Multi-Level Condition-Based Maintenance Planning for Railway Infrastructures

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Outline

1. Background
2. Multilevel Maintenance Planning
   - High-Level Problem
   - Low-Level Problem
3. Case Study
4. Conclusions & Future Work

1 Background

2 Multilevel Maintenance Planning
High-Level Problem
Low-Level Problem

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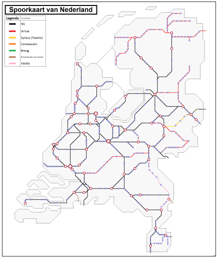
4 Conclusions & Future Work

Dutch Railway Network



Overview

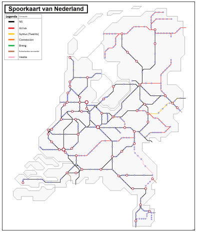
- One of the most intensive railway networks in Europe
- 6830 km of track, 388 stations, 7508 switches, 4500 km catenary
- Maintenance managed by ProRail, performed by contractors

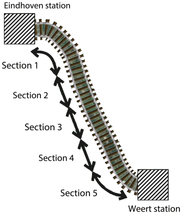
Track Defects & Interventions

Squats & Grinding

Ballast Defects& Tamping

Optimal Maintenance Planning





Preliminaries

- A railway network is divided into multiple sections
- Deterioration dynamics of each section is stochastic & independent

Performance Criteria

- Cost-efficiency
- Robustness
- Scalability

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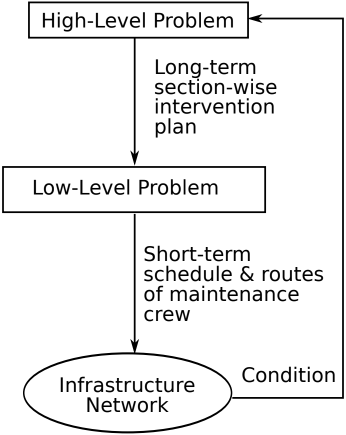
Multilevel Scheme for Maintenance Planning

Motivation

- Computational tractability
- Different time scales



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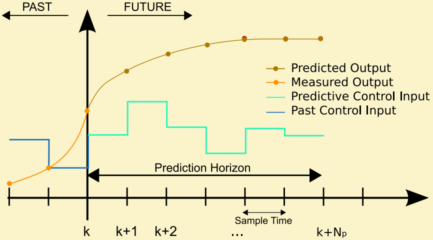
High-level Intervention Planning

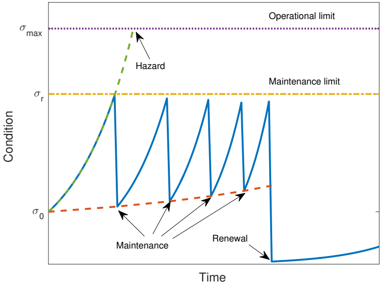
Problem Description

- Minimize expected condition deterioration & maintenance cost
- Subject to:
  - Safety constraints
  - Resource constraints

Model-Predictive Control (MPC)



Deterioration Dynamics



Maintenance

- Cannot restore to perfect condition
- Becomes less effective the more it is applied

Renewal

- "As good as new" condition
- Expensive

Deterioration Model

Notations

$x_{j,k} = \begin{bmatrix} x_{j,k}^{\mathrm{con}} & \text{Condition} \\ x_{j,k}^{\mathrm{aux}} & \text{"Memory"} \end{bmatrix}$: state

$u_{j,k} \in \{\underbrace{a_1,}_{\text{No maintenance}} \underbrace{a_2 \dots, a_N}_{\text{Interventions}}\}$: N maintenance options

$\theta_{j,k} \in \Theta_j$: **bounded** uncertain parameters with unknown probability distribution

Stochastic Deterioration Model

$$x_{j,k+1} = f_j(x_{j,k},\, u_{j,k},\, \theta_{j,k})$$
$$= \begin{cases} f_j^1(x_{j,k},\, \theta_{j,k}) & \text{if } u_{j,k} = 1 \text{ Natural degradation} \\ f_j^q(x_{j,k},\, \theta_{j,k}) & \text{if } u_{j,k} = q \text{ Effect of maintenance } \forall q \in \{2, \dots, N-1\} \\ f_j^N(\theta_{j,k}) & \text{if } u_{j,k} = N \text{ Effect of renewal} \end{cases}$$

Stochastic Local MPC Problem

Chance-Constrained MPC Problem

$$\min_{\tilde{u}_{j,k},\, \tilde{x}_{j,k}} \mathbb{E}_{\tilde{\theta}_{j,k}}[J_j(\tilde{x}_{j,k},\, \tilde{u}_{j,k},\, \tilde{\theta}_{j,k})]$$

subject to: $\tilde{x}_{j,k} = \tilde{f}_j(\tilde{u}_{j,k},\, \tilde{\theta}_{j,k};\, x_{j,k})$

$$\mathbb{P}_{\tilde{\theta}_{j,k}} \left[\max_{l=1,\ldots,N_{\mathrm{P}}} \hat{x}^{\mathrm{con}}_{j,k+l|k}(\tilde{u}_{j,k},\, \tilde{\theta}_{j,k};\, x_{j,k}) \le x_{\max} \right] \ge 1 - \epsilon_j$$ Chance Constraint

where ϵ_j is the violation level

Robust MPC Problem

$$\min_{\tilde{u}_{j,k},\, \tilde{x}_{j,k}} \max_{\tilde{\theta}_{j,k} \in \tilde{\Theta}_j} J_j(\tilde{x}_{j,k},\, \tilde{u}_{j,k},\, \tilde{\theta}_{j,k})$$

subject to: $\tilde{x}_{j,k} = \tilde{f}_j(\tilde{u}_{j,k},\, \tilde{\theta}_{j,k};\, x_{j,k})$

$$\max_{l=1,\ldots,N_{\mathrm{P}}} \max_{\tilde{\theta}_{j,k} \in \tilde{\Theta}_j} \hat{x}^{\mathrm{con}}_{j,k+l|k}(\tilde{u}_{j,k},\, \tilde{\theta}_{j,k};\, x_{j,k}) \le x_{\max}$$ Robust Constraint

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where ϵ_j is the violation level

Robust MPC Problem

$$\min_{\tilde{u}_{j,k},\,\tilde{x}_{j,k}} \max_{\tilde{\theta}_{j,k}\in\tilde{\Theta}_j} J_j(\tilde{x}_{j,k},\, \tilde{u}_{j,k},\, \tilde{\theta}_{j,k})$$

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$$\max_{l=1,\ldots,N_{\mathrm{P}}} \max_{\tilde{\theta}_{j,k}\in\tilde{\Theta}_j} \hat{x}^{\mathrm{con}}_{j,k+l|k}(\tilde{u}_{j,k},\, \tilde{\theta}_{j,k};\, x_{j,k}) \leq x_{\max} \quad \text{Robust Constraint}$$

We choose chance-constrained MPC in order to avoid conservatism

Scenario-based Approach

Scenario-based Approach

Approximate chance constraint by set of **deterministic constraints**:

$$\max_{l=1,\ldots,N_{\mathrm{P}}} \hat{x}^{\mathrm{con}}_{j,k+l|k}(\tilde{u}_{j,k},\, \tilde{\theta}^{(h)}_{j,k};\, x_{j,k}) \le x_{\max} \quad \forall h \in \mathcal{H}_j$$

$\mathcal{H}_j$: set of random scenarios for section j; $\tilde{\theta}^{(h)}_{j,k}$: realization of $\tilde{\theta}^{(h)}_{j,k}$ in scenario h

Remarks

- Sufficiently large $|\mathcal{H}_j|$ gives the **probabilistic guarantee**:

$$\mathbb{P}_h\left[\mathbb{P}_{\tilde{\theta}_{j,k}}\left[\max_{l=1,\ldots,N_{\mathrm{P}}} \hat{x}^{\mathrm{con}}_{j,k+l|k}(\tilde{u}_{j,k},\, \tilde{\theta}_{j,k};\, x_{j,k}) \le x_{\max}\right] \ge 1-\epsilon_j\right] \ge 1-\beta_j$$

- Integer decision variables → non-convex chance-constrained MPC problem
- Existing bounds on $|\mathcal{H}_j|$ for non-convex chance-constrained problem are conservative

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- Integer decision variables $\rightarrow$ non-convex chance-constrained MPC problem
- Existing bounds on $|\mathcal{H}_j|$ for non-convex chance-constrained problem are conservative

We choose the two-stage approach from Margellos et al. (2014)

Two-Stage Approach

Stage 1

Generate $\mathcal{H}_j$ satisfying

$$|\mathcal{H}_j| \geq \left\lceil \frac{1}{\epsilon_j} \cdot \frac{e}{e-1} \left(2|\tilde{\Theta}_j| - 1 + \ln \frac{1}{\beta_j} \right) \right\rceil$$

and solve the convex **scenario-based optimization problem**

$$\min_{\{(\underline{\tau}_i, \overline{\tau}_i)\}_{i=1}^{|\tilde{\Theta}_j|}} \sum_{i=1}^{|\tilde{\Theta}_j|} \overline{\tau}_i - \underline{\tau}_i$$

$$\text{subject to: } (\tilde{\theta}_{j,k})_i^{(h)} \in [\underline{\tau}_i, \overline{\tau}_i] \quad \forall h \in \mathcal{H}, \forall i \in \{1, \ldots, |\tilde{\Theta}_j|\}$$

to obtain the smallest hyperbox $\mathcal{B}^*_{j,k}$ covering all scenarios in $\mathcal{H}_j$.

Probabilistic Guarantee

$$\mathbb{P}_h \left[\mathbb{P}_{\tilde{\theta}_{j,k}} \left[\tilde{\theta}_{j,k} \in \mathcal{B}^*_{j,k} \right] \geq 1 - \epsilon_j \right] \geq 1 - \beta_j$$

Two-Stage Approach

Stage 2

Solve the resulting **robust optimization problem**

$$\min_{\tilde{u}_{j,k},\, \tilde{x}^{(h)}_{j,k}} \frac{1}{|\mathcal{H}_j|} \sum_{h \in \mathcal{H}_j} J_j(\tilde{x}^{(h)}_{j,k},\, \tilde{u}_{j,k})$$

$$\text{subject to:} \quad \max_{l=1,\ldots,N_{\mathrm{P}}} \; \max_{\tilde{\theta}_{j,k} \in \mathcal{B}^*_{j,k} \cap \tilde{\Theta}_j} \hat{x}^{\mathrm{con}}_{j,k+l|k}(\tilde{u}_{j,k},\, \tilde{\theta}_{j,k};\, x_{j,k}) \leq x_{\max} \quad \text{Robust Constraint}$$

$$\tilde{x}^{(h)}_{j,k} = \tilde{f}_j(\tilde{u}_{j,k},\, \tilde{\theta}^{(h)}_{j,k};\, x_{j,k}) \quad \forall h \in \mathcal{H}_j$$

Remarks

- Less conservative than standard robust approach
- Tractability depends on the robust problem

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Worst-Case Scenario

Worst-Case Scenario

- Define the worst-case scenario
$$\tilde{\theta}^{(w)}_{j,k} \in \underset{\tilde{\theta}_{j,k} \in \mathcal{B}^*_{j,k} \cap \tilde{\Theta}_j}{\arg\max} \max_{l=1\ldots N_{\mathrm{P}}} \hat{x}^{\mathrm{con}}_{j,k+l|k}(\tilde{u}_{j,k}, \tilde{\theta}_{j,k}; x_{j,k})$$
- $\tilde{\theta}^{(w)}_{j,k} i$ is easy to obtain if $\hat{x}^{\mathrm{con}}_{j,k+l|k}$ is concave w.r.t. $\tilde{\theta}_{j,k}$.

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Worst-Case Scenario

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$$\tilde{\theta}^{(w)}_{j,k} \in \arg\max_{\tilde{\theta}_{j,k} \in \mathcal{B}^*_{j,k} \cap \tilde{\Theta}_j} \max_{l=1\ldots N_{\mathrm{P}}} \hat{x}^{\mathrm{con}}_{j,k+l|k}(\tilde{u}_{j,k}, \tilde{\theta}_{j,k}; x_{j,k})$$

- $\tilde{\theta}^{(w)}_{j,k} i$ is easy to obtain if $\hat{x}^{\mathrm{con}}_{j,k+l|k}$ is concave w.r.t. $\tilde{\theta}_{j,k}$.

Sufficient Condition on Concavity

For $f_j(x_{j,k}, u_{j,k}, \theta_{j,k}) = \begin{bmatrix} f^{\mathrm{con}}_j(x_{j,k}, u_{j,k}, \theta_{j,k}) \\ f^{\mathrm{aux}}_j(x_{j,k}, u_{j,k}, \theta_{j,k}) \end{bmatrix}$, if f^{con}_j and f^{aux}_j are

- concave in $\tilde{\theta}_{j,k}$
- concave and non-decreasing in every dimension of $x_{j,k}$

then $\hat{x}^{\mathrm{con}}_{j,k+l|k}$ is concave in $\tilde{\theta}_{j,k}$ for any $l = 1, \ldots, N_{\mathrm{P}}$.

Scenario-based Robust MPC

Deterministic MPC Problem

$$\min_{\tilde{u}_{j,k},\, \tilde{x}_{j,k}^{(h)}} \frac{1}{|\mathcal{H}_j|} \sum_{h \in \mathcal{H}_j} J_j(\tilde{x}_{j,k}^{(h)},\, \tilde{u}_{j,k})$$

$$\text{subject to: } P_j \tilde{x}_{j,k}^{(w)} \leq x_{\max} \quad \forall w \in \mathcal{W}_j$$

$$\underbrace{\tilde{x}_{j,k}^{(s)} = \tilde{f}_j(\tilde{u}_{j,k},\, \tilde{\theta}_{j,k}^{(s)};\, x_{j,k})}_{\textbf{Deterministic}\text{ prediction model for scenario } s} \qquad \forall s \in \mathcal{H}_j \cup \{\tilde{\theta}_{j,k}^{(w)}\}$$

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Scenario-based Robust MPC

Deterministic MPC Problem

$$\min_{\tilde{u}_{j,k},\, \tilde{x}^{(h)}_{j,k}} \frac{1}{|\mathcal{H}_j|} \sum_{h \in \mathcal{H}_j} J_j(\tilde{x}^{(h)}_{j,k},\, \tilde{u}_{j,k})$$

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Remark

- Original stochastic dynamics is replaced by a set of deterministic dynamics
- Each deterministic dynamics follows a distinctive sequence of realizations of uncertainties

We still need to deal with hybrid dynamics

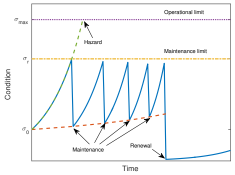
Frameworks for Hybrid MPC

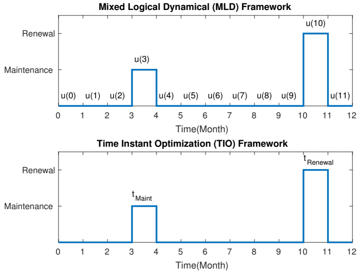
MLD-MPC

- Optimizes a sequence of **discrete** control inputs
- Mixed integer programming problem

TIO-MPC

- Optimizes **continuous** time instants at which each intervention takes place
- Time instants rounded to nearest steps
- Non-smooth continuous optimization problem





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Centralized MPC Problem

Centralized MLD-MPC Problem

$$\min_{\tilde{\delta}_k, \tilde{z}_k} \sum_{j=1}^{n} c_{j,1}^{\mathrm{T}} \tilde{\delta}_{j,k} + c_{j,2}^{\mathrm{T}} \tilde{z}_{j,k}$$ Summation of local objective functions

subject to: $$\sum_{j=1}^{n} R_j \tilde{\delta}_{j,k} \leq r$$ Global linear constraints on resources

$$F_{j,1}\tilde{\delta}_{j,k} + F_{j,2}\tilde{z}_{j,k} \leq l_j \quad \forall j \in \{1, \ldots, n\}$$ Local constraints

$$\tilde{\delta}_k \in \bigtimes_{j=1}^{n} \{0, 1\}^{n_{\tilde{\delta}_j}}$$ Binary variables

$$\tilde{z}_k \in \bigtimes_{j=1}^{n} \tilde{\mathcal{Z}}_j \subset \bigtimes_{j=1}^{n} \mathbb{R}^{n_{\tilde{z}_j}}$$ Continuous variables

Remark on Complexity

Scenario-based Deterioration Model

$$x_{j,k+1}^{(s)} = f_j(x_{j,k},\, u_{j,k},\, \theta_{j,k}^{(s)})$$
$$= \begin{cases} f_j^1(x_{j,k},\, \theta_{j,k}^{(s)}) & \text{if } u_{j,k} = 1 \\ f_j^q(x_{j,k},\, \theta_{j,k}^{(s)}) & \text{if } u_{j,k} = q \quad \forall q \in \{2, \dots, N-1\} \\ f_j^N(\theta_{j,k}^{(s)}) & \text{if } u_{j,k} = N \end{cases}$$

Size of Centralized MLD-MPC Problem

- Linear dynamics
  - # binary variables $\propto$ # sections
- Piecewise-affine dynamics
  - # binary variables $\propto$ # sections & # scenarios

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Distributed Optimization

Motivation

Centralized problem is intractable for large-scale networks with high-dimensional uncertainties

Distributed Optimization

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Decomposition Methods

- The centralized problem is only coupled by global constraints
- Dantzig-Wolfe Decomposition

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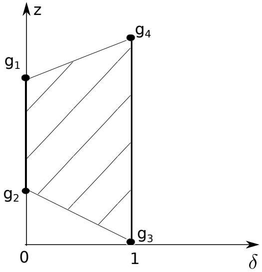
Dantzig-Wolfe Decomposition

Basic Idea

- Define $\mathcal{P}_{j,k} = \{(\tilde{\delta}_{j,k}, \tilde{z}_{j,k}) \in \{0, 1\}^{n_{\tilde{\delta}_j}} \times \tilde{\mathcal{Z}}_j : F_{j,1}\tilde{\delta}_{j,k} + F_{j,2}\tilde{z}_{j,k} \leq l_j\}$ as the local feasible region for section j
- Define generating set $\mathcal{G}_{j,k}$ containing extreme points (columns) of $\mathrm{Conv}(\mathcal{P}_{j,k})$
- (Minkowski's Theorem) Each point in $\mathrm{Conv}(\mathcal{P}_{j,k})$ can be written as a convex combination of columns $g \in \mathcal{G}_{j,k}$



Dantzig-Wolfe Reformulation

Dantzig-Wolfe Reformulation

$$\min_{\mu} \sum_{j=1}^{n} \sum_{g \in \mathcal{G}_{j,k}} (c_{j,1}\tilde{\delta}_{j,k}^{[g]} + c_{j,2}\tilde{z}_{j,k}^{[g]})\mu_{j,g}$$

subject to: $\sum_{j=1}^{n} \sum_{g \in \mathcal{G}_j} (R_j \tilde{\delta}_{j,k}^{[g]})\mu_{j,g} \leq r$ Global constraint

$\sum_{g \in \mathcal{G}_j} \mu_{j,g} = 1 \quad \forall j \in \{1, \dots, n\}$ Convexity constraints

$\mu_{j,g} \in \{0, 1\} \quad \forall g \in \mathcal{G}_{j,k}, \forall j \in \{1, \dots, n\}$ Binary condition

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$$\min_{\mu} \sum_{j=1}^{n} \sum_{g \in \mathcal{G}_{j,k}} (c_{j,1}\tilde{\delta}_{j,k}^{[g]} + c_{j,2}\tilde{z}_{j,k}^{[g]})\mu_{j,g}$$

subject to: $\sum_{j=1}^{n} \sum_{g \in \mathcal{G}_j} (R_j \tilde{\delta}_{j,k}^{[g]})\mu_{j,g} \leq r$ Global constraint

$\sum_{g \in \mathcal{G}_j} \mu_{j,g} = 1 \quad \forall j \in \{1, \ldots, n\}$ Convexity constraints

$\mu_{j,g} \in \{0, 1\} \quad \forall g \in \mathcal{G}_{j,k}, \forall j \in \{1, \ldots, n\}$ Binary condition

Remarks

- Reformulation is equivalent
- Master problem: linear relaxation of Dantzig-Wolfe reformulation
- Generating set $\mathcal{G}_{j,k}$ can be huge

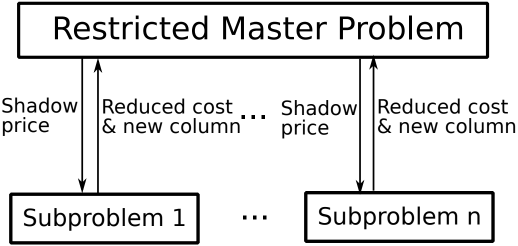
Column Generation

Restricted Master Problem

- Master problem with partial generating sets $\mathcal{G}^{s}_{j,k} \subset \mathcal{G}_{j,k}$
- Linear programming problem
- Its dual gives the shadow prices

Subproblem

- Pricing problem giving the most "attractive" column
- MILP
- Its optimum gives the reduced cost



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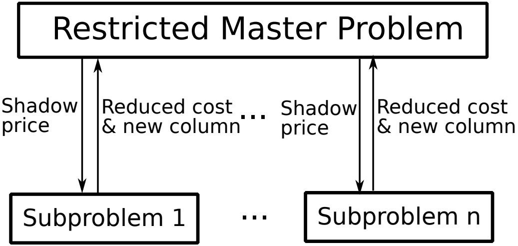

Column generation terminates when all reduced costs are 0

Column Generation

Bounds

- Upper & Lower bounds can be used to accelerate the procedure
- Binary solution of restricted master problem $\rightarrow$ upper bound
- Lagrangian dual function of centralized MPC problem $\rightarrow$ lower bound

Column Generation

Bounds

- Upper & Lower bounds can be used to accelerate the procedure
- Binary solution of restricted master problem $\rightarrow$ upper bound
- Lagrangian dual function of centralized MPC problem $\rightarrow$ lower bound

Solution Quality

- Upper bound =Lower bounds: exact solution of Dantzig-Wolfe reformulation
- Fractional solution with zero reduced costs
  - Solve restricted master problem as an integer programming problem with resulting partial generating sets
  - Suboptimal solution

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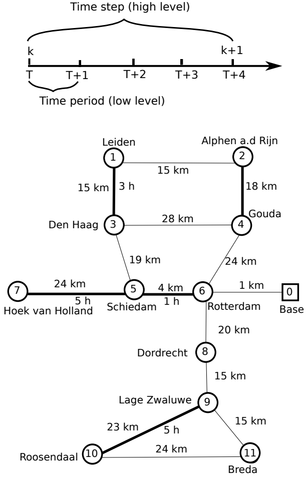
Problem Description



Preliminaries

- Base: storage place of machinery
- Maintenance operation: one round tour of maintenance crew
- One operation per time period
- One time budget per period
- An estimated maintenance time for each line can be obtained from high level

Goal

- Optimal schedule for the maintenance crew
- Minimize setup costs & total travel costs

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Physical Network to Virtual Graph

Transformation

Railway Network	**Undirected Graph**
Maintenance base	Depot
Lines to be maintained	Required edges
Estimated maintenance time	Edge demand
Time period	Virtual vehicle
Maintenance time budget	Vehicle capacity
Setup cost per operation	Fixed costs per vehicle
Line length	Travel cost

Arc Routing Problem

Capacitated Arc Routing Problem with Fixed Costs (CARPFC)

Finding optimal set of routes for a fleet of vehicles

- Minimize fixed setup costs & travel costs
- Cover all required edges
- Satisfy demands
- Not exceed vehicle capacity

Settings

- Periods with same time budget & setup costs → Homogeneous CARPFC
- Periods with different time budget & setup costs → Heterogeneous CARPFC

Solution Approach

- Transformation into equivalent node routing problems
- $\#$ nodes (new graph) $= 2 \times \#$ required edges

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Case Study: Treatment of Ballast Defects

Deterioration Model

$$x_{j,k+1}^{\mathrm{con}} = \begin{cases} a_j x_{j,k} & \text{if } u_{j,k} = 1 \text{ No maintenance} \\ x_{j,k}^{\mathrm{aux}} & \text{if } u_{j,k} = 2 \text{ Tamping} \\ \underline{x} & \text{if } u_{j,k} = 3 \text{ Renewal} \end{cases}$$

$$x_{j,k+1}^{\mathrm{aux}} = \begin{cases} x_{j,k}^{\mathrm{aux}} & \text{if } u_{j,k} = 1 \text{ No maintenance} \\ x_{j,k}^{\mathrm{aux}} + \alpha_j & \text{if } u_{j,k} = 2 \text{ Tamping} \\ \underline{x} & \text{if } u_{j,k} = 3 \text{ Renewal} \end{cases}$$

$$\theta_{j,k} = [a_j \; \alpha_j]^{\mathrm{T}}$$

Settings

Sampling time: 3 months
Prediction & Control horizon: 6 steps (18 months)

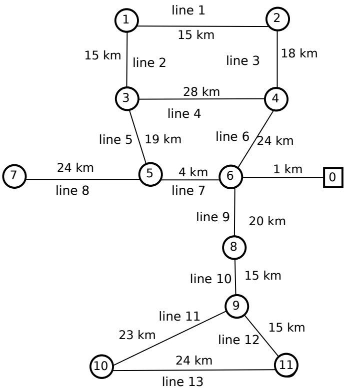
Settings Low Level

Physical Network

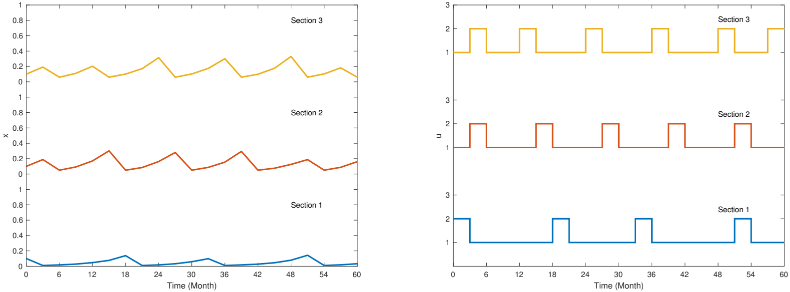
- Part of Dutch railway network including Randstadt Zuid and the middle-south region
- Each line divided into 5-km section
- 13 lines, 53 sections
- A line is to be tamped if any section of it is suggested by the high-level controller

Time Periods for Tamping

- One long period (6 h), two short periods (4 h)
- 120 kEuro for long period, 100 kEuro for short period



Simulation Results: High Level



- x: condition
- u: maintenance option (1 for no maintenance, 2 for tamping, 3 for ballast renewal)

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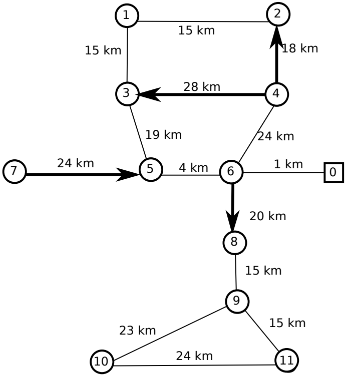
Simulation Results: Low Level



Optimal Routes

0→6⇒8→6→5→7 ⇒5→3→4⇒3→4→2 ⇒4→6→0

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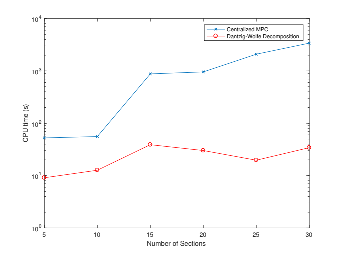
Comparison with Centralized MPC

CPU Time



Settings

- Desktop computer with Quad Core CPU and 64 GB RAM
- Matlab 2016B on SUSE Linux Enterprise Desktop 12
- CPLEX 12.7 as MILP & LP solver

1 Background

2 Multilevel Maintenance Planning
High-Level Problem
Low-Level Problem

3 Case Study

4 Conclusions & Future Work

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Conclusions & Future Work

Conclusions

- Integrated multi-level approach for track maintenance planning
- Tractable, robust and scalable

Future work

- Improved Dantzig-Wolfe decomposition
- Comparison with other distributed optimization method for MILP