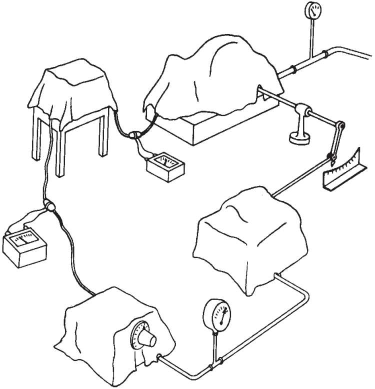
Distributed Verification of Sparse Systems

Anders Rantzer

Automatic Control LTH, Lund University

Can Systems be Certified Distributively?



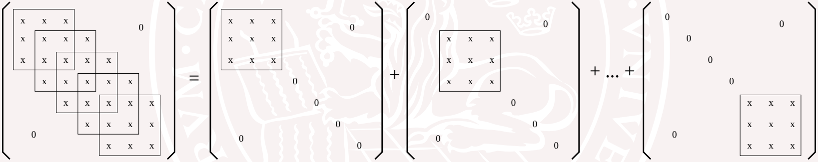
Componentwise performance verification without global model?

Outline

- ◦ Introduction
- • **Distributed Positive Test for Matrices**
- ◦ Distributed Performance Verification of Linear Systems
- ◦ Open problems

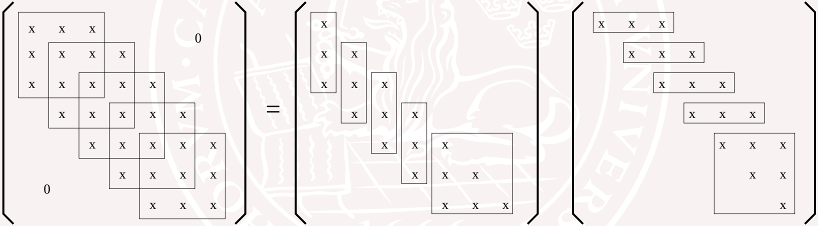
A Matrix Decomposition Theorem

The sparse matrix on the left is positive semi-definite if and only if it can be written as a sum of positive semi-definite matrices with the structure on the right.



Proof idea

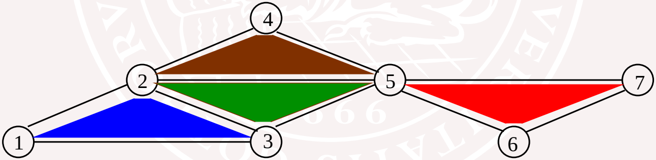
The decomposition follows immediately from the band structure of the Cholesky factors:



Generalization

Cholesky factors inherit the sparsity structure of the symmetric matrix if and only if the sparsity pattern corresponds to a “chordal” graph.

$$\begin{pmatrix} * & * & * & & & & \\ * & * & * & * & * & & \\ * & * & * & & * & & \\ & * & & * & * & & \\ & * & * & * & * & * & * \\ & & & & * & * & * \\ & & & & * & * & * \end{pmatrix} = \begin{pmatrix} * & * & * & & & & \\ * & * & * & & & & \\ * & * & * & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{pmatrix} + \begin{pmatrix} & & & & & & \\ & * & * & & * & & \\ & * & * & & * & & \\ & & & & & & \\ & * & * & & * & & \\ & & & & & & \\ & & & & & & \end{pmatrix} + \begin{pmatrix} & & & & & & \\ & * & & * & * & & \\ & & & & & & \\ & * & & * & * & & \\ & * & & * & * & & \\ & & & & & & \\ & & & & & & \end{pmatrix} + \begin{pmatrix} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & * & * & * \\ & & & & * & * & * \\ & & & & * & * & * \end{pmatrix}$$



[Blair & Peyton, An introduction to chordal graphs and clique trees, 1992]

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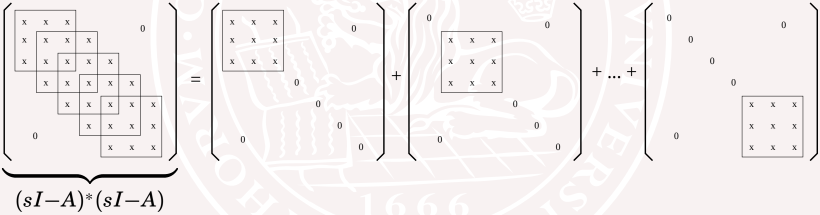
A Sparse Stability Test

For the sparse matrix A, let the left hand side illustrate the structure of $(sI-A)^*(sI-A)$. Then the matrix is stable if and only if the right hand side split can be done with all squares positive definite for s in the right half plane.



Hence global stability can always be verified by local tests!

A Sparse Passivity Test

Suppose

$$\dot{x} = Ax + Bx + w \qquad\qquad x(0) = 0$$
$$y = Cx$$

Then

$$\int_0^T \left(\gamma^2 u(t)y(t) + |w(t)|^2\right) dt \geq 0 \qquad \text{for all } u, w, T$$

if and only if the matrix

$$\begin{bmatrix} (sI-A)^*(sI-A) & \gamma^2 C^T - (sI-A)^* B \\ \gamma^2 C - B^*(sI-A) & B^T B \end{bmatrix}$$

is positive semi-definite for $\mathrm{Re}\, s \geq 0$.

Passivity can be tested componentwise without conservatism!

Conclusions

- A “general” distributed non-conservative performance test
- Relation to existing sufficient criteria
- Storage function interpretation
- Non-linear versions?