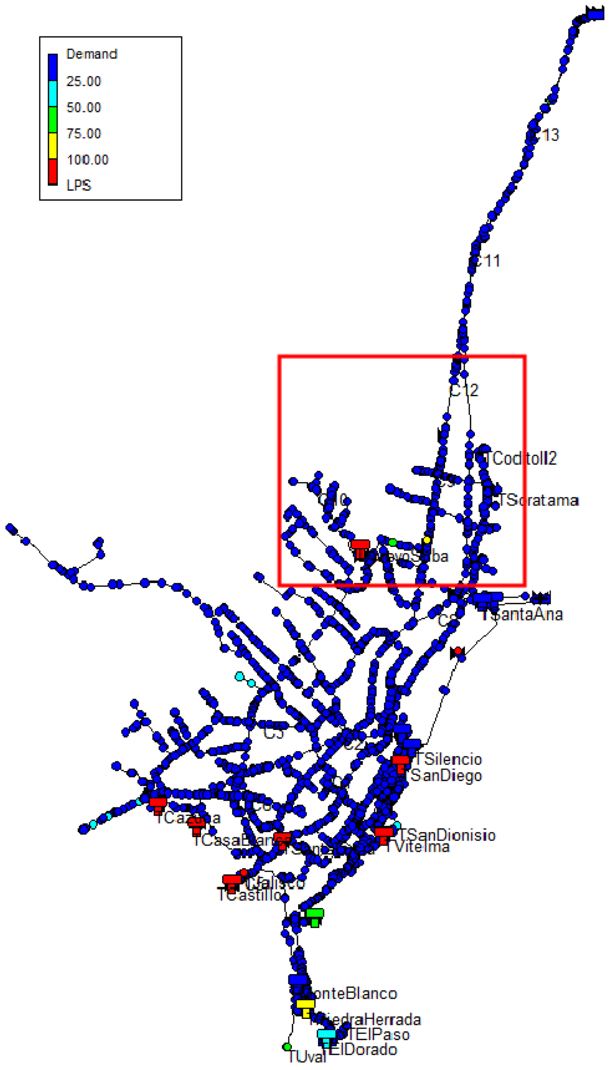
First step: **Control of the Rijnmond area**

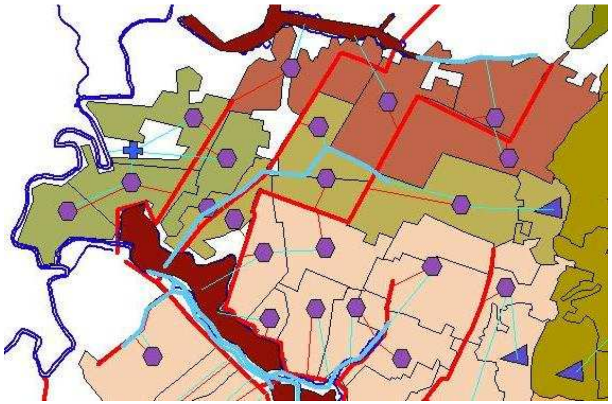


Maintain water levels in cities by controlling gates, subject to
tidal sea water level, varying river inflows, safety and actuator constraints

discrete (actuators) + continuous dynamics (partial differential equations)
$\longrightarrow$ hybrid MPC approach using mixed-integer nonlinear programming

3.3. Water supply and sewers



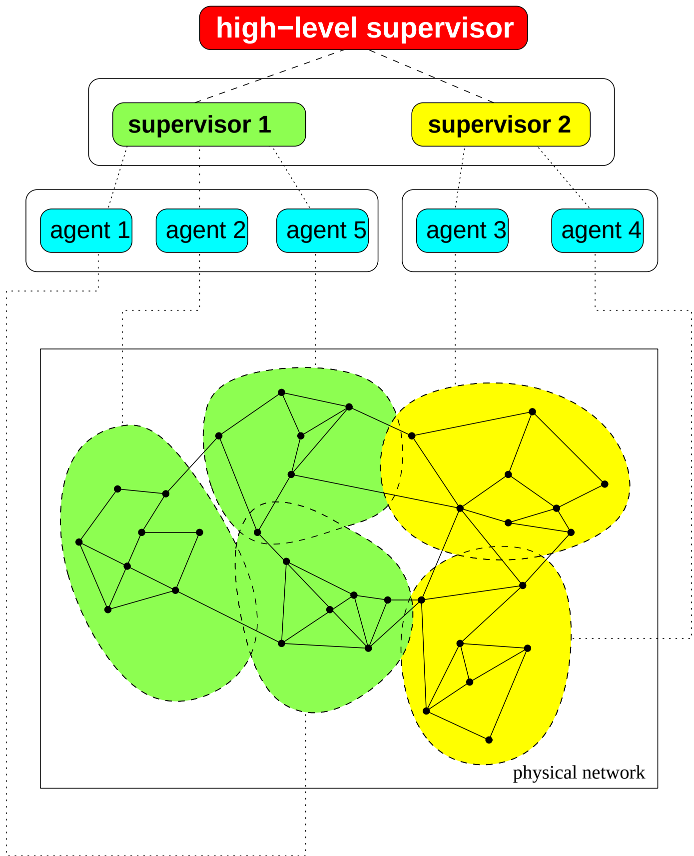


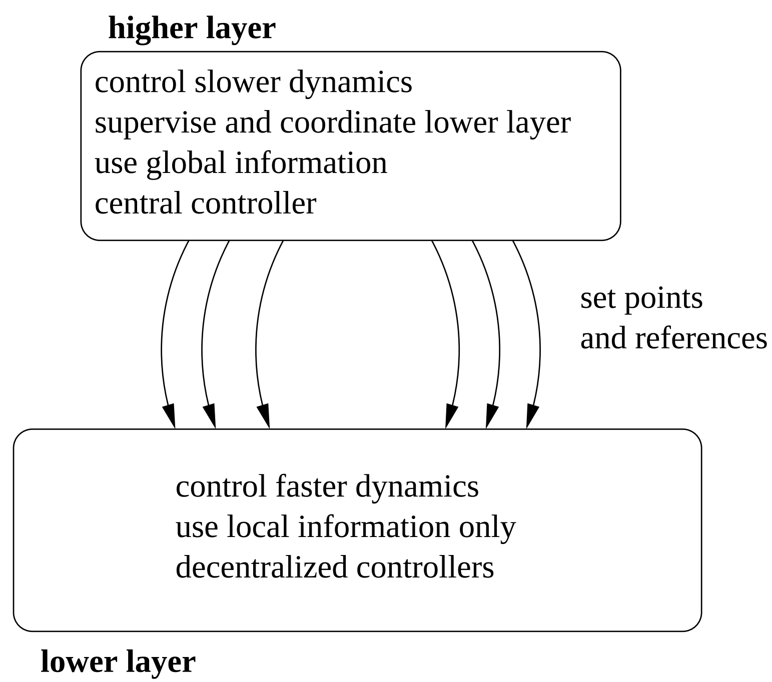
distributed MPC for water supply networks
control pumps and valves taking into account
water flow and pressure constraints
to optimally supply varying water demands

pattern-search MPC for sewer systems
coordination of *on* and *off* switching
of pumps to prevent sewer saturation and overflows

4. A multi-scale approach for HD-MPC

Time-based separation into layers





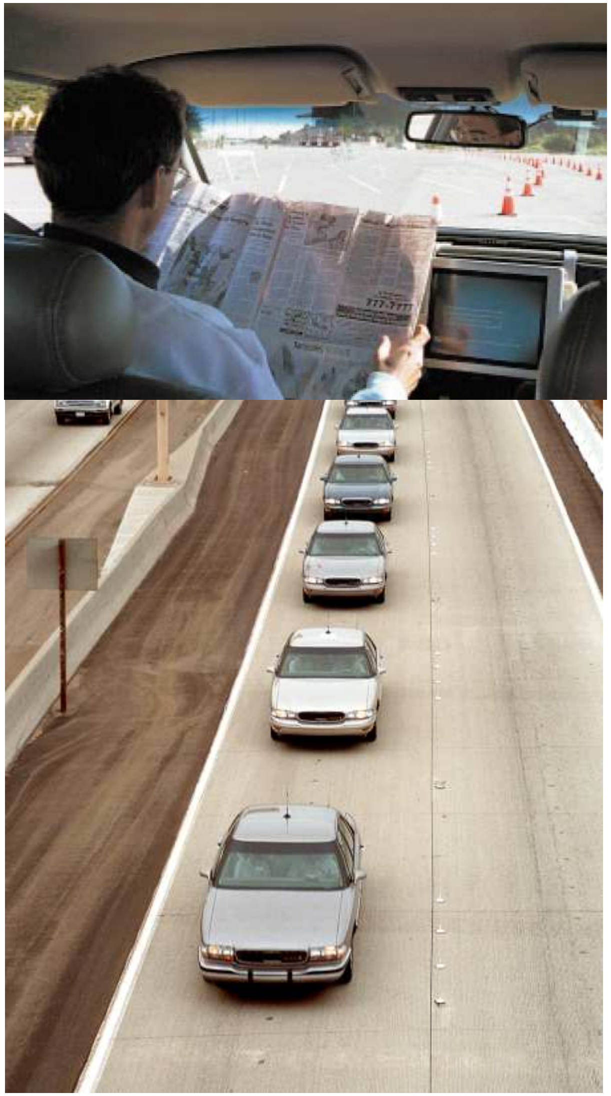
→ HD-MPC (http://www.ict-hd-mpc.eu)

4. A multi-scale approach for HD-MPC

Application: Intelligent Vehicle-Highway Systems

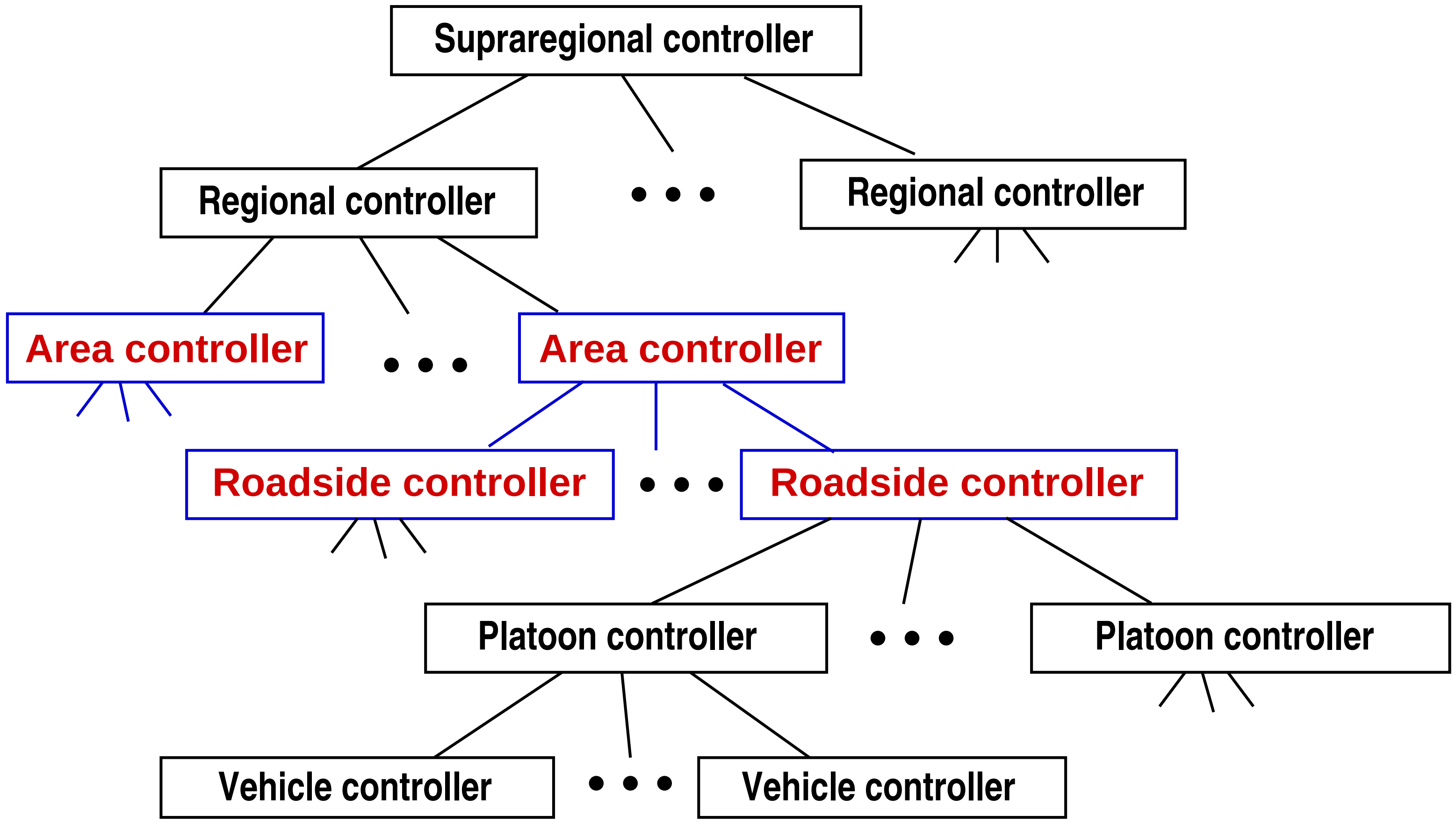
→ next generation traffic control and management system

- Use in-car telematics (navigation, telecommunication, information, . . .) systems
  → autonomous vehicles organized in platoons
- Vehicle-vehicle + vehicle-roadside communication
  → cooperative vehicle-infastructure systems
- Control via
  - cooperative adaptive cruise control,
  - intelligent speed adaption,
  - route guidance, . . .



4. A multi-scale approach for HD-MPC

$\longrightarrow$ hierarchical multi-layer control approach ($\sim$ California PATH)



4. A multi-scale approach for HD-MPC

Controller	Unit	Control	Time scale	Type of controller
Area	flows of platoons	routing	$>$ min	hybrid MPC (MILP)
Roadside	platoons	lanes & speeds, split & merge	s–min	nonlinear MPC
Platoon	vehicles	distances & speeds, trajectories	$<$ s	adaptive PID
Vehicle	vehicle	throttle, brake, steering	$\ll$ s	basic (PID, logic, . . .)

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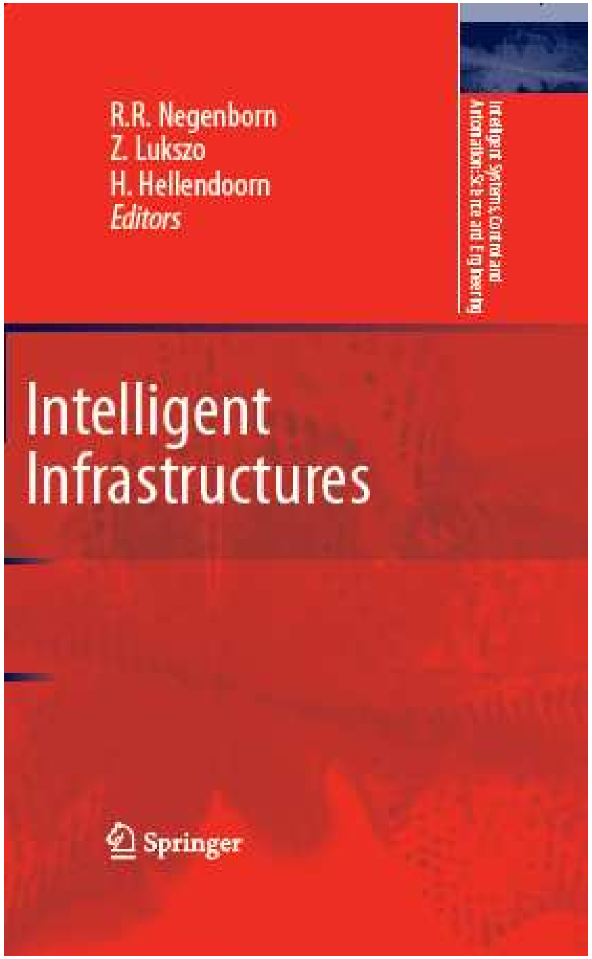
$\longrightarrow$ multi-agent approach possible

5. Concluding remarks

- Large-scale water networks: distributed control required
  $\longrightarrow$ multi-agent approach
- Coordination for achieving agreement on mutual interaction, i.e., deciding on the inflows and outflows among subsystems
- Multi-objective control + constraints
  $\longrightarrow$ model-based predictive control (DMPC/HD-MPC)
- **Open issues**:
  - How to obtain tractable prediction models?
  - What is the best division into subnetworks?
  - How can existing approaches be extended to hybrid systems?
  - How can the computation/iteration time be reduced?
  - How should the higher control layers be designed?

Intelligent Infrastructures



Multi-agent control for:

- electricity infrastructures
- road traffic infrastructures
- water infrastructures

`http://IntelligentInfrastructures.net/`

(vol. 42 in Springer Series *Intelligent Systems, Control and Automation*, 2010)

More info: Rudy Negenborn <r.r.negenborn@tudelft.nl>

Open issues

- How to obtain tractable prediction models?
- How can the computation/iteration time be reduced?
- How can existing approaches be extended to hybrid systems?
- What is the best division into subnetworks?
- How should the higher control layers be designed?
- How should coordination and interaction between control layers be organized?