Two Problems on Minimality in RLC Circuit Synthesis

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Problem 1 (Minimum number of resistors.)

Consider the set $\mathscr{W}$ of RLC circuits containing n reactive elements (inductors/capacitors). Let $\mathscr{U}$ be a generating subset of $\mathscr{W}$, namely all the impedances that can be synthesised by $\mathscr{W}$ can be synthesised by $\mathscr{U}$. Let m be the maximum number of resistors in any circuit in $\mathscr{U}$. What is the least value of m among generating sets $\mathscr{U}$?

Write

$$Z(s) = \frac{a(s)}{b(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_0}.$$

A lower bound is $n+1$.

Lin (1965) showed that, for $n = 2$, for SP (series-parallel) realisations, 3 resistors suffice.

Reichert (1969), Jiang and Smith (2012) showed, for $n = 2$, that 3 resistors suffice in general.

Unsolved for $n > 2$.

Problem 2 (Minimum number of reactive elements.)

Let m be maximum number of reactive elements among the members of a set $\mathscr{V}$ (of RLC circuits) which generates all positive-real functions of McMillan degree n. What is the least value of m among such generating sets $\mathscr{V}$?

For $n = 2$, the Bott-Duffin construction (SP) shows 6 reactive elements are sufficient. Pantell simplification (bridge network) shows 5 reactive elements are sufficient.

For $n = 2$, Hughes and Smith (2014) showed that 6 reactive elements are needed for an SP realisation (of a minimum function). For $n = 2r$, $4r$ reactive elements are necessary for an SP realisation (of some minimum functions).

For $n = 2$, Hughes (2014) showed showed that 5 reactive elements are needed for general (not necessarily SP) realisations (for almost all minimum functions).

Unsolved for $n > 2$.