

# **Stability of Passivity-Based Control for Power Systems and Power Electronics**

Kevin Bacovchin and Marija D. Ilic Department of ECE Carnegie Mellon University Pittsburgh, PA 15213 USA milic@ece.cmu.edu

Open problem LCCC Workshop
Lund, Sweden
October 2014

## The problem

- Passivity-based control
  - Nonlinear control method
  - Exploits the intrinsic energy properties of the system dynamics
  - Robust due to the avoidance of exact cancellation of nonlinearities
- Challenge for under-actuated systems
  - Not all state variables can be regulated
  - Desired state variables cannot be all arbitrarily selected
  - Non-directly controlled desired state variables have dynamics
  - Desired dynamics need to be stable for control to work
  - Stability of desired dynamics can depend on
    - which state variables are chosen to be directly controlled
    - parameters and set points

## **Description of the Problem**

State space model:  $\dot{\mathbf{x}} = \mathbf{f}(x, u)$ 

Closed-loop energy functions:  $\tilde{W}_m$ '( $\tilde{\mathbf{x}}$ ),  $\tilde{W}_e$ ( $\tilde{\mathbf{x}}$ )

where  $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}^D$ 

 $\frac{\text{Closed-loop}}{\text{dissipation function}}: \qquad \tilde{R}(\tilde{\mathbf{x}})$ 

Set point equations:  $\mathbf{f}_r(x^D) = \mathbf{r}^*$ 

can derive control law in an automated manner

#### **Lyapunov function**:

$$V\left(\tilde{\mathbf{X}}\right) = \tilde{W}_{m}'\left(\tilde{\mathbf{X}}\right) + \tilde{W}_{e}\left(\tilde{\mathbf{X}}\right)$$

$$\frac{dV(\tilde{\mathbf{x}})}{dt} = \frac{dV(\tilde{\mathbf{x}})}{d\tilde{\mathbf{x}}} \frac{d\tilde{\mathbf{x}}}{dt}$$

If  $\begin{cases} V(\tilde{\mathbf{x}}) \text{ is positive definite} \\ \frac{dV(\tilde{\mathbf{x}})}{dt} \text{ is negative definite} \end{cases}$ 

Then  $\tilde{\mathbf{x}} \to 0, \ \mathbf{x} \to \mathbf{x}^D$ 

Passivity-Based

$$\mathbf{u} = \mathbf{g}_1(x, x^{Dn}, r^*)$$

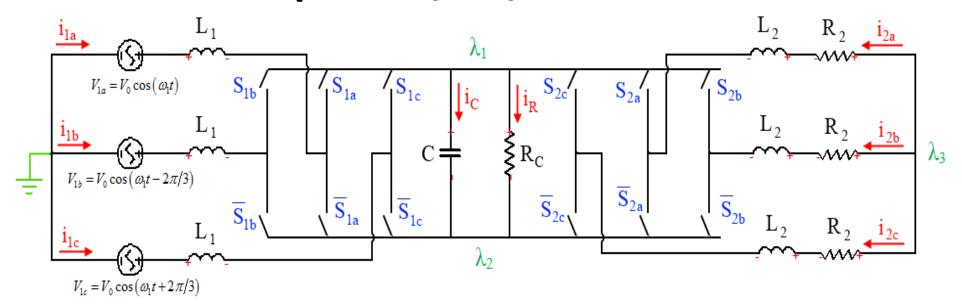
**Control Law**:

$$\dot{\mathbf{x}}^{Dn} = \mathbf{g}_2(x, x^{Dn}, r^*) \boldsymbol{\angle}$$

Non-directly controlled desired state variables have dynamics, which can go unstable

Source: K. D. Bachovchin, M. D. Ilić, "Automated Passivity-Based Control Law Derivation for Electrical Euler-Lagrange Systems and Demonstration on Three-Phase AC/DC/AC Converter," EESG Working Paper No. R-WP-5-2014, August 2014.

## **Example: AC/DC/AC Converter**



- Choose to directly regulate the direct and quadrature components of the load and source currents
- Desired capacitor charge has dynamics

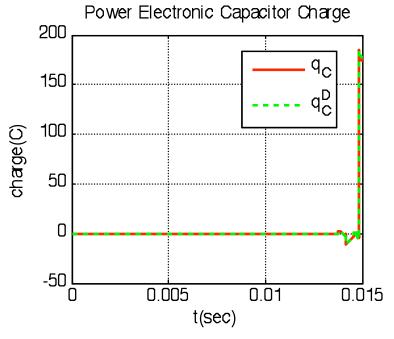
$$\frac{dq_{C}^{D}}{dt} = -\frac{\left(q_{C}^{D}\right)^{2} - C^{2}R_{C}\left(V_{1d}i_{1d}^{*} + V_{1q}i_{1q}^{*}\right) + C^{2}R_{C}R_{2}\left(i_{2d}^{*2} + i_{2q}^{*2}\right) + C^{2}R_{C}R_{1}\left(i_{1d}^{*2} + i_{1q}^{*2} - i_{1d}i_{1d}^{*} - i_{1q}i_{1q}^{*}\right)}{CR_{C}q_{C}^{D}}$$

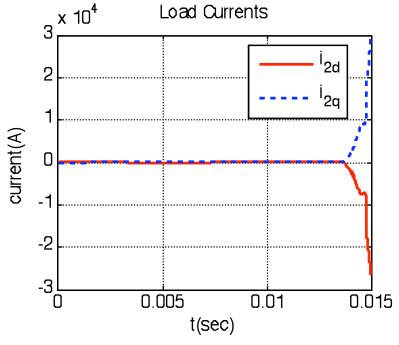
## **Example: AC/DC/AC Converter**

A stable equilibrium for  $q_{C}^{\ \ D}$  only exists when

$$\underbrace{V_{1d}i_{1d}^* + V_{1q}i_{1q}^* \ge R_2i_{2d}^{*2} + R_2i_{2q}^{*2}}_{\text{power input by source}} + R_2i_{2q}^{*2}$$

If this condition is violated, then the passivity-based controller will be unstable

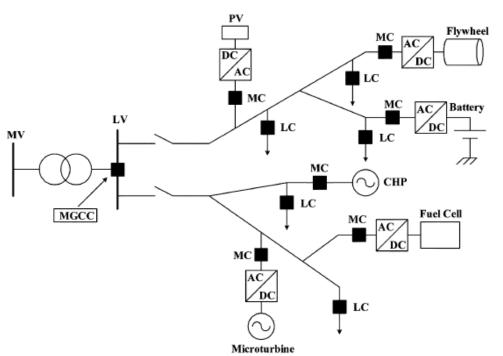


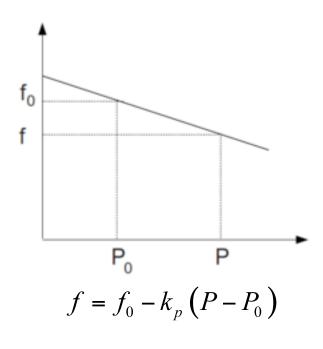


Source: K. D. Bachovchin, M. D. Ilić, "Automated Passivity-Based Control Law Derivation for Electrical Euler-Lagrange Systems and Demonstration on Three-Phase AC/DC/AC Converter," EESG Working Paper No. R-WP-5-2014, August 2014.

### **Motivation**

Analysis and control of microgrids often begin with droop characteristic





Before droop characteristic analysis can be used, the fast dynamics must be stabilized using control with provable performance