Adaptive Control - A Perspective

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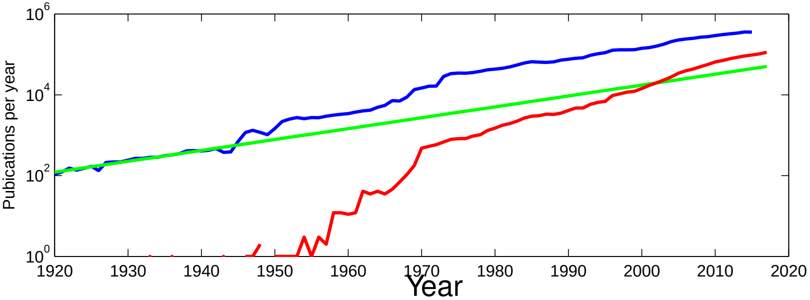
Adaptive Control - A Perspective

1. Introduction
2. Model Reference Adaptive Control
3. Self-Tuning Regulators
4. Dual Control
5. Summary

A Brief History of Adaptive Control

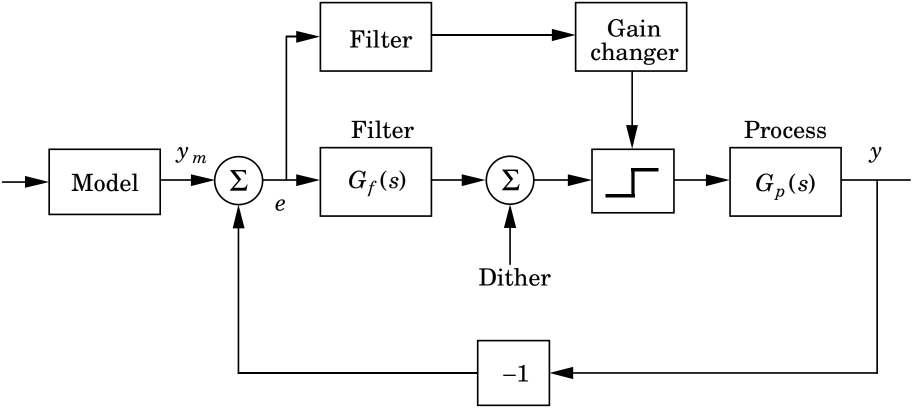
- Adaptive Cpmtrol: Learn enough about a process and its environment for control – restricted domain, prior info
- Development similar to neural networks
  - Many ups and downs
  - Lots of strong egos
- Early work driven adaptive flight control 1950-1970.
  - The brave era: Develop an idea, hack a system and fly it!
  - Several adaptive schemes emerged no analysi
  - Disasters in flight tests - the X-15 crash nov 15 1967
  - Gregory P. C. ed, Proc. Self Adaptive Flight Control Systems. Wright Patterson Airforce Base, 1959
- Emergence of adaptive theory 1970-1980
  - Model reference adaptive control emerged from flight control stability theory
  - The self tuning regulator emerged from process control and stochastic control theory
- Microprocessor based products 1980
- Robust adaptive control 1990
- L1-adaptive control - Flight control 2006

Publications in Scopus



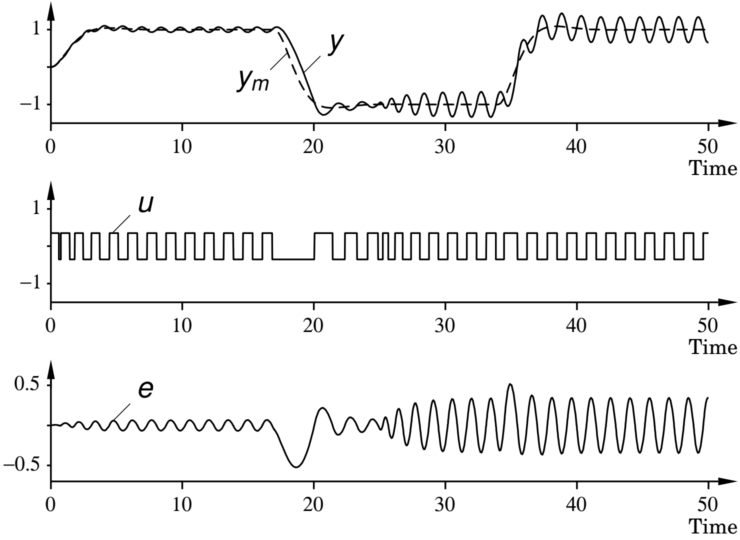
Blue control red adaptive control

The Self-Oscillating Adaptive System



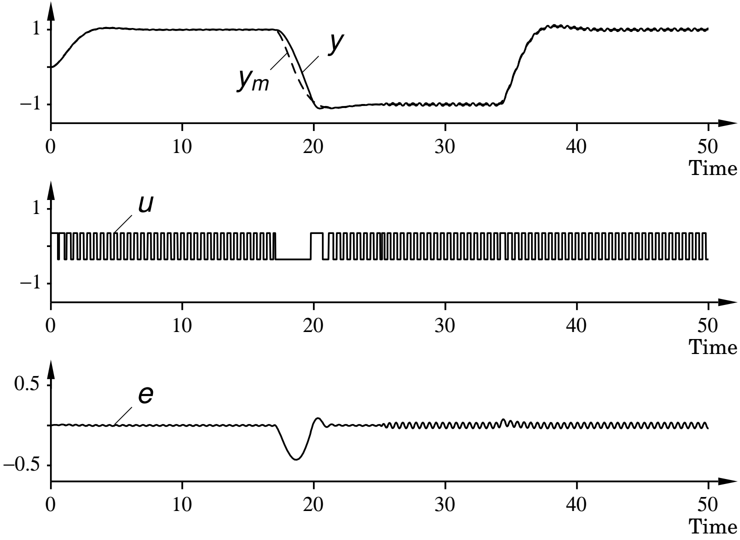
- Oscillation at high frequency governed by relay and filter
- Automatically adjusts to gain margin $g_m = 2$!
- Dual input describing functions

SOAS Simulation 1



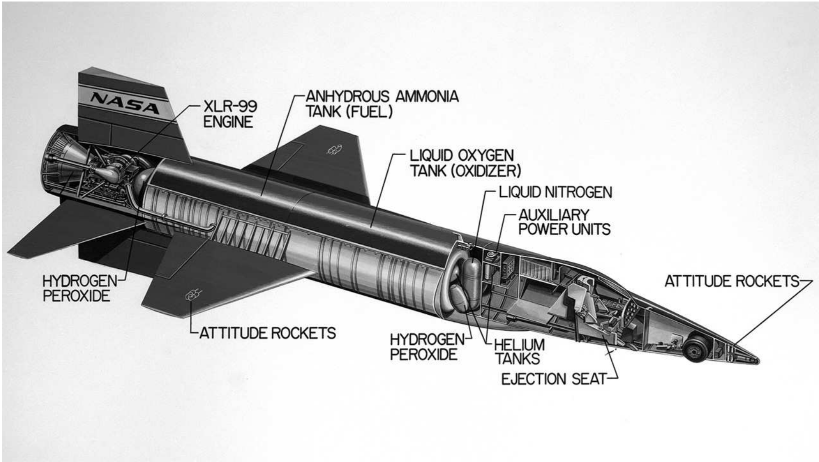
Gain increases by a factor of 5 at time $t = 25$

SOAS Simulation 2



Gain increases by a factor of 5 at time $t = 25$

The X-15 Crash Nov 11 1967



Adaptive Control - A Perspective

1. Introduction
2. Model Reference Adaptive Control
   - The MIT rule -sensitivity derivatives
   - Direct MARS - update parameters of a process model
   - Indirect MRAS - update controller parameters directly
   - L1 adaptive control - avoid dividing with estimates
3. Self-Tuning Regulators
4. Dual Control
5. Summary

MRAS - The MIT Rule

Process

$$\frac{dy}{dt} = -ay + bu$$

Model

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c$$

Controller

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t)$$

Ideal controller parameters

$$\theta_1 = \theta_1^0 = \frac{b_m}{b}$$
$$\theta_2 = \theta_2^0 = \frac{a_m - a}{b}$$

Find a feedback that changes the controller parameters so that the closed loop response is equal to the desired model

MRAS - The MIT Rule

The error

$$e = y - y_m, \qquad y = \frac{b\theta_1}{p + a + b\theta_2} u_c \qquad p = \frac{dx}{dt}$$

$$\frac{\partial e}{\partial \theta_1} = \frac{b}{p + a + b\theta_2} u_c$$

$$\frac{\partial e}{\partial \theta_2} = -\frac{b^2\theta_1}{(p + a + b\theta_2)^2} u_c = -\frac{b}{p + a + b\theta_2} y$$

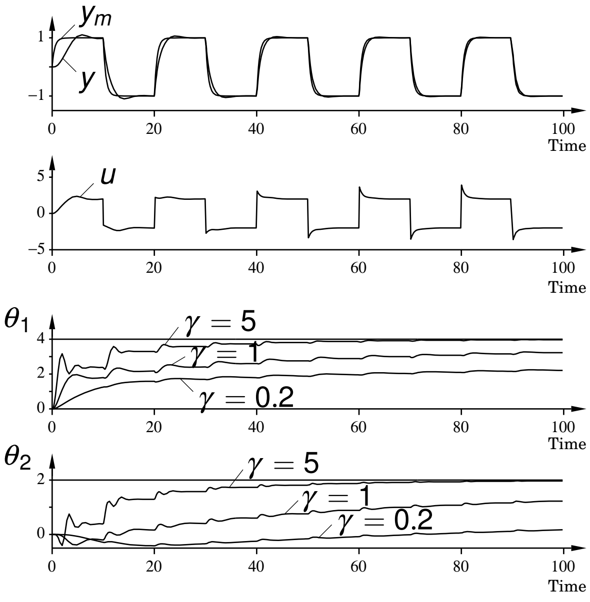
Approximate

$$p + a + b\theta_2 \approx p + a_m$$

The MIT rule: Minimize $e^2(t)$

$$\frac{d\theta_1}{dt} = -\gamma \left(\frac{a_m}{p + a_m} u_c \right) e, \qquad \frac{d\theta_2}{dt} = \gamma \left(\frac{a_m}{p + a_m} y \right) e$$

Simulation $a = 1, b = 0.5, a_m = b_m = 2.$



Adaptation Laws from Lyapunov Theory

Replace ad hoc with desings that give guaranteed stability

- Lyapunov function $V(x) > 0$ positive definite

$$\frac{dx}{dt} = f(x),$$
$$\frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} = \frac{DV}{dx} f(x) < 0$$

- Determine a controller structure
- Derive the Error Equation
- Find a Lyapunov function
- $\frac{dV}{dt} \leq 0$ Barbalat's lemma
- Determine an adaptation law

First Order System

Process model and desired behavior

$$\frac{dy}{dt} = -ay + bu, \qquad \frac{dy_m}{dt} = -a_m y_m + b_m u_c$$

Controller and error

$$u = \theta_1 u_c - \theta_2 y, \qquad e = y - y_m$$

Ideal parameters

$$\theta_1 = \frac{b}{b_m}, \qquad \theta_2 = \frac{a_m - a}{b}$$

The derivative of the error

$$\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)\, u_c$$

Candidate for Lyapunov function

$$V(e, \theta_1, \theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)^2\right)$$

Derivative of Lyapunov Function

$$V(e,\theta_1,\theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)^2\right)$$

Derivative of error and Lyapunov function

$$\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)u_c$$

$$\frac{dV}{dt} = e\frac{de}{dt} + \frac{1}{\gamma}(b\theta_2 + a - a_m)\frac{d\theta_2}{dt} + \frac{1}{\gamma}(b\theta_1 - b_m)\frac{d\theta_1}{dt}$$

$$= -a_m e^2 + \frac{1}{\gamma}(b\theta_2 + a - a_m)\left(\frac{d\theta_2}{dt} - \gamma ye\right)$$

$$+ \frac{1}{\gamma}(b\theta_1 - b_m)\left(\frac{d\theta_1}{dt} + \gamma u_c e\right)$$

Adaptation law

$$\frac{d\theta_1}{dt} = -\gamma u_c e, \qquad \frac{d\theta_2}{dt} = \gamma ye \Rightarrow \frac{de}{dt} = -e^2$$

Error will always go to zero, what about parameters, Barbara's lemma!

Indirect MRAS - Estimate Process Model

Process and estimator

$$\frac{dx}{dt} = ax + bu, \qquad \frac{d\hat{x}}{dt} = \hat{a}\hat{x} + \hat{b}u$$

Nominal controller gains:
$k_x = k_x^0 = (a - a_m)/b, \quad k_r = k_r^0 = b_m/b.$
Estimation error $e = \hat{x} - x$ has the derivative

$$\frac{de}{dt} = \hat{a}x + \hat{b}u - ax - bu = ae + (\hat{a} - a)\hat{x} + (\hat{b} - b)u = ae + \tilde{a}\hat{x} + \tilde{b}u,$$

where $\tilde{a} = \hat{a} - a$ and $\tilde{b} = \hat{b} - a$. Lyapunov function

$$2V = e^2 + \frac{1}{\gamma}\left(\tilde{a}^2 + \tilde{b}^2\right).$$

Its derivative becomes

$$\frac{dV}{dt} = e\frac{de}{dt} + \frac{1}{\gamma}\left(\tilde{a}\frac{d\hat{a}}{dt} + \tilde{b}\frac{d\hat{b}}{dt}\right) = ae^2 + \left(e\hat{x} + \frac{1}{\gamma}\frac{d\tilde{a}}{dt}\right)\tilde{a} + \left(eu + \frac{1}{\gamma}\frac{d\tilde{b}}{dt}\right)\tilde{b}$$

L1 Adaptive Control - Hovkimian and Cao 2006

Replace

$$u = -\frac{\hat{a} - a_m}{\hat{b}} x + \frac{b_m}{\hat{b}} r$$
$$\hat{b} u + (\hat{a} - a_m) x - b_m r = 0$$

with the differential equation

$$\frac{du}{dt} = K\left(b_m r - (\hat{a} - a_m) x - \hat{b} u\right)$$

Avoid division by $\hat{b}$, can loosely speaking be interpreted as sending the signal $\hat{b}_m r + (a_m - \hat{a}) x$ through a filter with the transfer function

$$G(s) = \frac{K}{s + K\hat{b}}$$

Adaptive Control - A Perspective

Steady State Regulation

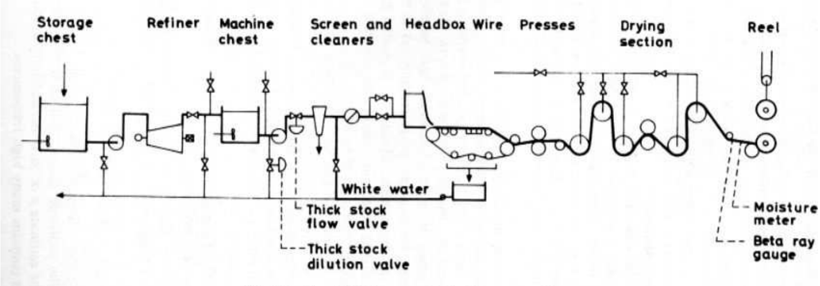
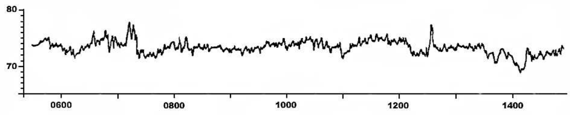
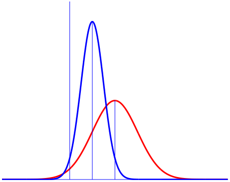


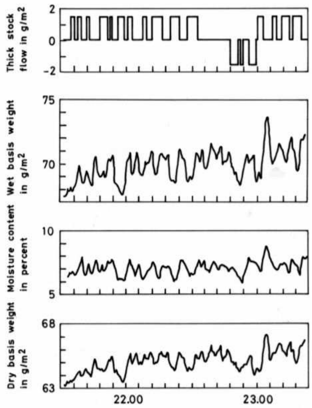
Fig. 6.9. Simplified diagram of a kraft paper machine.





Modeling from Data (Identification)

- Experiments in normal production
- To perturb or not to perturb
- Open or closed loop?
- Maximum Likelihood Method
- Model validation
- 20 min for two-pass compilation of Fortran program!
- Control design
- Skills and experiences



KJÅ and T. Bohlin, Numerical Identification of Linear Dynamic Systems from Normal Operating Records. In Hammond, *Theory of Self-Adaptive Control Systems, Plenum Press, January 1966.*

Minimum Variance Control

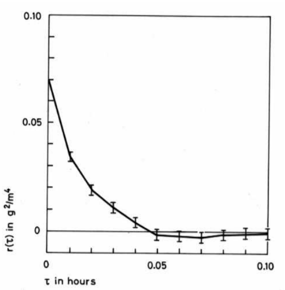
Process model

$$y_t + a_1 y_{t-1} + ... = b_1 u_{t-k} + ... + e_t + c_1 e_{t-1} + ...$$

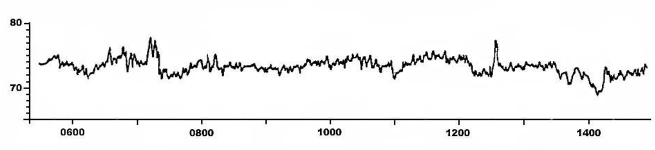
$$Ay_t = Bu_{t-k} + Ce_t$$

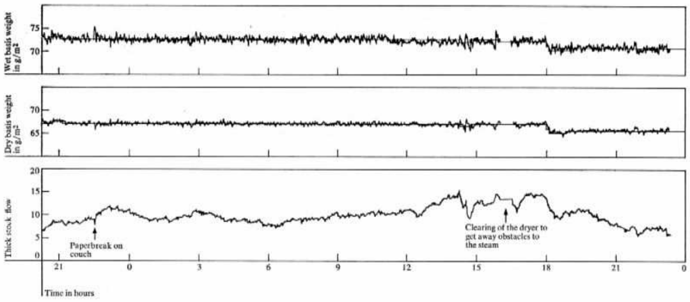
- Ordinary differential equation with time delay
- Disturbances are statinary stochastic process with rational spectra
- The predition horizon: tru delay and one samling period
- Control law $Ru = -Sy$
- Output becomes a moving averate of white noise $y_{t+k} = Fe_t$
- Robustness and tuning

The output is a moving average $y_{t+j} = Fe_t$, which is easy to validate!



Experiments





KJÅ Computer Control of a Paper Machine : An Application of Linear Stochastic Control Theory. IBM J of Research and Development, **11:4**, pp. 389–405, 1967.

Can we find an adaptive regulator that regulates as well?

The Self-Tuning Regulator STR

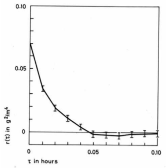
Process model, estimation model and control law

$$y_t + a_1 y_{t-1} + \cdots + a_n y_{t-n} = b_0 u_{t-k} + \cdots o b_m u_{t-n}$$
$$+ e_t + c_1 e_{t-1} + \cdots + c_n e_{t-n}$$
$$y_{t+k} = s_0 y_t + s_1 y_{t-1} + \cdots + s_m y_{t-m} + r_0 (u_t + r_1 u_{t-1} + \cdots r_n u_{t-\ell})$$
$$u_t + \hat{r}_1 u_{t-1} + \cdots \hat{r}_n u_{t-\ell} = -(\hat{s}_0 y_t + \hat{s}_1 y_{t-1} + \cdots + \hat{s}_m y_{t-m})/r_0$$

If estimate converge and $0.5 < r_0/b_0 < \infty$

$$r_y(\tau) = 0, \tau = k, k+1, \cdots k+m+1$$
$$r_{yu}(\tau) = 0, \tau = k, k+1, \cdots k+\ell$$

If degrees sufficiently large $r_y(\tau) = 0, \forall \tau \geq k$



- The self-tuning regulator (STR) automates identification and minimum variance control in about 35 lines of code.
- Easy to check if minimum variance control is achieved!
- A controller that drives covariances to zero

KJÅ and B. Wittenmark On Self-Tuning Regulators, Automatica **9** (1973),185-199

Convergence Analysis

Process model $Ay = Bu + Ce$

$$y_t + a_1 y_{t-1} + \cdots + a_n y_{t-n} = b_0 u_{t-k} + \cdots b_m u_{t-n}$$
$$+ e_t + c_1 e_{t-1} + \cdots + c_n e_{t-n}$$

Estimation model

$$y_{t+k} = s_0 y_t + s_1 y_{t-1} + \cdots + s_m y_{t-m} + r_0 (u_t + r_1 u_{t-1} + \cdots r_n u_{t-\ell})$$

Theorem: Assume that

- Time delay k of the sampled systemis known
- Upper bounds of the degrees of A, B and C are known
- Polynomial B has all its zeros inside the unit disc
- Sign of b_0 is known

The the sequences u_t and y_t are bounded and the parameters converge to the minimum variance controller

G. C. Goodwin, P. J. Ramage, P. E. Caines, Discrete-time multivariable adaptive control. IEEE **AC-25** 1980, 449–456

Convergence Analysis

Markov processes and differential equations

$$dx = f(x)dt + g(x)dw, \qquad \frac{\partial p}{\partial t} = -\frac{\partial p}{\partial x}\left(\frac{\partial fp}{\partial ix}\right) + \frac{1}{2}\frac{\partial^2}{\partial x^2}g^2 f = 0$$

$$\theta_{t+1} = \theta_t + \gamma_t \varphi e, \qquad \frac{d\theta}{d\tau} = f(\theta) = E\varphi e$$

Method for convergence of recursive algorithms. Global stability of STR ($Ay = Bu + Ce$) if $G(z) = 1/C(z) - 0.5$ is SPR

L. Ljung, Analysis of Recursive Stochastic Algorithms IEEE Trans **AC-22** (1967) 551–575.

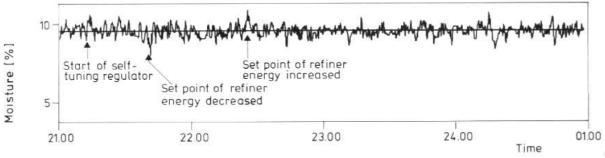
Converges locally if $\Re C(z_k) > 0$ for all z_k such that $B(z_k) = 0$

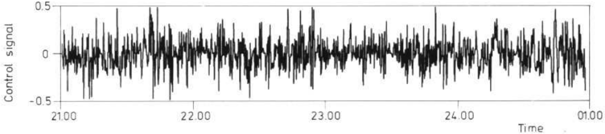
Jan Holst, Local Convergence of Some Recursive Stochastic Algorithms. 5th IFAC Symposium on Identification and System Parameter Estimation, 1979

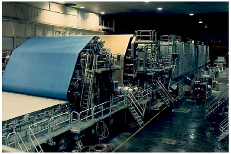
General convergence conditions

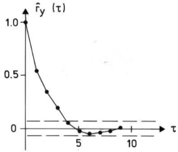
Lei Gui and Han-Fu Chen, The Åström-Wittenmbark Self-tuning Regulator Revisited and ELS-Based Adaptive Trackers. IEEE Trans **AC36:7** 802–812.

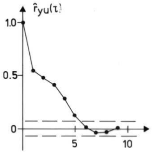
Paper Machine Control











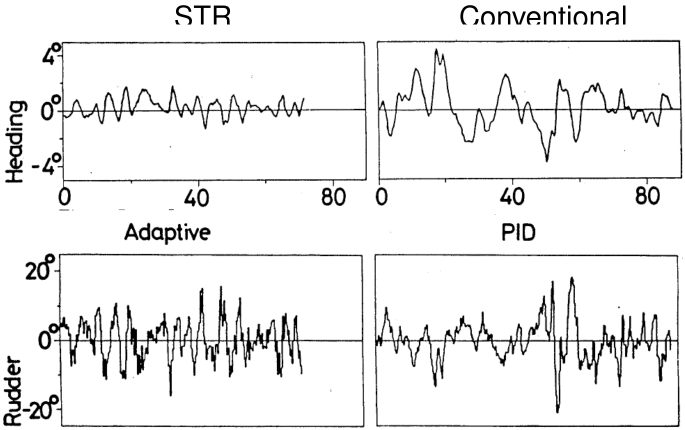
U. Borisson and B. Wittenmark An Industrial Application of a Self-Tuning Regulator, 4th IFAC/IFIP Symposium on Digital Computer Applications to Process Control 1974

Steermaster

- Ship dynamics
- SSPA Kockums
- Full scale tests on ships in operation

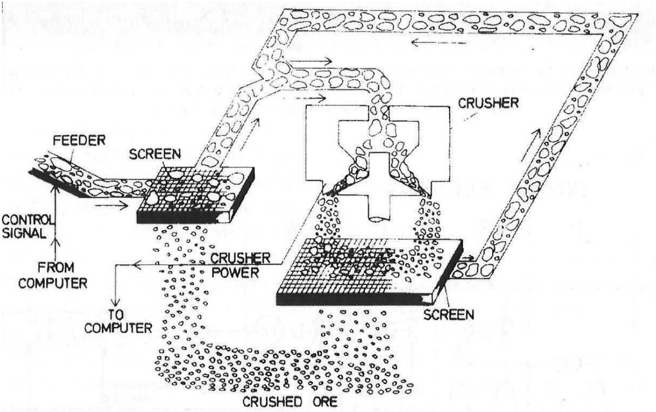
NORTHROP GRUMMAN

Ship Steering - Performance



C. Källström, KJÅ, N. E. Thorell, J. Eriksson, L. Sten, Adaptive Autopilots for Tankers, Automatica, **15** 1979, 241-254

Control of Orecrusher 1973



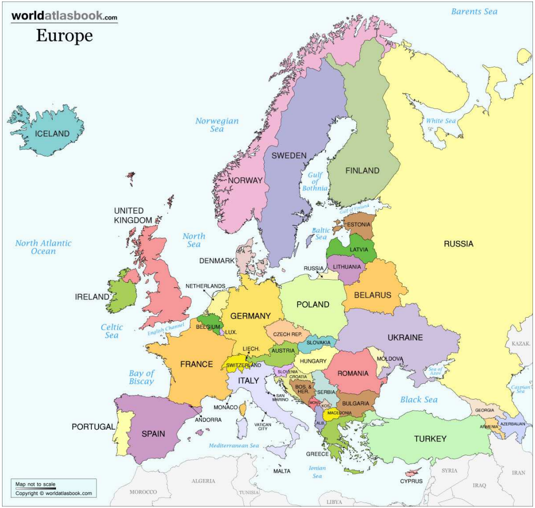
Forget Physics! - Hope an STR can work!

Power increased from 170 kW to 200 kW

U. Borisson, and R. Syding, Self-Tuning Control of an Ore Crusher, Automatica 1976, **12:1**, 1–7

Control of Orecrusher 1973

Distance Lund-Kiruna 1400 km, home made modem, supervision over phone, sampling period 20s.



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Dual Control

A. A. Feldbaum
Control should be probing as well as directing

Dual control theory I A. A. Feldbaum Avtomat. i Telemekh., 1960, 21:9, 1240–1249

Dual control theory II A. A. Feldbaum Avtomat. i Telemekh., 1960, 21:11, 1453–1464

R. E. Bellman Dynamic Programming Academic Press 1957

Stochastic control theory - Adaptive control

Decisionmaking under uncertainty - Economics

Optimization Hamilton Jacobi Bellman

Curse of dimensionality - Bellman

The Problem

Consider the system

$$y_{t+1} = y_t + bu_t + e_{t+1}$$

where e_t is a sequence of independent normal $(0, \sigma^2)$ random variables and b a constant but unknown parameter with a normal $\hat{b}, P(0)$ prior or a random wai.
Find a control llaw such that u_t based on the information available at time t

$$\mathcal{X}_t = y_t, y_{t-1}, \ldots, y_0, u_{t-1}, u_{t-2}, \ldots, u_0,$$

that minimizes the cost function

$$V = E \sum_{k=1}^{T} y^2(k).$$

KJÅ and A. Helmersson. *Dual Control of an Integrator with Unkown Gain*, Computers and Mathematics with Applications 12:6A, pp 653–662, 1986.

The Hamilton-Jakobi-Bellman Equation

The solution to the problem is given by the Bellman equation

$$V_t(\mathcal{X}_t) = E_{\mathcal{X}_t} \min_{u_t} E\left(y_{t+1}^2 + V_{t+1}(\mathcal{X}_{t+1}) \Big| \mathcal{X}_t\right)$$

The state is $\mathcal{X}_t = y_t, y_{t-1}, y_{t-2}, \ldots, y_0, u_{t-1}, u_{t-2}, \ldots, u_0$. The derivation is general applies also to

$$\begin{aligned} x_{t+1} &= f(x_t, u_t, e_t) \\ y_t &= g(x_t, u_t, v_t) \\ \min E &\sum q(x_1, u_t) \end{aligned}$$

How to solve the optimization problem?
The curse of dimensionality: X_t has high dimension

A Sufficient Statistic - Hyperstate

It can be shown that a sufficient statistic for estimating future outputs is y_t and the conditional distribution of b given $\mathcal{X}_t$. In our setting the conditional distribution is gaussian $N(\hat{b}_t, P_t)$

$$\hat{b}_t = E(b|\mathcal{X}_t), \qquad P_t = E[(\hat{b}_t - b)^2|\mathcal{X}_t]$$

$$\hat{b}_{t+1} = \hat{b}_t + K_t[y_{t+1} - y_t - \hat{b}_t u_t] = \hat{b}_t + K_t e_{t+1}$$

$$K_t = \frac{u_t P_t}{\sigma^2 + u_t^2 P_t}$$

$$P_{t+1} = [1 - K_t u_t] P_t = \frac{\sigma^2 P_t}{\sigma^2 + u_t^2 P_t}$$

In our particular case the conditional distrubution depens only on by y, $\hat{b}$ and P - a significant reduction of dimensionality!

The Bellman Equation

$$V_t(\mathcal{X}_t) = E_{\mathcal{X}_t} \min_{u_t} E\left(y_{t+1}^2 + V_{t+1}(\mathcal{X}_{t+1}) \middle| \mathcal{X}_t\right)$$

Use hyperstate to replace
$\mathcal{X}_t = y_t, y_{t-1}, y_{t-2}, \ldots, y_0, u_{t-1}, u_{t-2}, \ldots, u_0$ with $y_t, \hat{b}_t, P_t$.
Introduce

$$V_t(y_t, \hat{b}_t, P_t) = \min_{\mathcal{U}_t} \left(E \sum_{k=t+1}^{T} y_k^2 \middle| y_t, \hat{b}_t, P_t \right)$$

$$y_{t+1} = y_t + \hat{b}_t u_t + e_{t+1}, \quad \hat{b}_{t+1} = \hat{b}_t + K_t e_{t+1}, \quad P_{t+1} = \frac{\sigma^2 P_t}{\sigma^2 + u_t^2 P_t}$$

and the Bellman equation becomes

$$V_t(y, \hat{b}, P) = \min_{u} E\left(y_t^2 + V_{t+1}\left(y_{t+1}, \hat{b}_{t+1}, P_{t+1}\right) \middle| y, \hat{b}_t, P_t\right)$$

Short Time Horizon - 1 Step Ahead

Consider situation at time t and look one step ahead

$$V_{T-1}(y, \hat{b}, P) = \min_u E \sum_{k=T}^{T} y_k^2 = \min_u y_T^2$$

$$y_T = y_{T-1} + b u_{T-1} + e_T$$

We know y_t have an estimate $\hat{b}$ of b with covariance P

$$\begin{aligned} V_T(y, \hat{b}, P) &= \min_u E y_T^2 = \min_u \Big((y + \hat{b}u)^2 + u^2 P + \sigma^2 \Big) \\ &= \min_u \Big(y^2 + 2y\hat{b}u + u^2(\hat{b}^2 + P) + \sigma^2 \Big) = \sigma^2 + \frac{P y^2}{\hat{b}^2 + P} \end{aligned}$$

where minimum occurs for

$$u = -\frac{\hat{b}}{\hat{b}^2 + P} y \qquad \Rightarrow \qquad u = -\frac{1}{\hat{b}} y \quad \text{as} \quad P \to 0$$

These control laws are called cautious control and certainty equivalence control (Herbert Simon).

The Solution and Scaling

$$V_t(y, \hat{b}, P) = \min_u \Big((y + \hat{b}u)^2 + \sigma^2 + u^2 P + V_{t+1}\left(y_{t+1}, \hat{b}_{t+1}, P_{t+1}\right) \Big)$$

$$V_T(y, \hat{b}, P) = \sigma^2 + \frac{Py^2}{\hat{b}^2 + P}$$

Iterate backward in time. An important observation, $V_T(y, \hat{P}, P)$ does not depend on y, state is thus two-dimensional!!

Scaling

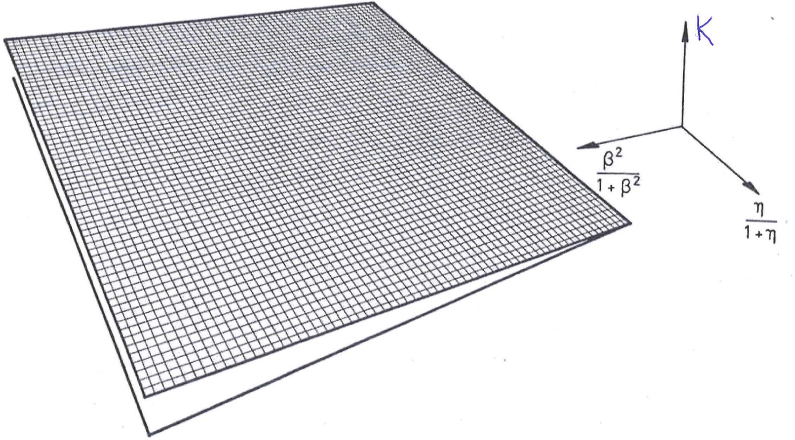
$$\eta = \frac{y}{\sigma}. \qquad \beta = \frac{\hat{b}}{\sqrt{P}}, \qquad \mu = \frac{u\sqrt{P}}{\sigma}$$

Introduce

Two functions: the value function and the policy function

Controller Gain - Cautious Control

$$u = -\frac{\hat{b}}{\hat{b}^2 + P} y = Ky, \eta = \frac{y}{\sigma}. \qquad \beta = \frac{\hat{b}}{\sqrt{P}},$$



Solving the Bellman Equation Numerically

The scaled Bellman equation

$$W_t(\eta, \beta) = \min_{\mu} U_t(\eta, \beta, \mu), \qquad \varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

where

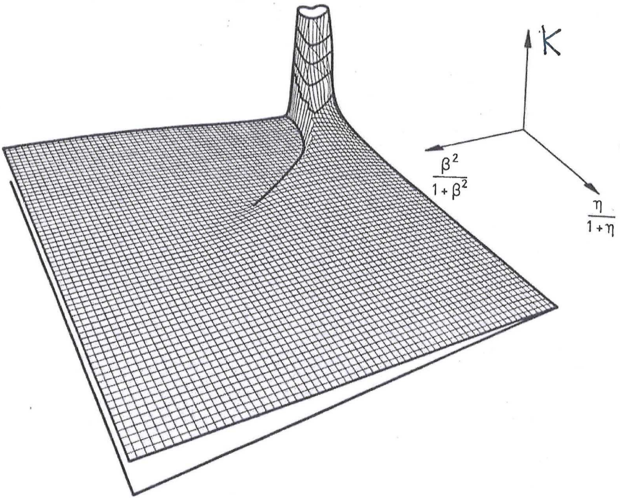
$$U_t(\eta, \beta, \mu) = (\eta + \beta\mu)^2 + 1 + \mu^2 + \int_{-\infty}^{\infty} \Big(W_{t+1}(\eta + \beta\mu + \epsilon\sqrt{1+\mu^2}, \beta\sqrt{1+\mu^2} + \mu\epsilon \Big) \varphi(\epsilon) d\epsilon$$

Solving minimization gives control law $\mu = \Pi(\eta, \beta)$, $\mu = \frac{u\sqrt{P}}{\sigma}$, $u = \frac{\sigma}{\sqrt{P}\,\Pi(\eta,\beta)}$

Numerics:

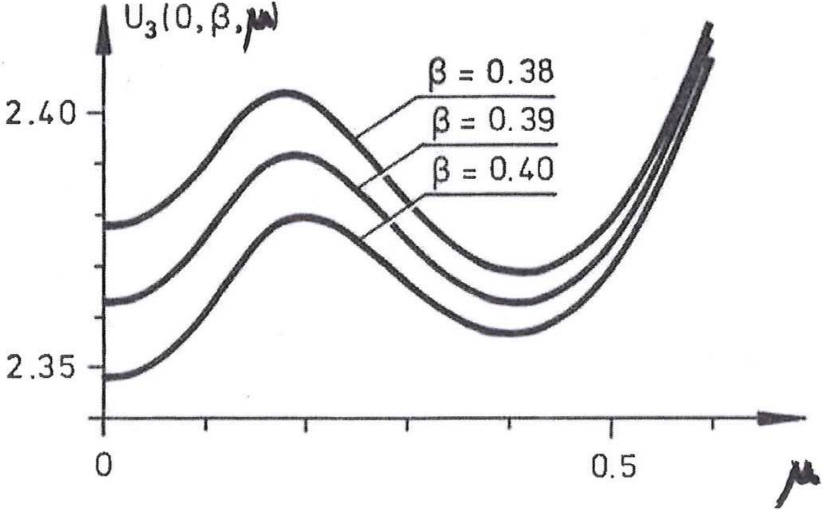
- Transform to the interval (0 1), quantize U function 128×128
- Store the a gridded version of the function $U(\eta, \beta, mu)$
- Evaluate the function $W(\eta, \beta, \mu)$ by extrapolation, and numeric integration
- Minimize $W(\eta, \beta, \mu)$ with respet to μ

Controller Gain - 3 Steps



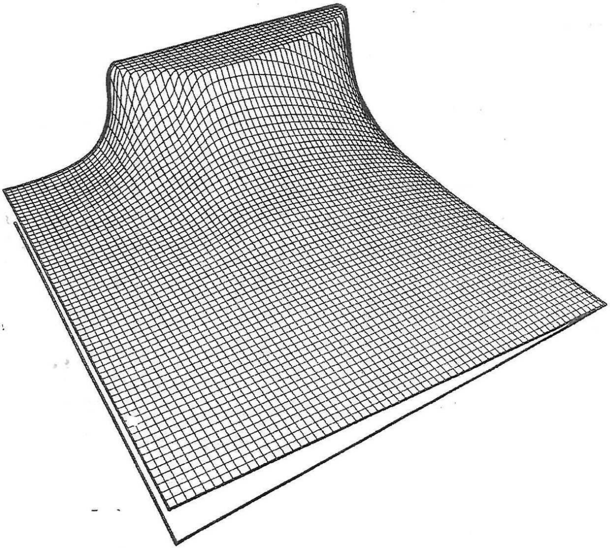
$K(\eta, \beta)$ larger than 3 not shown

Understanding Probing

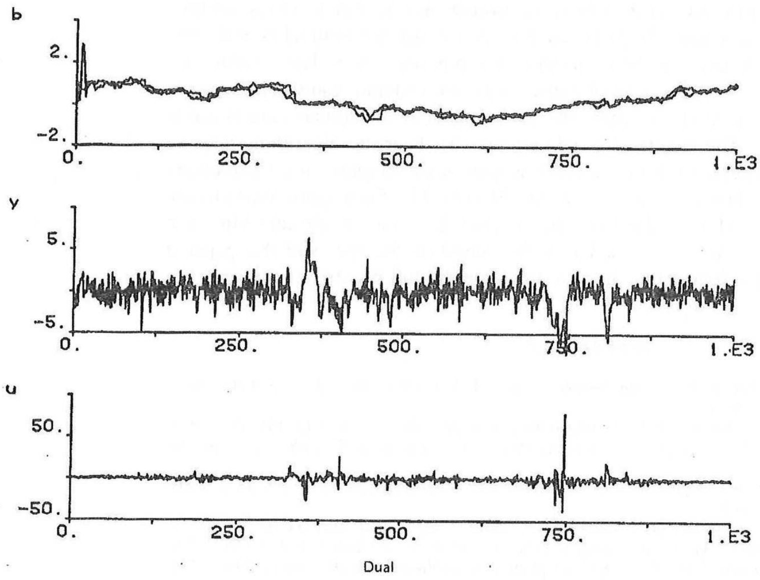


Notice jump!!

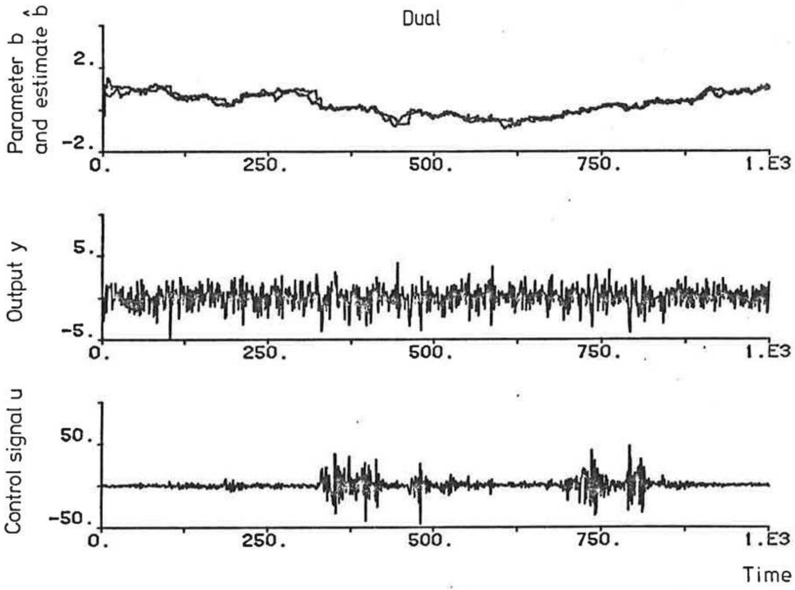
Controller gain for 30 Steps



Cautious Control - Drifting Parameters

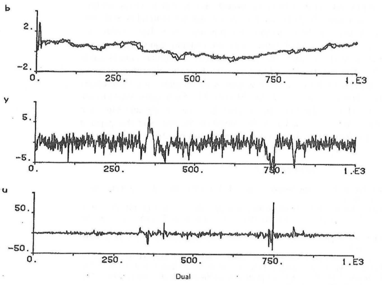


Dual Control - Drifting Parameters

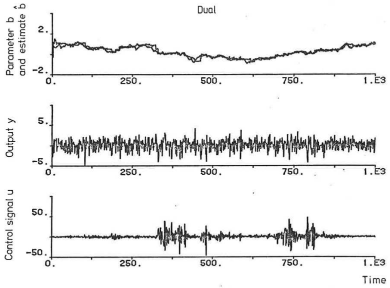


Comparison

Cautious Control



Dual Control



Adaptive Control - A Perspective

1. Introduction
2. Model Reference Adaptive Control
3. Self-Tuning Regulators
4. Dual Control
5. Summary

Summary

- A glimpse of an interesting and useful field of control
- Nonlinear and not trivial to analyse and design
- A turbulent history
- Now reasonably well understood
- A number of successful industrial applications
- Cnnections to learning
  - Dual control and probing - can we learn when to probe?
  - Representation of functions of many variables a key
  - Can neural be used to avoid curse of dimensionality?
- Many issues not covered
  - Identificaton in closed loop
  - The need for excitation
  - Robustness
  - Relay auto-tuning of PID controllers $> 10^5$ controllers

KJÅ and B. Wittenmark. Adaptive Control. Second Edition. Dover 2008.