



# Modeling for plug-and-play control in strongly coupled nonlinear networks

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# Intro

- ❖ Many real-world network systems (electric power grids, coupled robotic systems, biological systems) are becoming more strongly coupled than in the past; coupling is both temporal and spatial
- ❖ A basic question: Can coupling can be used for systematic design of cooperative control?
- ❖ Can't apply SSP modeling for near-optimal composite control design (temporal simplifications)
- ❖ Cant apply Nssp modeling for spatial simplifications
- ❖ Relevant because of implications on complexity and performance of control/communication designs
- ❖ Potential of controllers in the nodal components of the network, as well as potential of fast switched control of its branch components

# Problem description

- ❖ Three controllers (governor, Exciter and FACTS)

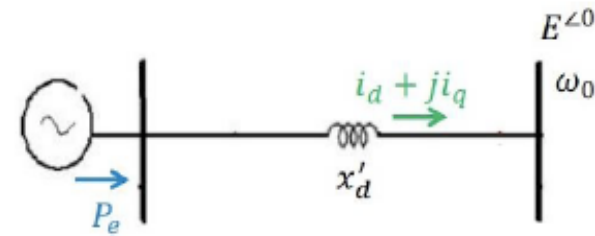


Fig.1 Electric power network

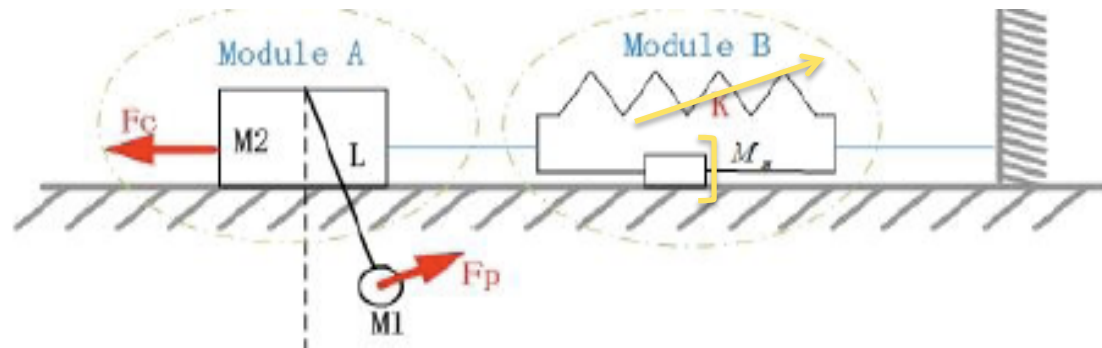


Fig. 2 Mechanical analogy of the electric power network

- ❖ Three controllers ( $F_p$ ,  $F_c$ , and controllable inverter)

## Modeling questions for plug-and-play control design

- ❖ Control problem—strong coupling of modules A and B; MIMO design so that
- ❖ 1) pendulum is synchronized; 2) states and control within the pre-specified limits

# Mechanical system modeling options

The standard state space model is

$$\begin{aligned}\dot{X}_A &= f(X_A, X_B, u_A) & X_A &= [\theta \quad \omega \quad x_2 \quad v_2]^T & U &= [u_A \quad u_B]^T & u_A &= [F_P \quad F_C]^T \\ \dot{X}_B &= f(X_B, X_A, u_B) & X_B &= [x_B \quad v_B]^T & & & u_B &= [K]\end{aligned}$$

❖ Acceleration as the coupling (interaction) variable of modules A and B

$$\dot{X}_{\text{pendulum}} = f(X_{\text{pendulum}}, \dot{v}_2, u_p) \quad \text{where } u_p \text{ is } F_P$$

$$\dot{X}_{M2} = f(X_{M2}, \dot{\omega}, \dot{v}_B, u_M) \quad \text{where } u_M \text{ is } F_C$$

## Stored energy and rate of change of stored energy coupling (interaction) variable—new state space

$$Z_A = \frac{1}{2} J \omega^2 + \frac{1}{2} m_2 v_2^2 + m_1 g l (\cos \theta - 1) \quad \text{where } J = m_1 l^2 \quad P_A = J \omega \dot{\omega} + m_2 \dot{v}_2 v_2 - m_1 g l \sin \theta$$

$$Z_B = \frac{1}{2} m_B v_B^2 + \frac{1}{2} k (x_B - x_0)^2 \quad \text{where } x_0 \text{ is initial length of spring}$$

$$P_B = m_B v_B \dot{v}_B + k (x_B - x_0) x_B$$

The new state space takes on the form

$$X_A^{new} = \left[ \bar{\bar{X}}_A \quad Z_A \quad P_A \right]^T \quad \bar{\bar{X}}_A = [\theta \quad \omega]^T \quad X_B^{new} = [Z_B \quad P_B]$$

$$\begin{aligned} \dot{\bar{\bar{X}}}_A &= f_A(\bar{\bar{X}}_A, Z_A, P_A, u_A) \\ \dot{Z}_A &= f_{ZA}(\bar{\bar{X}}_A, Z_A, P_B) \\ \dot{P}_A &= f_{PA}(\bar{\bar{X}}_A, P_A, \dot{P}_B) \end{aligned}$$

Internal dynamics module A

Zoom-out layer in terms of interaction variables

$$\begin{aligned} \dot{Z}_B &= f_{ZB}(Z_B, P_A, u_B) \\ \dot{P}_B &= f_{PB}(P_B, \dot{P}_A) \end{aligned}$$

# Comparison of modular models

- ❖ Acceleration as an interaction variable works w/o assumptions if actuator dynamics are neglected; otherwise, projection of centrifugal force effect on M2 needs to be ignored; conjecture—if dynamics of actuator accounted for only stabilization around stable pendulum position possible; also acceleration must be communicated, hard to do
- ❖ When stored energy and rate of stored energy used as coupling (interaction) variables –no approximations needed; can stabilize around inverted pendulum (Furuta, Astrom)
- ❖ only local power measurement needed, completely decentralized
- ❖ It is possible to specify interactions over several time horizons —important for complex networks

# Open problem

- ❖ Extend nonlinear control design to multi-layered strongly coupled complex networks.
- ❖ Provable performance precludes solutions in which position changes in an unbounded way
- ❖ New problem, when assumptions are not made (acceleration ideal input)
- ❖ Motivation--- functional specifications for interconnected smart grids with lots of fast power electronics switching; micro-grids; systems with wind power plants and delivery power electronically controlled to increase delivery