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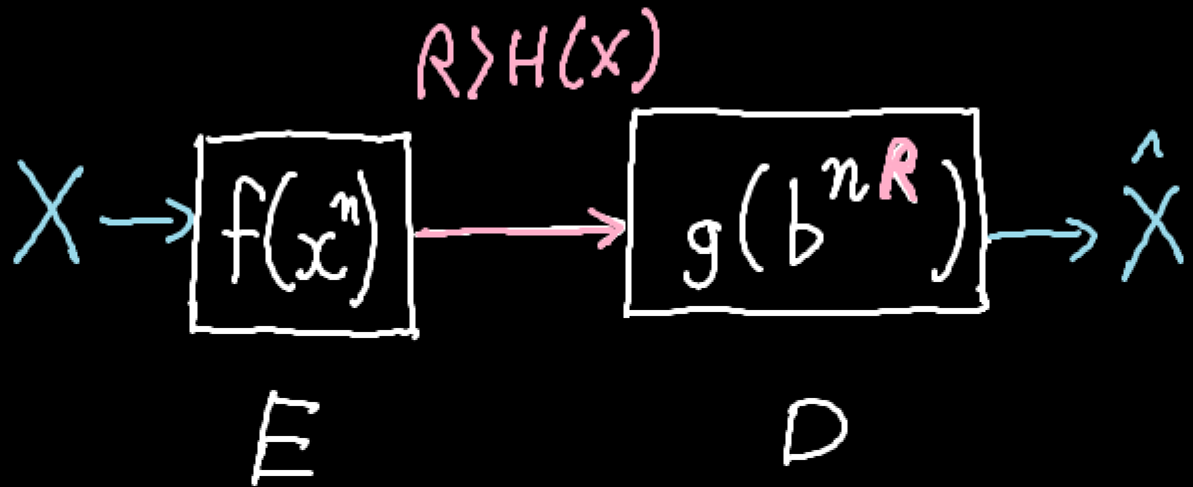
Fun with Porosity and Aliens! (in channel coding)

LCCC Focus Period on Information and Control in Networks

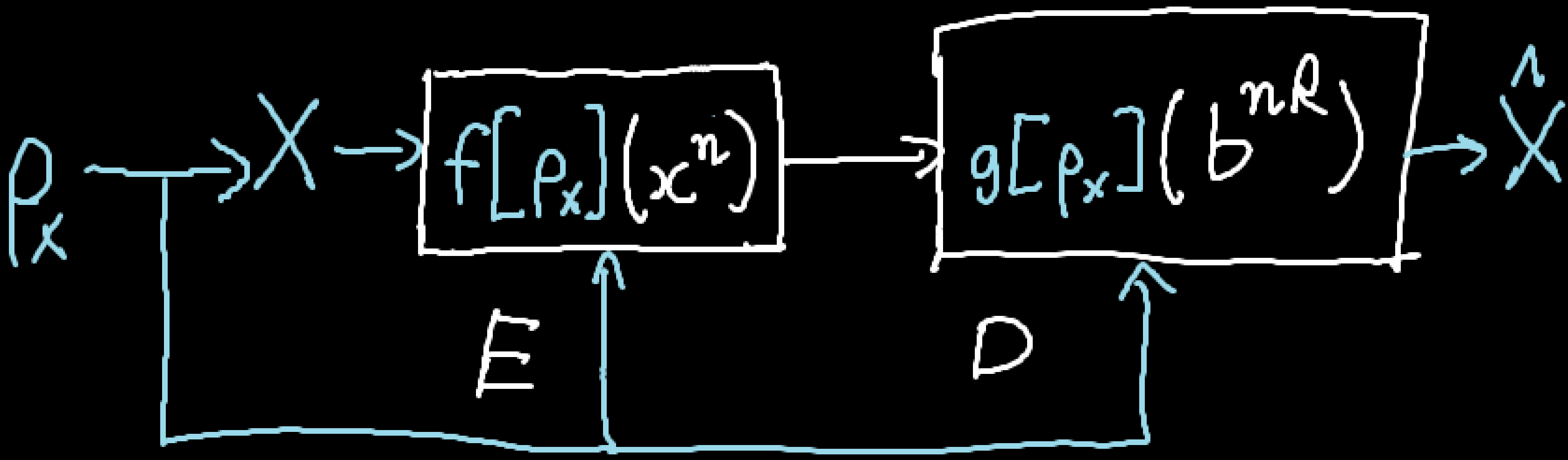
Outline

- **Universality!**
  - Universal source coding!
  - Universal channel coding?
- Universal channel *decoding*!
  - Traditional formulations!
  - Aliens!
- Universal channel coding, with feedback!

(Lossless) Source Coding

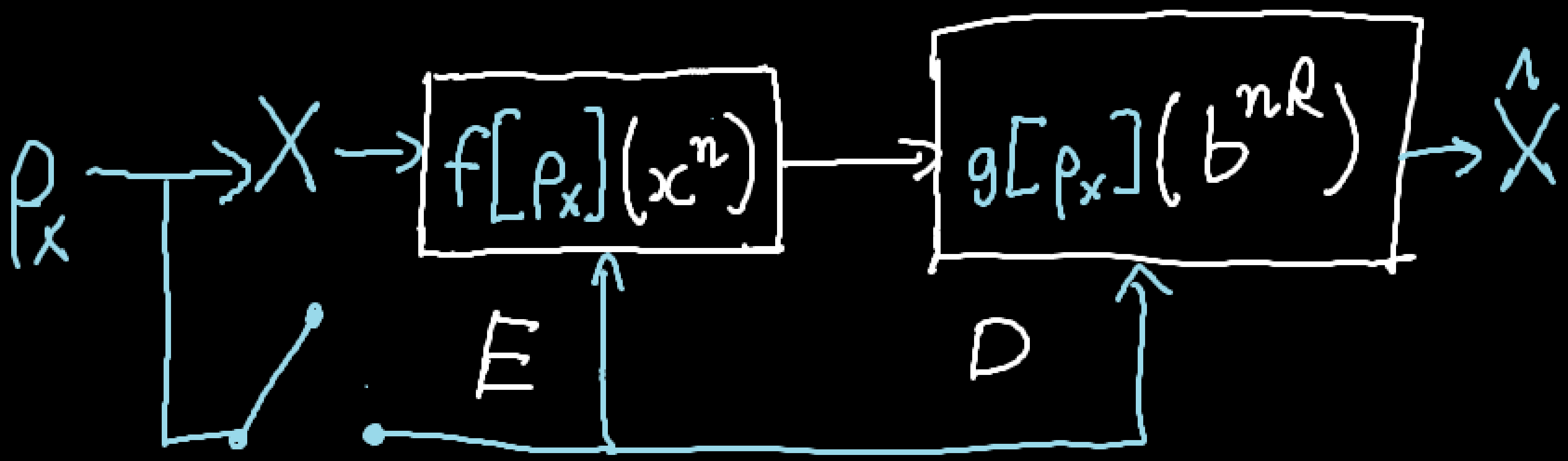


(Lossless) Source Coding



- Known source model!
- Encoder/Decoder optimized for source.

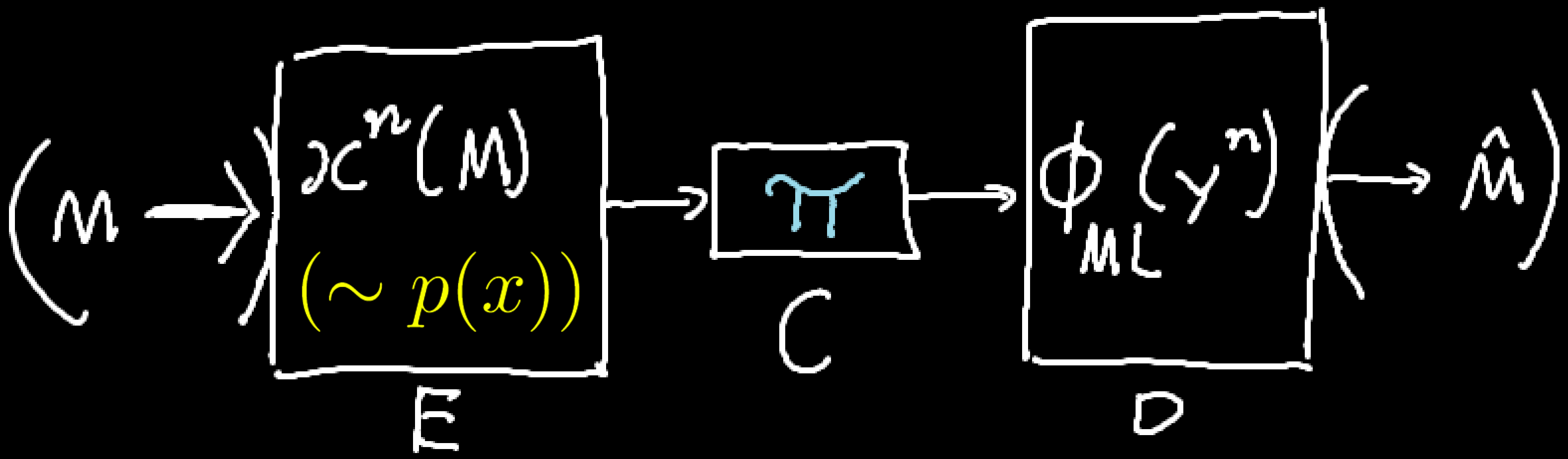
Universal Source Coding



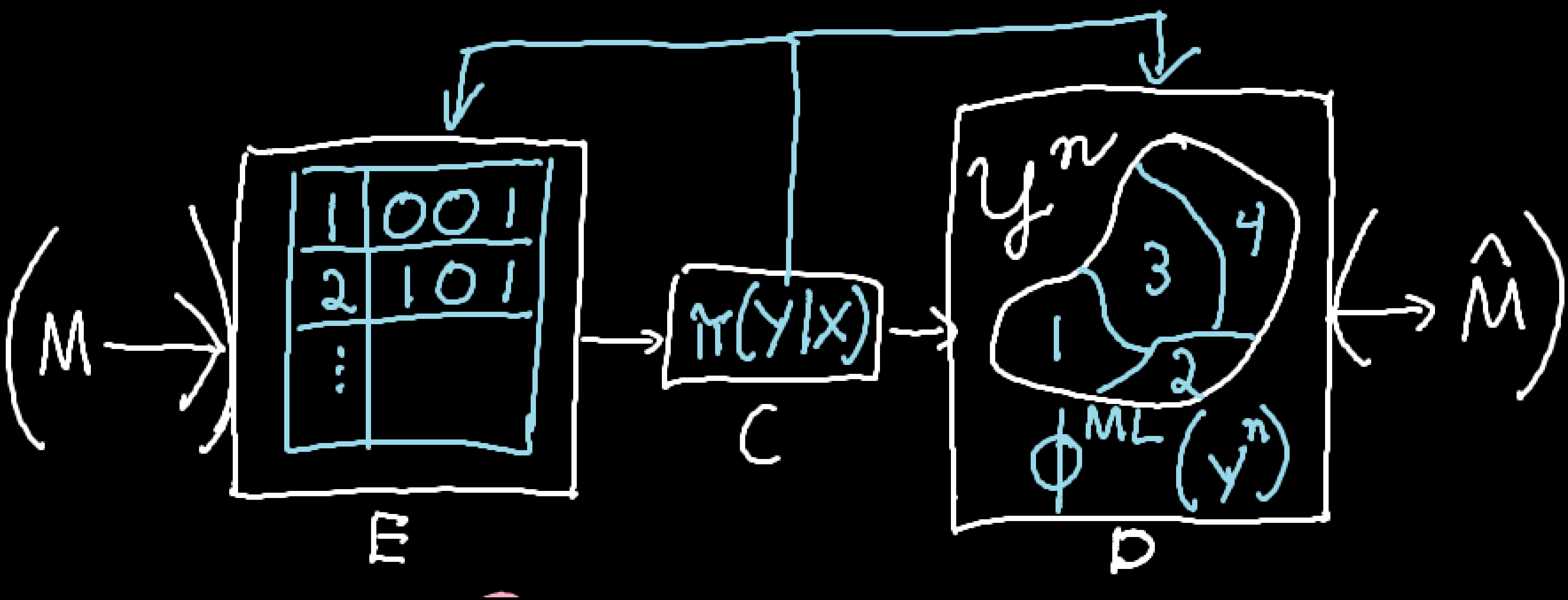
- Encoder and decoder can adapt.
- Strong sense of universality: optimal compression for *every* source model.

Channel Coding

$$R > I(X;Y)$$

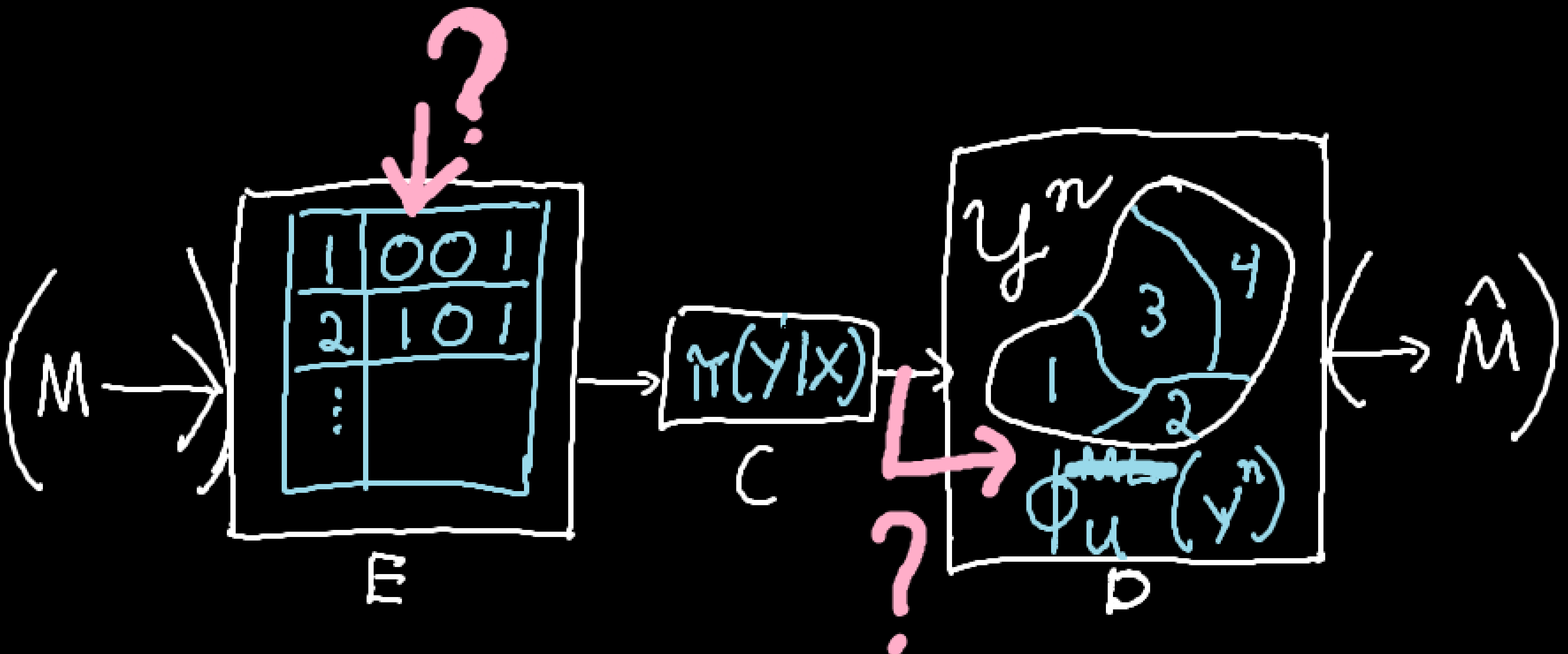


Channel Coding



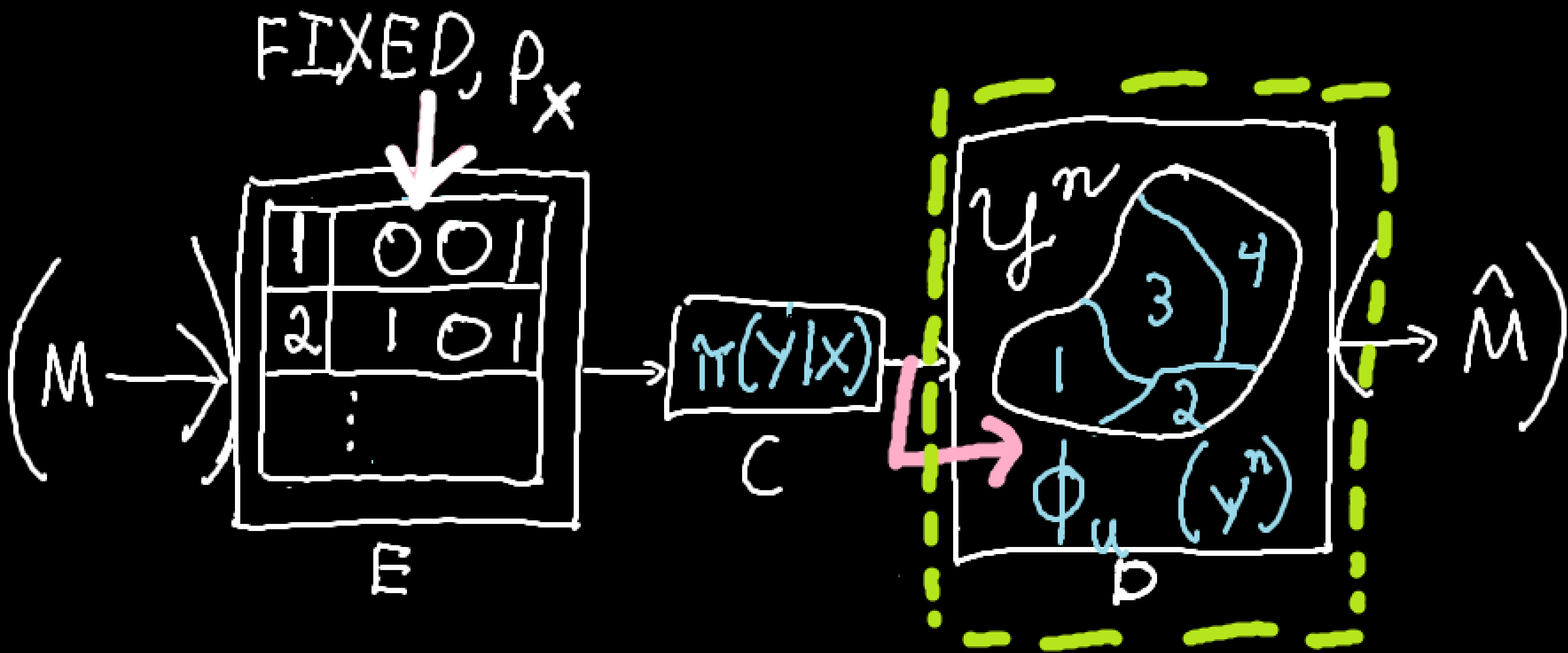
- Relevant component: channel model π.
- Codebook/Decoder can be optimized for given π.

Universal Channel Coding



- Encoder cannot adapt.
- Decoder *might* adapt.

Universal Channel *De*Coding



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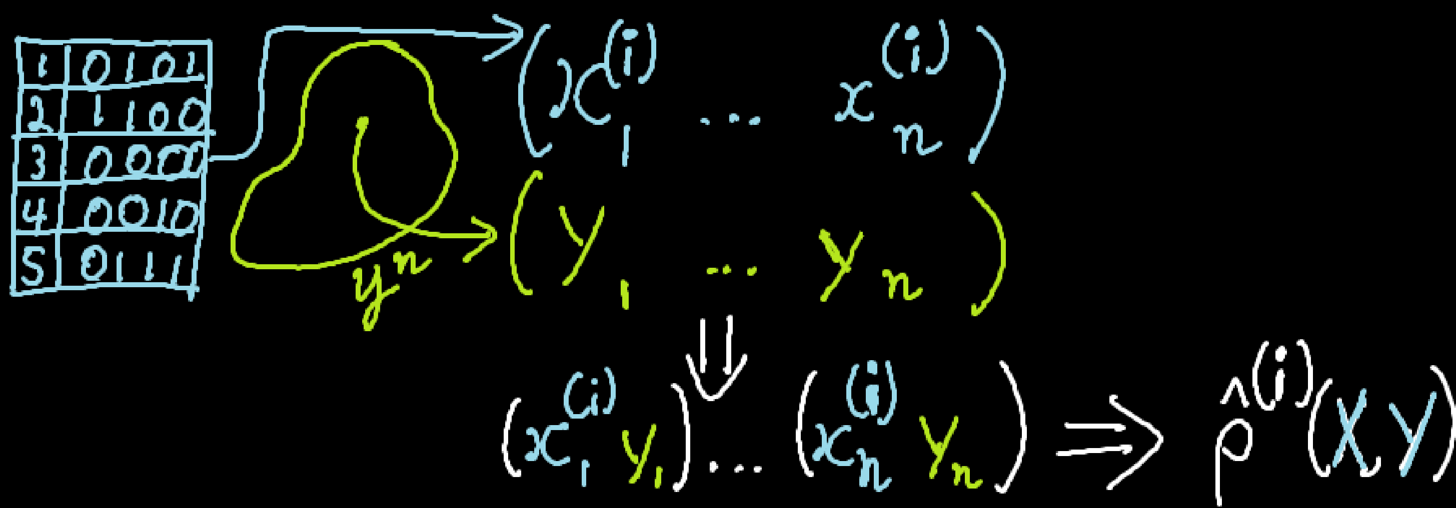
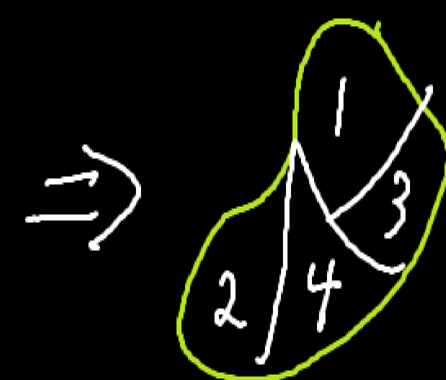
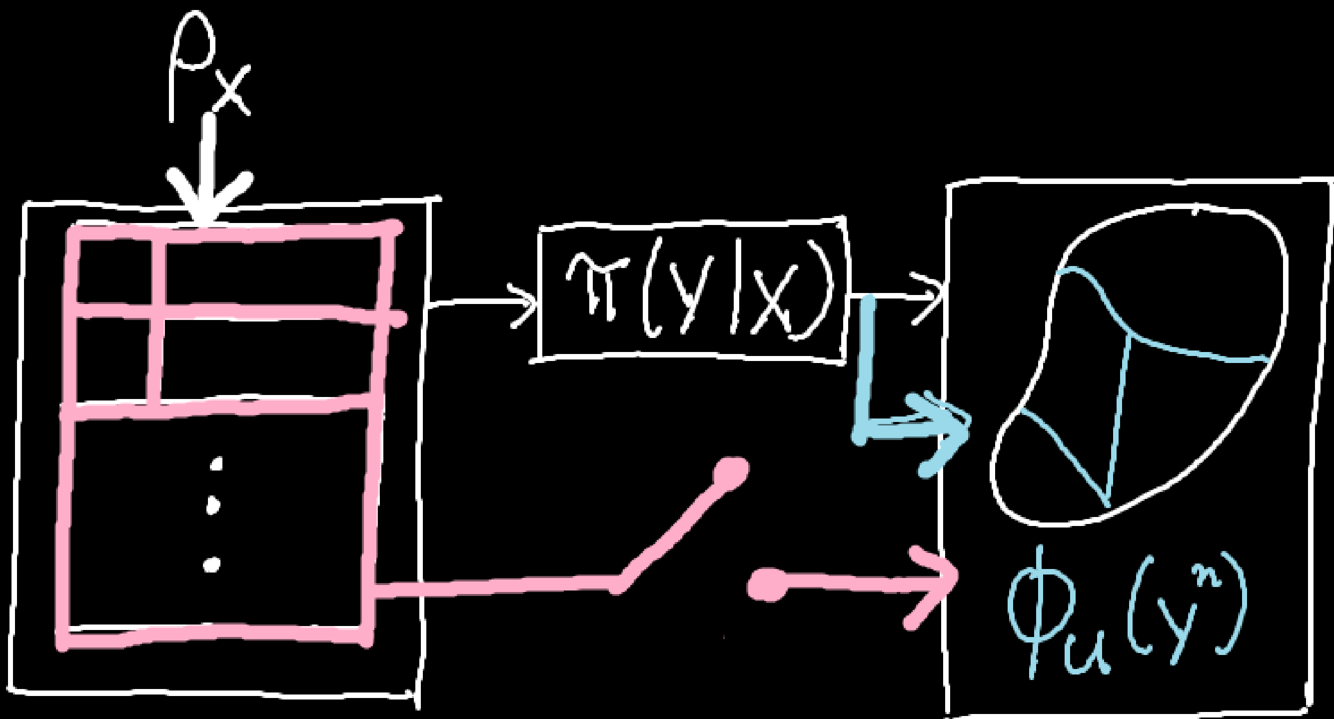
The max mutinf (MMI) decoder



MMI: maximize $I(X;Y)_{\hat{p}^{(i)}(X,Y)}$

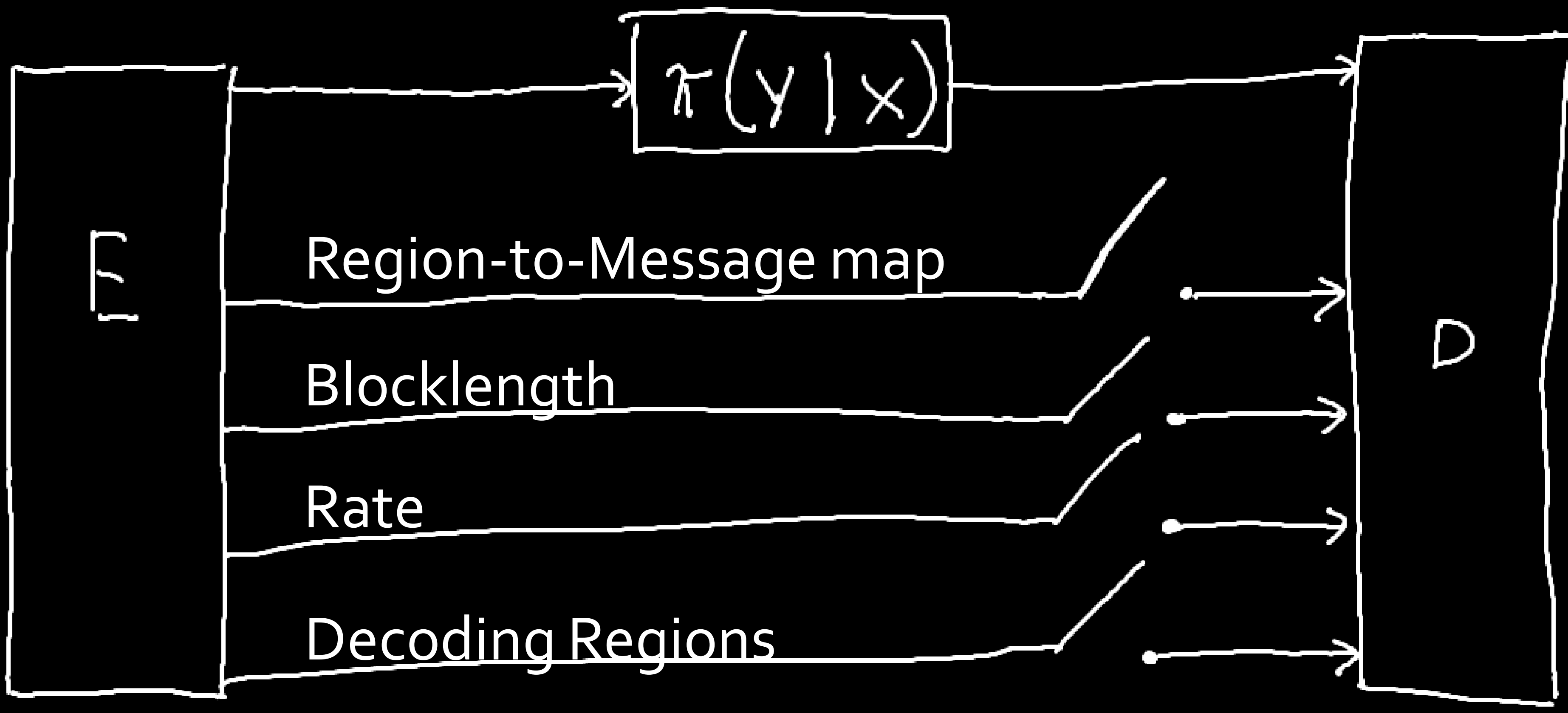


Codebook universality?

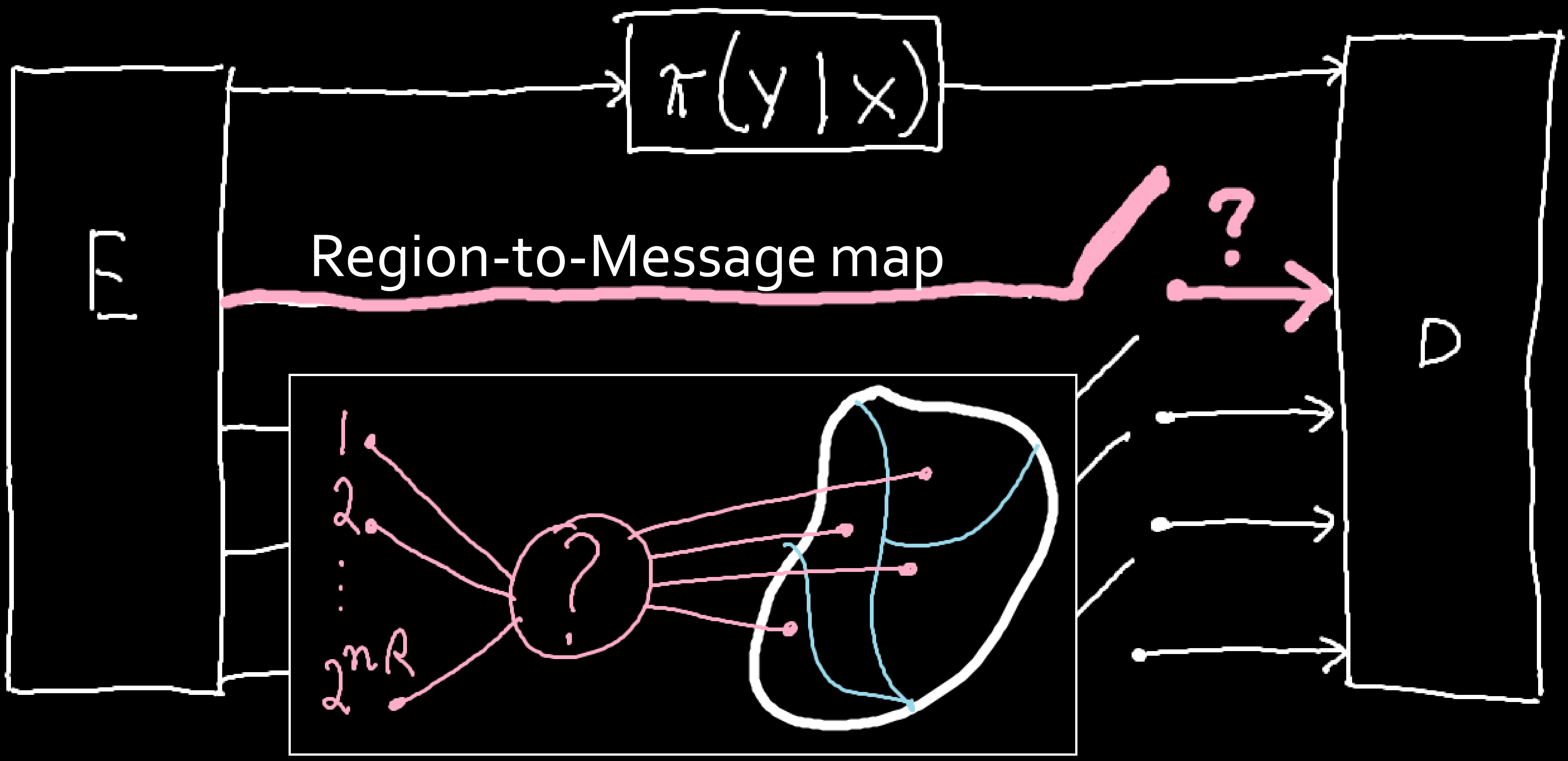


- Extreme universality: decoder doesn't know code!
- "Communicating with Aliens" --- Sudan et al.
- Eavesdropping, robustness, adaptive encoder…

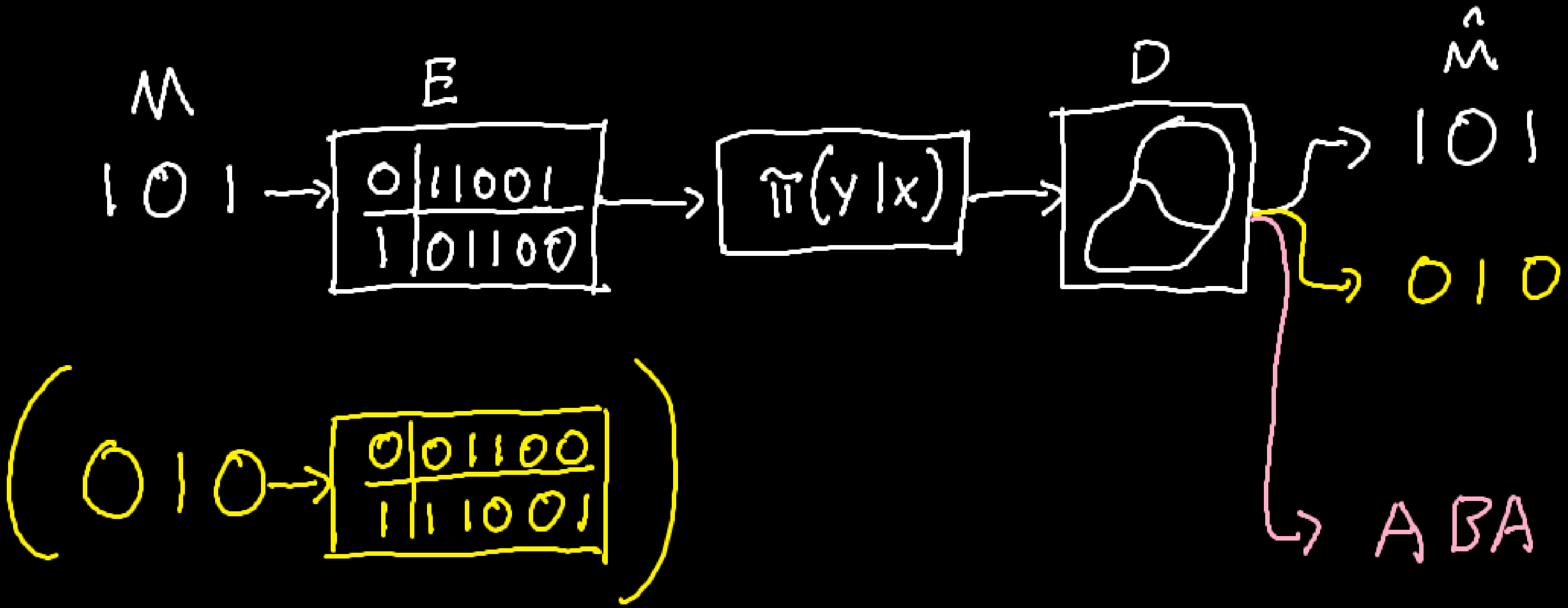
Codebook universality, in 4 parts



1: Guess the message map?

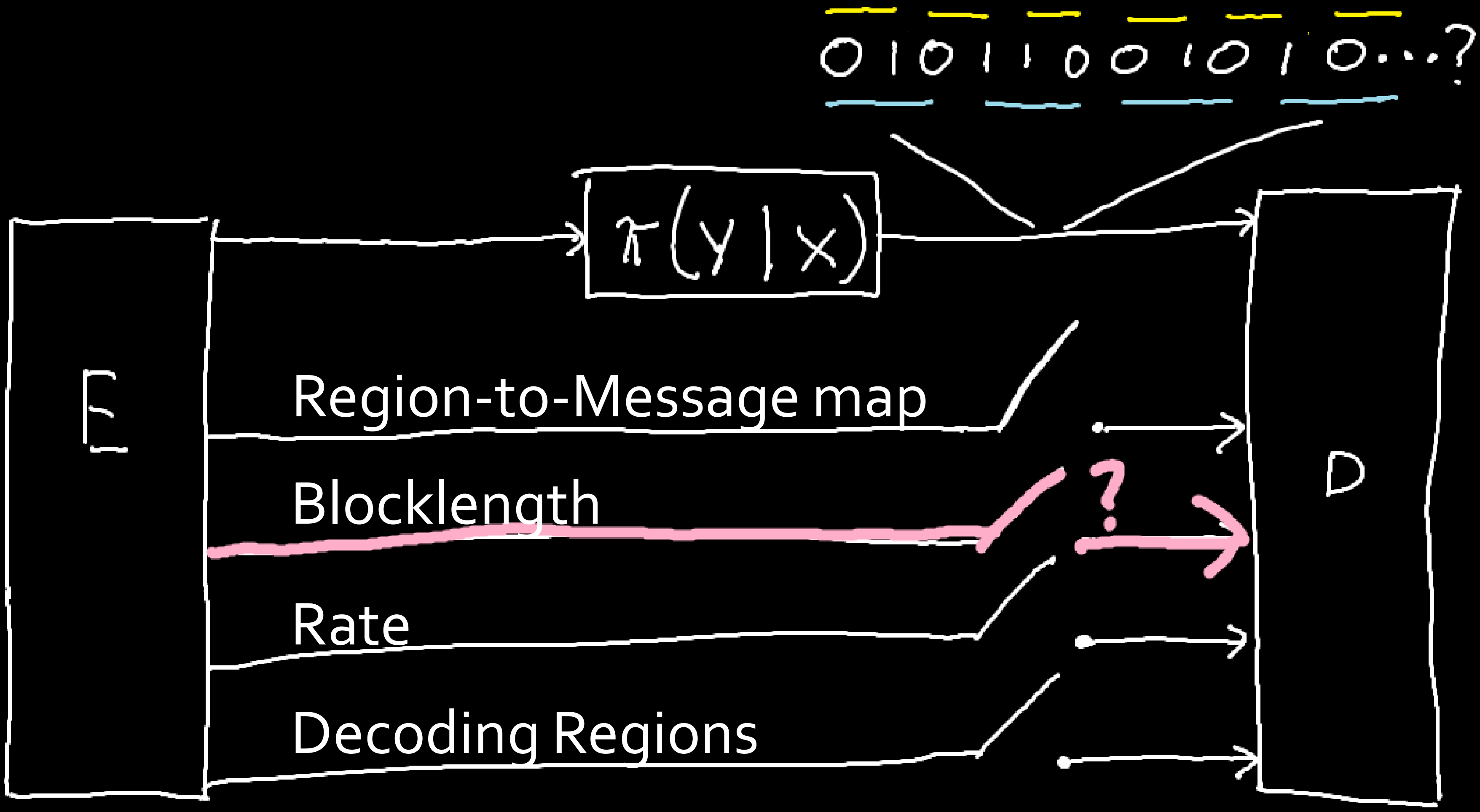


Pattern Decoding

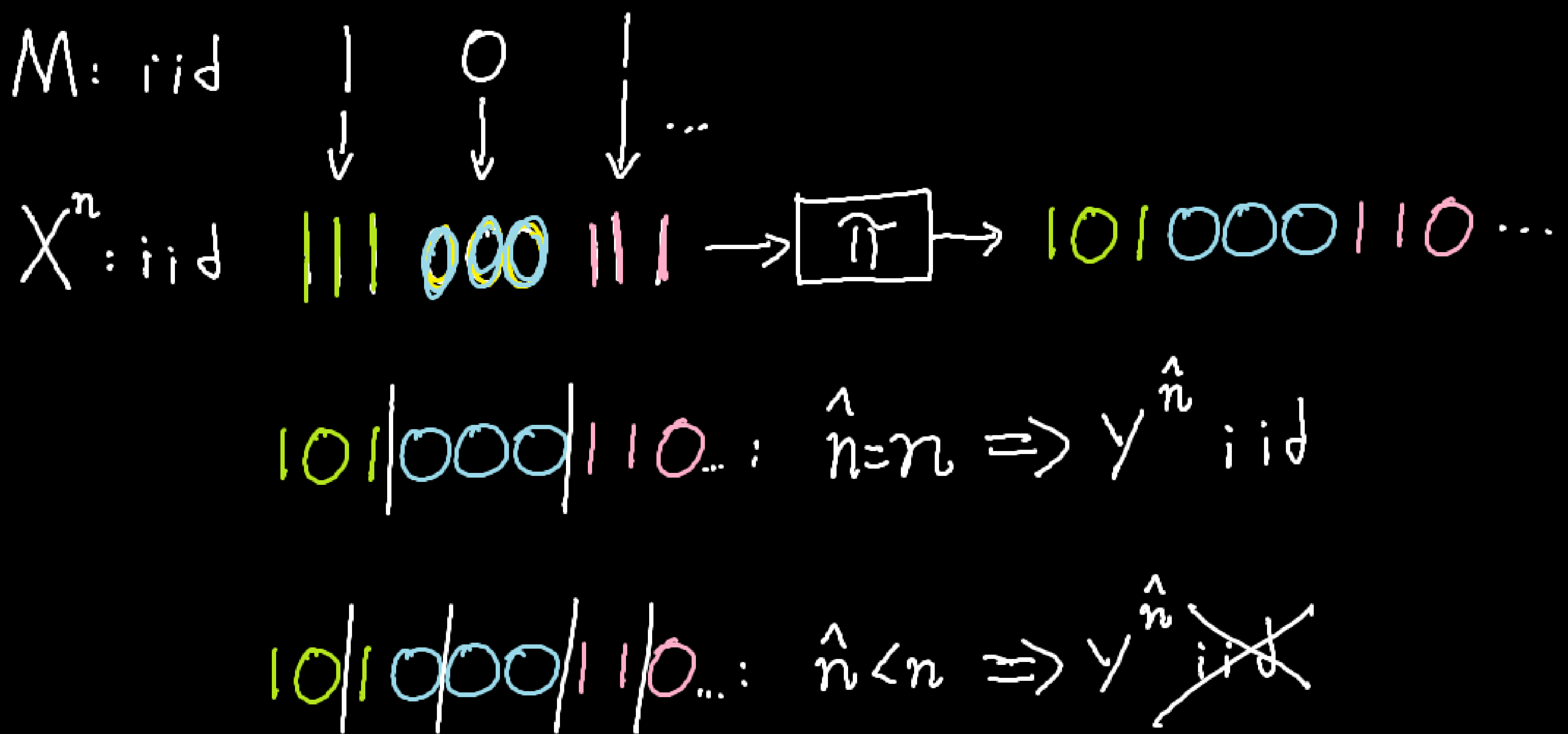


- Fundamental ambiguity.
- Decode "pattern" of message (Orlitsky et al.).
- Alternatively: minimum context is $\log(2^{nR}!)$ bits.

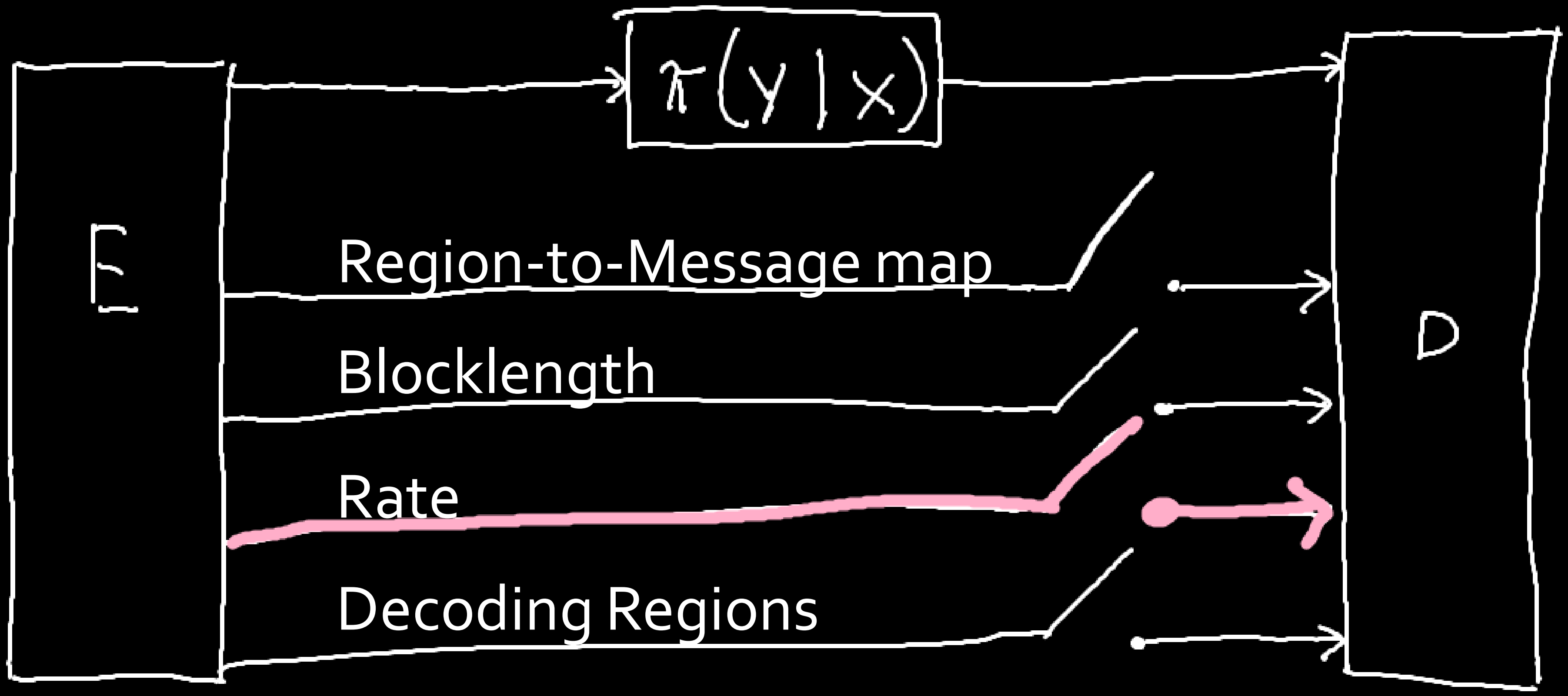
2: Guess the blocklength!



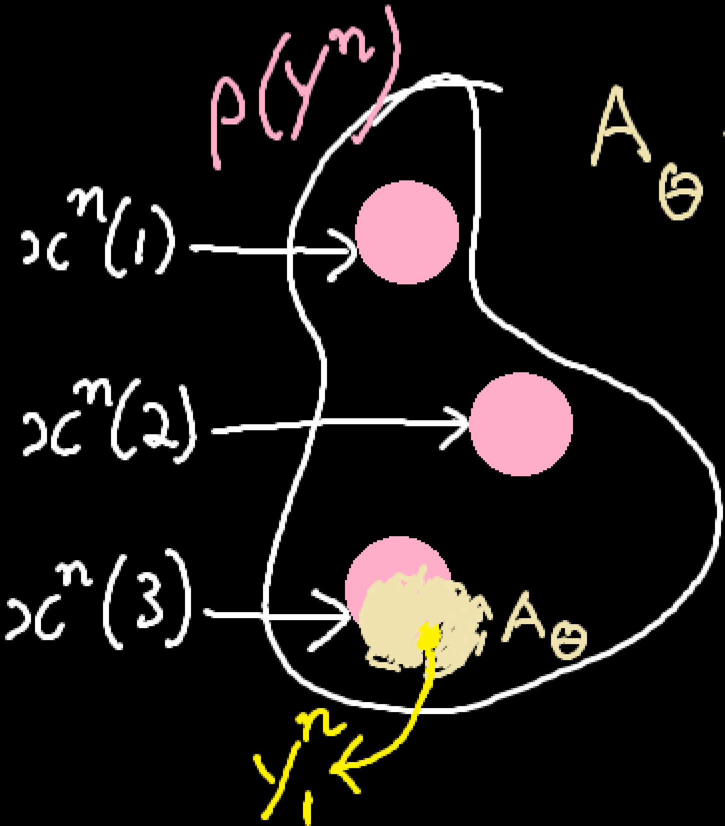
Look for independence!



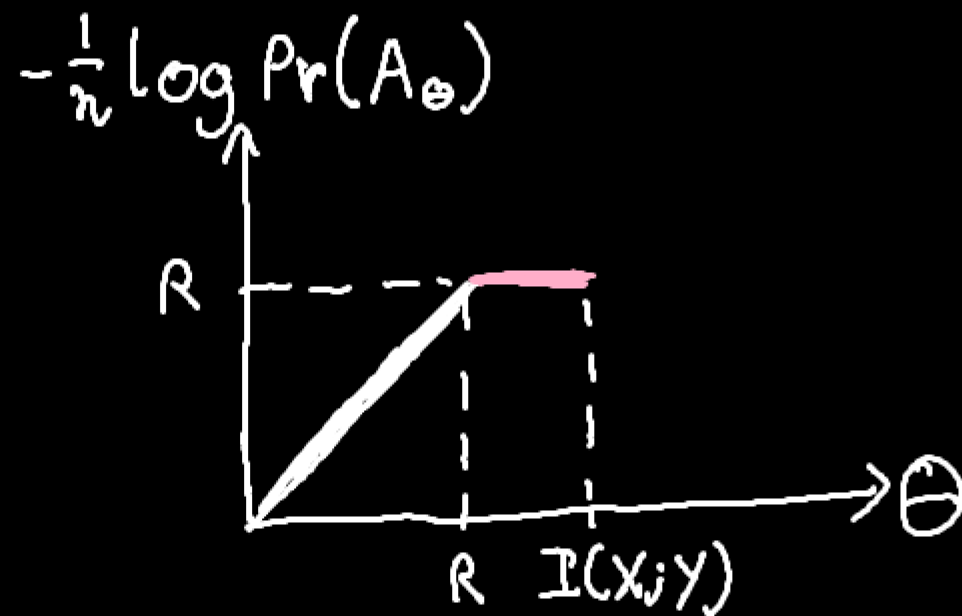
3: Guess the rate!



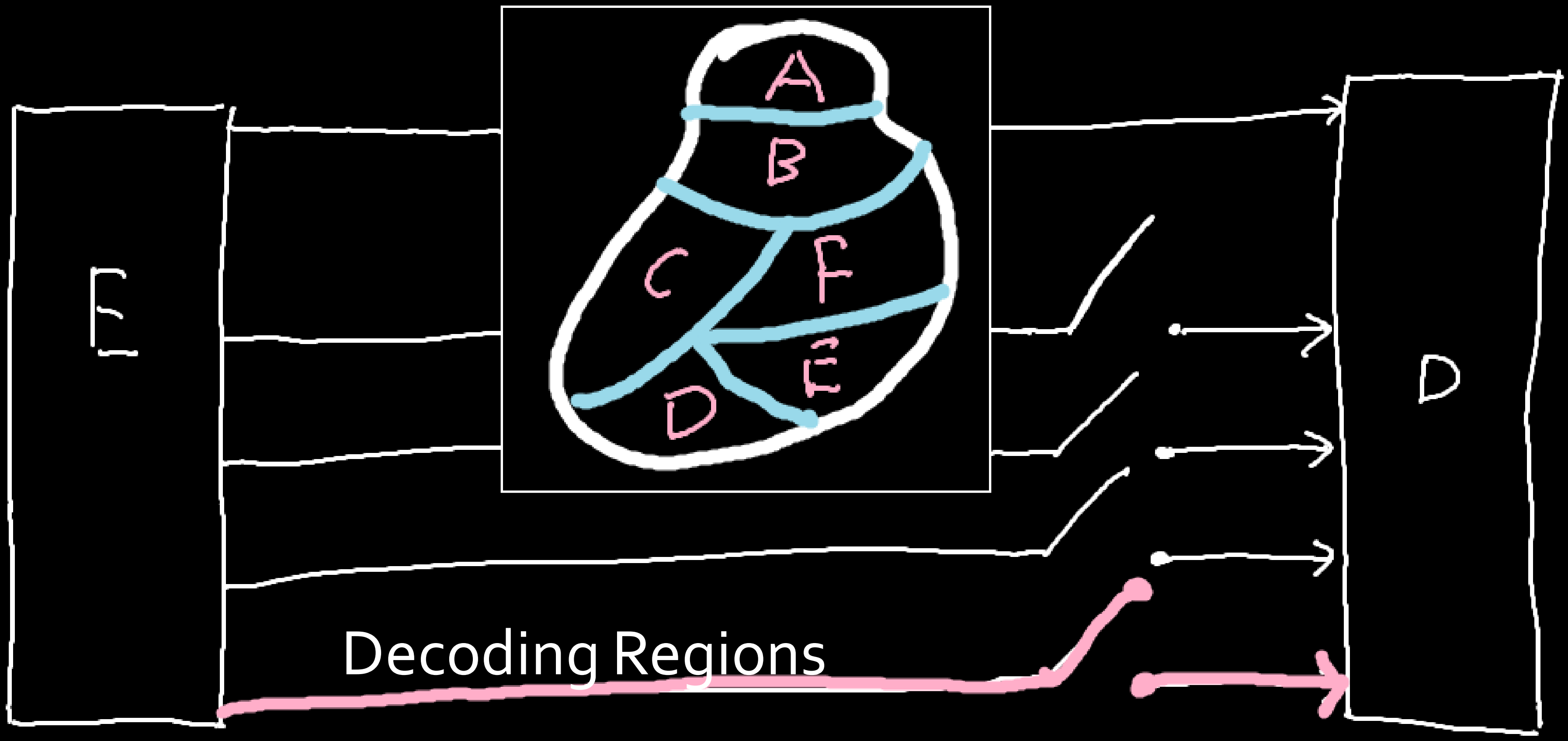
Idea: clustering



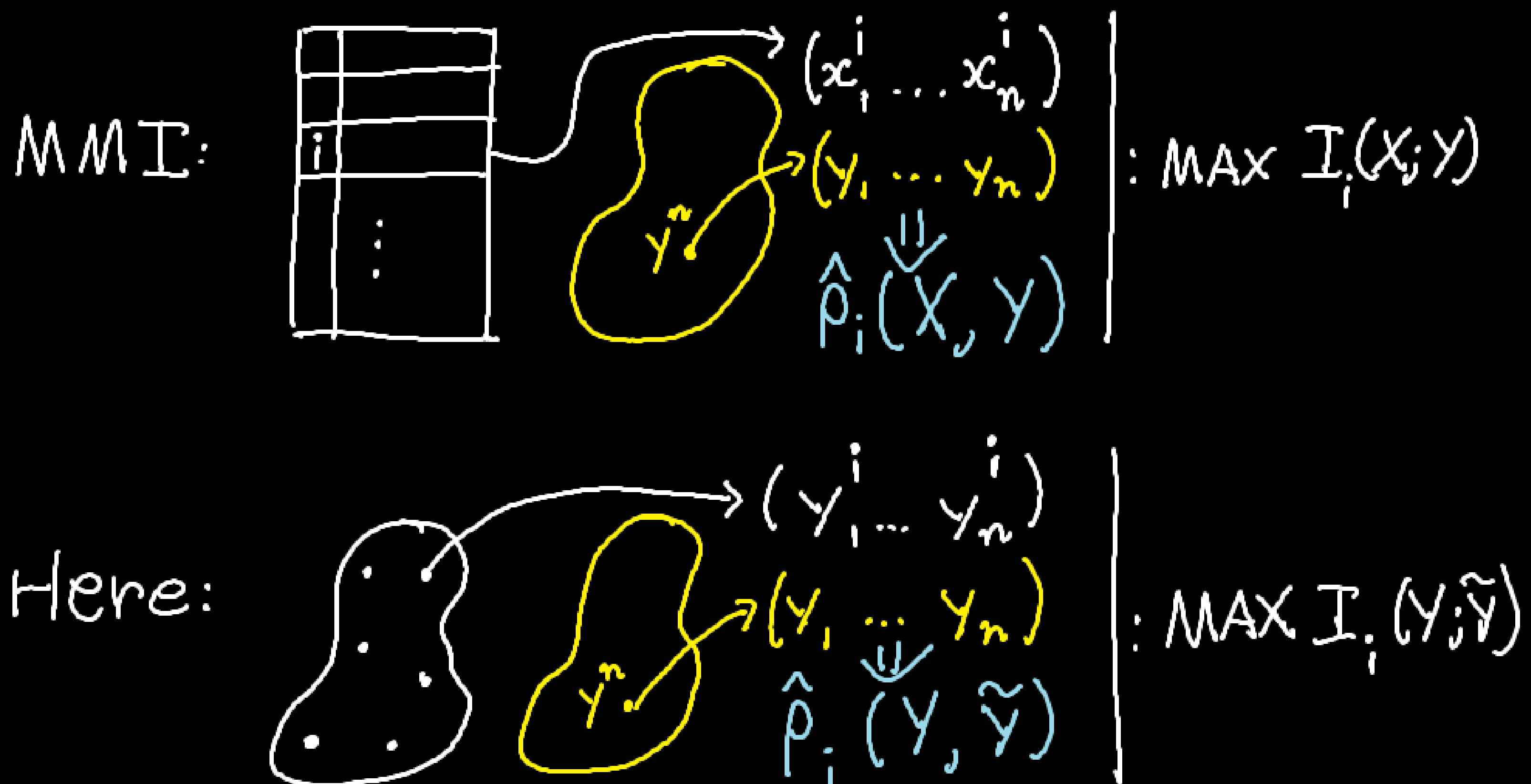
$$A_\Theta = \left\{ y^n : I(Y;\tilde{Y})_{\hat{p}[y^n,\tilde{y}^n]} \geq \Theta \right\}$$



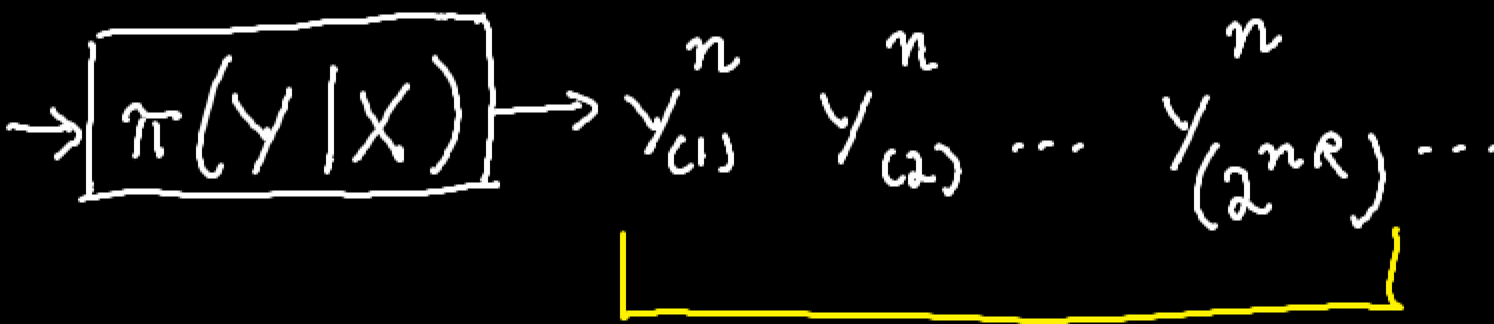
4: Guess the decoding regions!

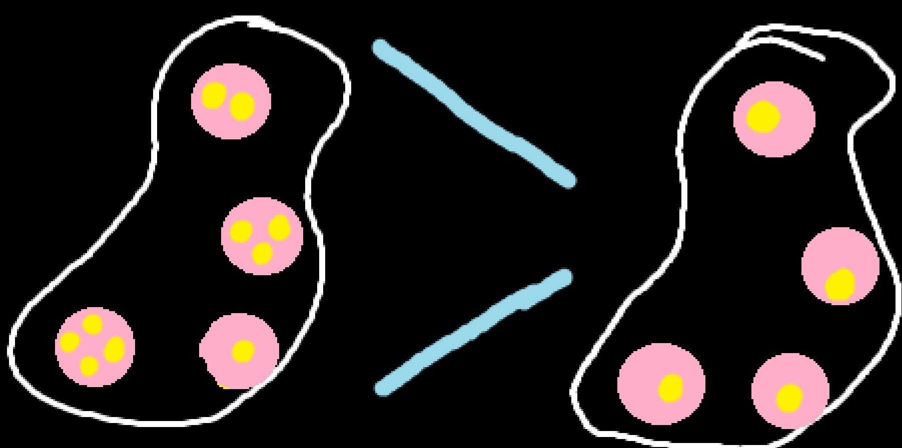


Dirty MMI Decoding:



Choosing a dirty codebook (DC)

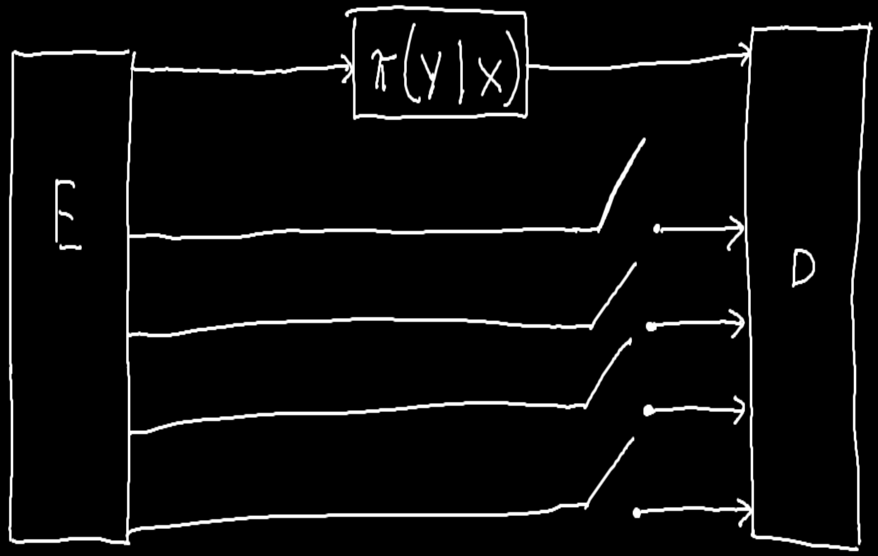




- Choose at random!
- Filtered DC (FDC): smallest uniquely decoding sub-DC.
- Works!

Aliens, in review:

- Universal pattern decoding for random codes!



- Conj: possible for deterministic codes with positive error exponent.
- (I lied! m-tuple version of decoder required.

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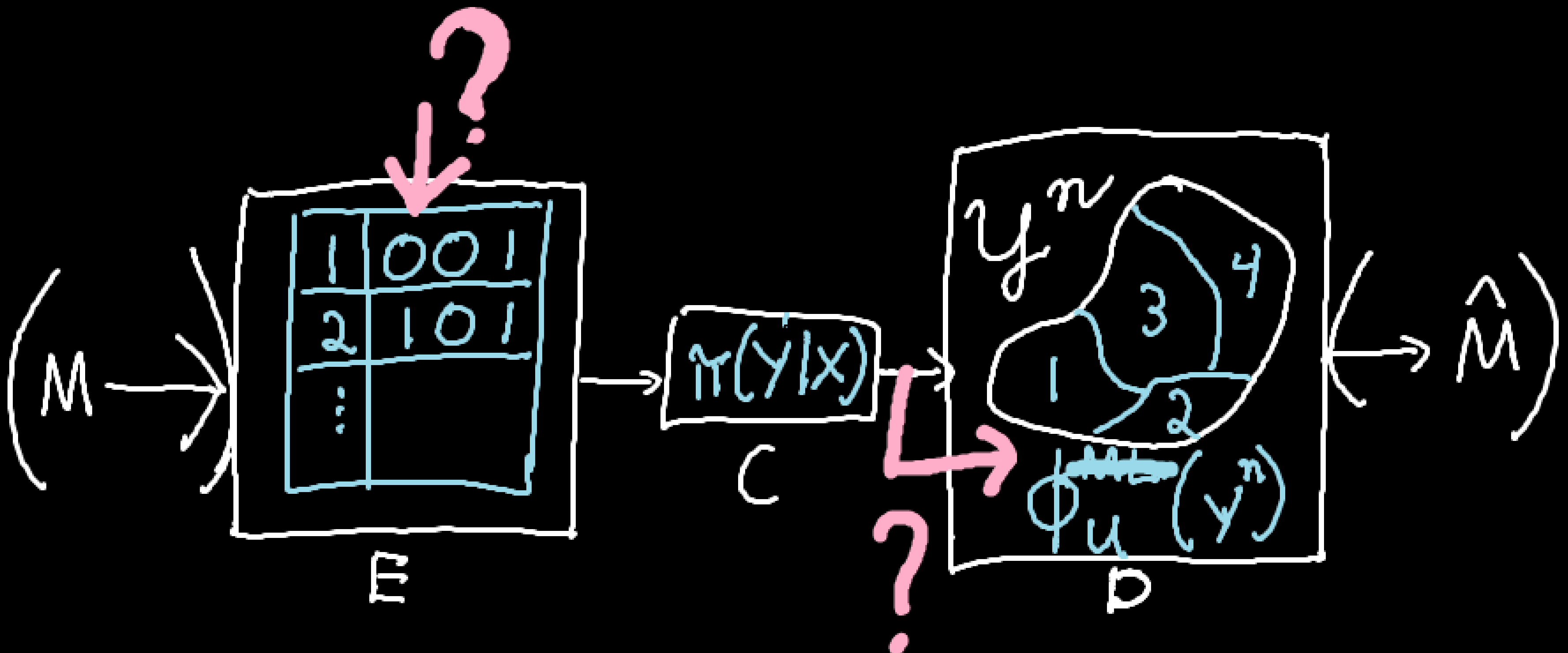
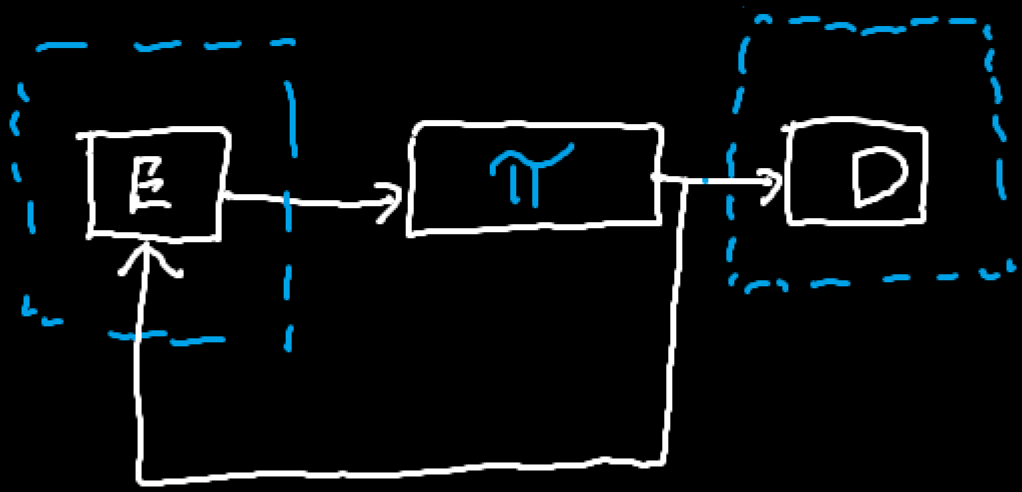
Universal Channel Coding



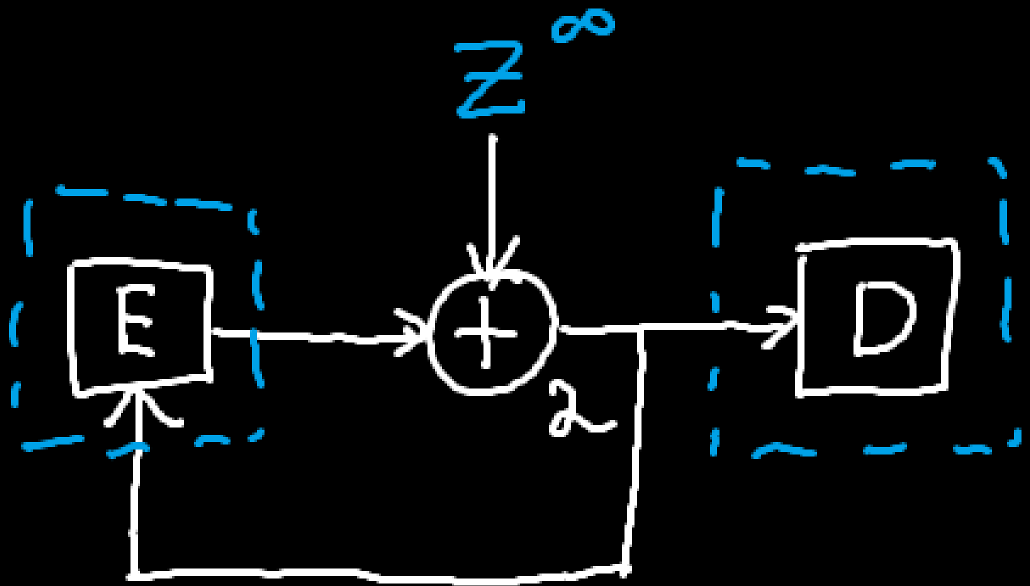
- Encoder cannot adapt.
- Decoder *might* adapt.

Feedback to the rescue?



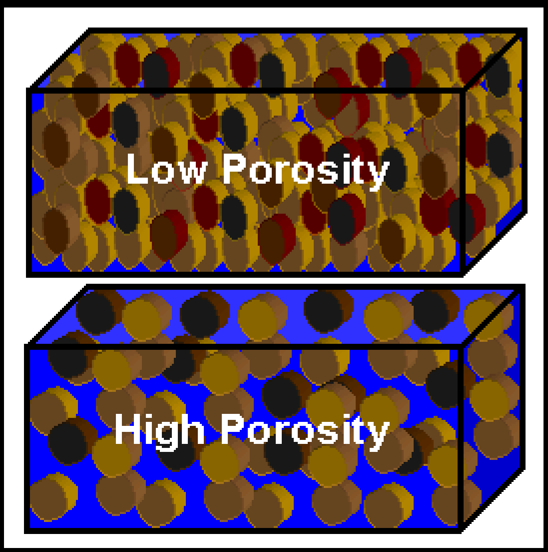
- Encoder and decoder can adapt.
- Stronger form of universality?
- (More fundamental channel coding problem?)

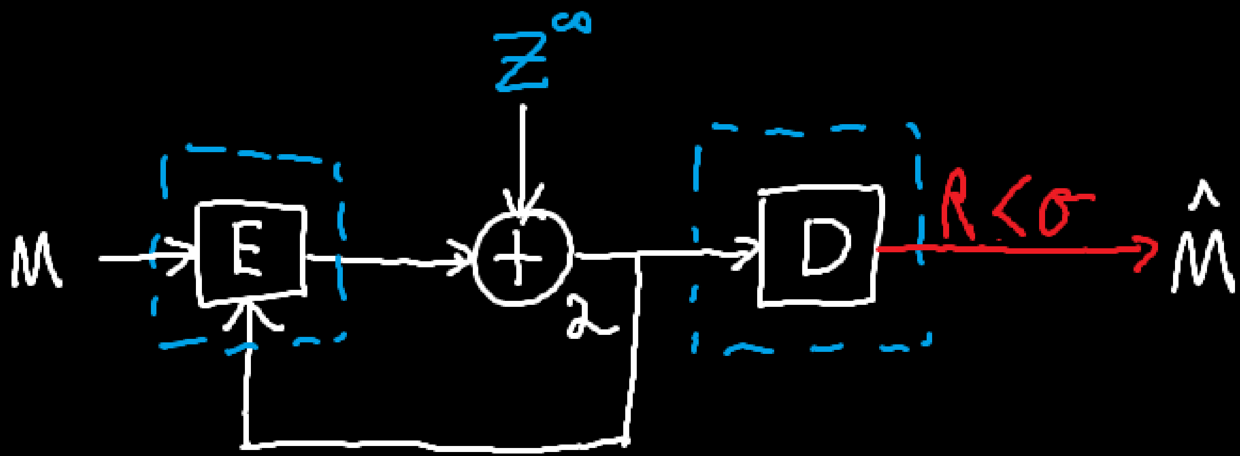
Modulo-additive channels



- Stronger analogy with source coding.
- Source process <-> Noise process.
- More general: individual noise sequence.

"Porosity," or $\sigma(z)$



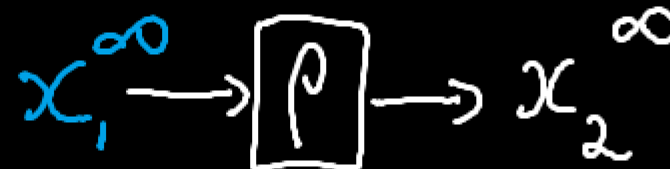


- How rapidly can encoder/decoder communicate?
- Best possible rate: $\sigma(z)$.

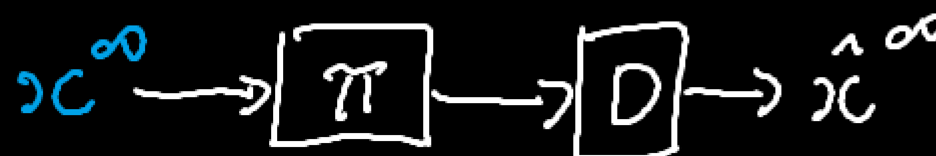
Individual sequence properties

Compressibility:
(Lempel/Ziv)

Predictability:
(Feder/Merhav/Gutman)



Denoisability:
(Weissman et al.)



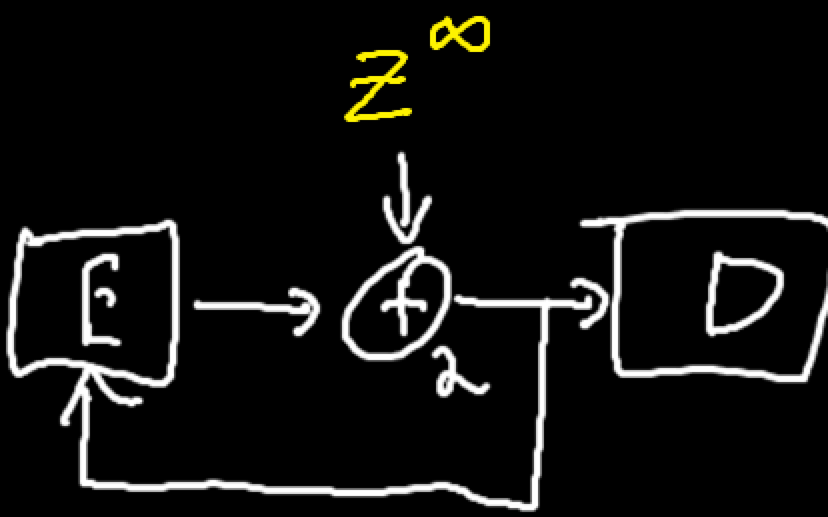
LZ Parallel #1: Individual sequences

LZ

Individual source sequence

Porosity

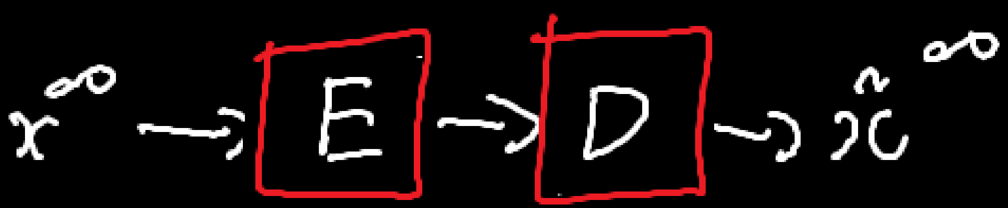
Individual noise sequence



LZ Parallel #2: Finite-state encoding/decoding

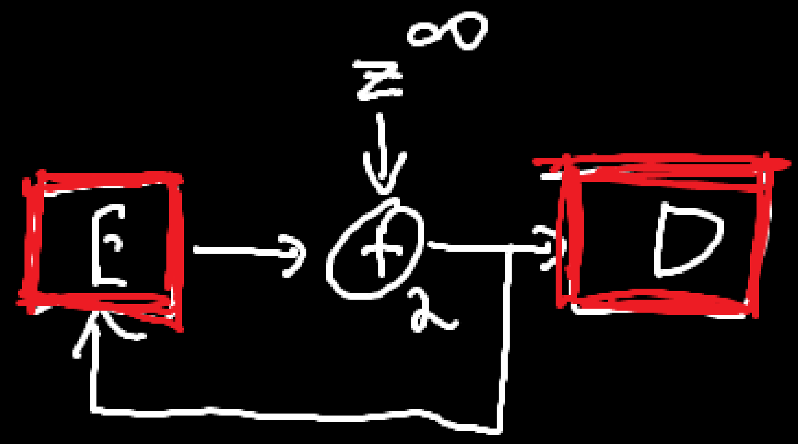
LZ

Finite state source encoder and decoder



Porosity

Finite state channel encoder and decoder

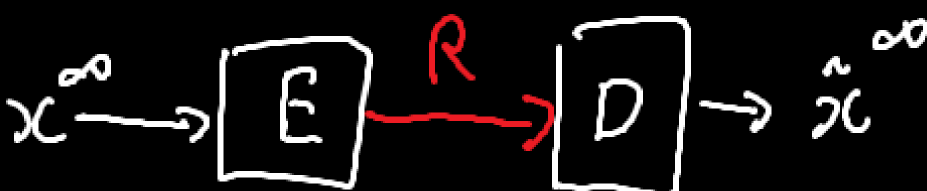


LZ Parallel #3: Finite-state converse

LZ

FSM can compress no better than compressibility.

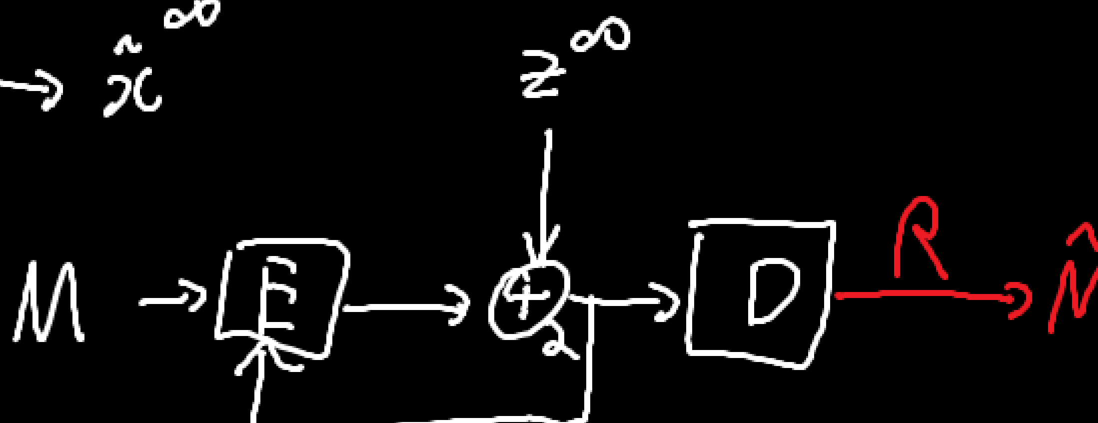
$$\rho(x) = \overline{\lim}_k \overline{\lim}_n \widehat{H}_k(x^n)$$



Porosity

FSM can transmit no faster than porosity.

$$\sigma(z) = ?$$



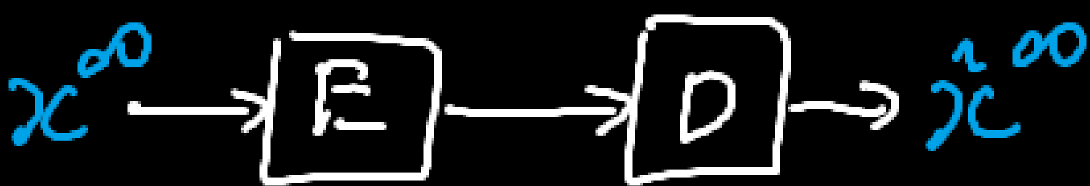
LZ Parallel #4: Universal achievability schemes

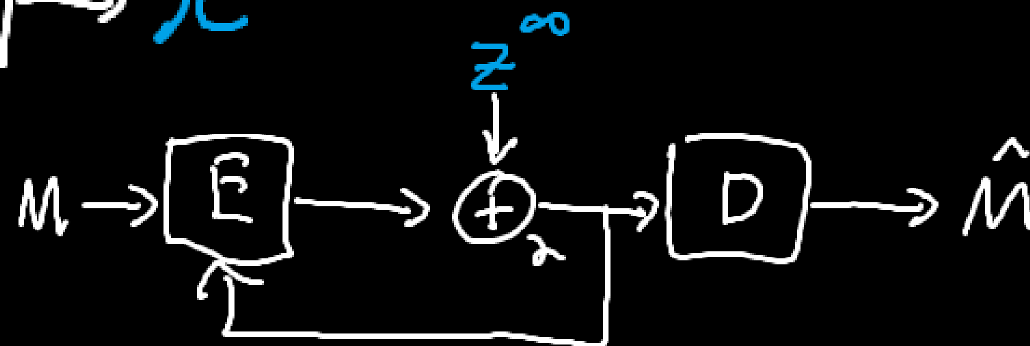
LZ

Sequence of FS schemes.
(simple!)

Porosity

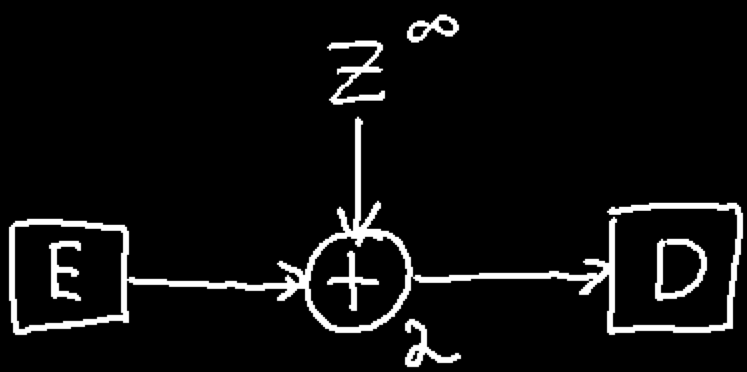
Sequence of FS schemes.
(not simple!)
Suboptimal FS schemes
(simple!)





Lomnitz/Feder (2011)

- Competitive universality introduced.
- Rate-adaptive scheme achieves $1 - \rho(z)$.
- No "iterated fixed-blocklength" scheme does better.



Shayevitz/Feder (2009)

- Achieves first-order "empirical capacity."

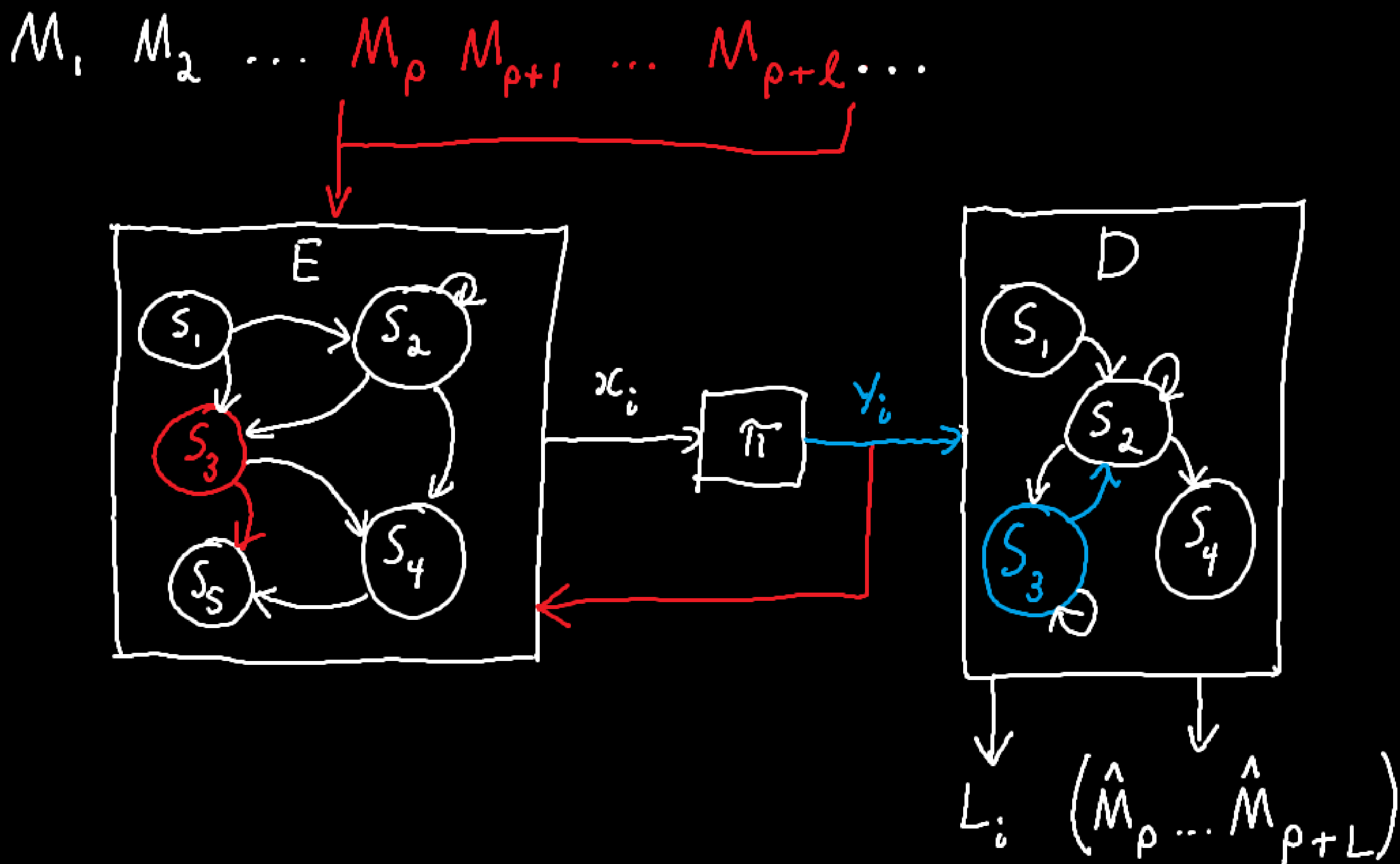
$$R_n \approx 1 - \widehat{H}_1(z^n)$$

- Can generalize to m-order empirical capacity.

$$R_n \approx 1 - \widehat{H}_m(z^n)$$

- Related: Eswaran/Sarwate/Sahai/Gastpar [2010].

Finite-state (FS) schemes



Converse

Suppose an FS scheme achieves rate R and error ϵ with positive probability.

Then $R < 1 - \rho(z) + h_b(\epsilon)$.

i.e. $\sigma(z) = 1 - \rho(z)$.

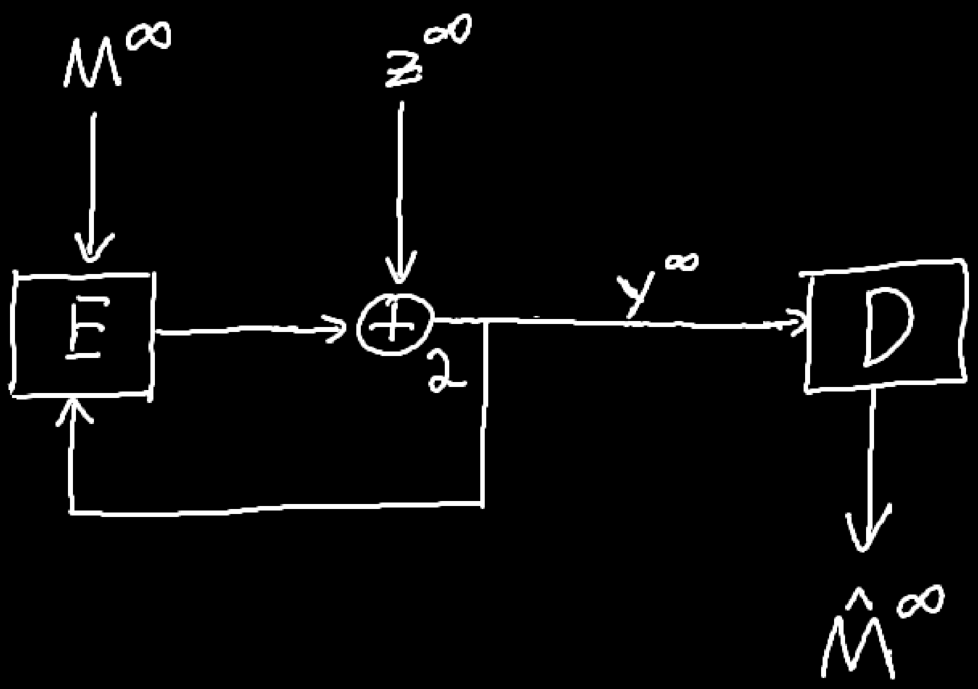
Achievability

There exists a sequence of FS schemes $\{\mathcal{F}_m\}$

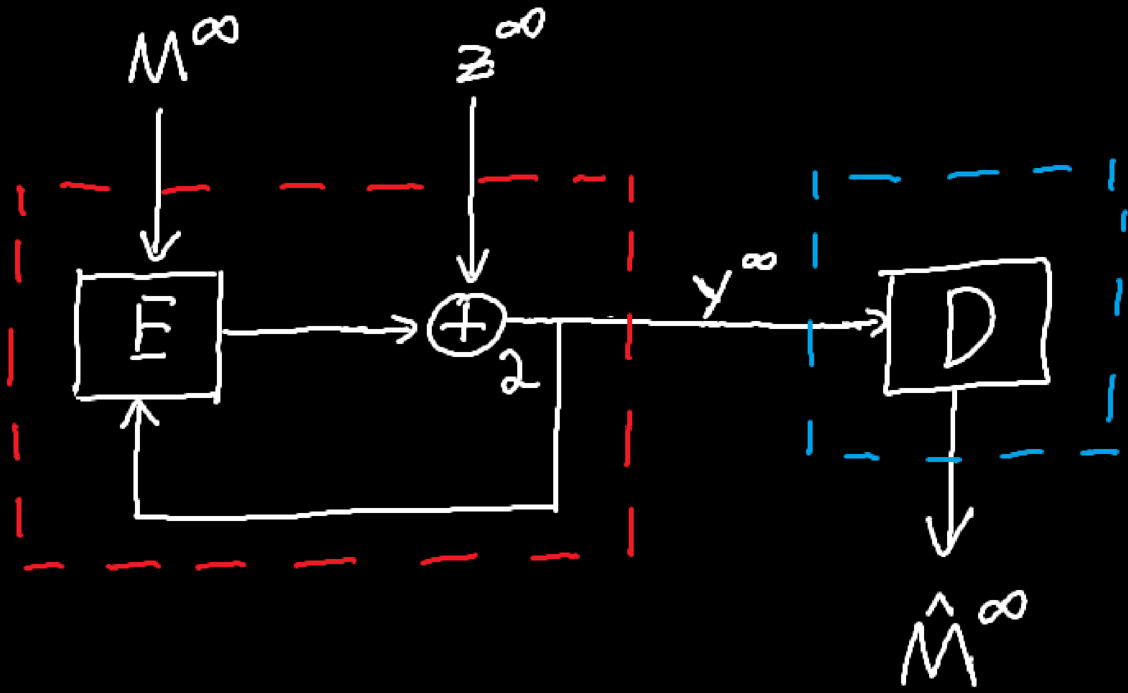
such that $(R_m, \epsilon_m) \to (\sigma(z), 0)$

for all noise sequences z .

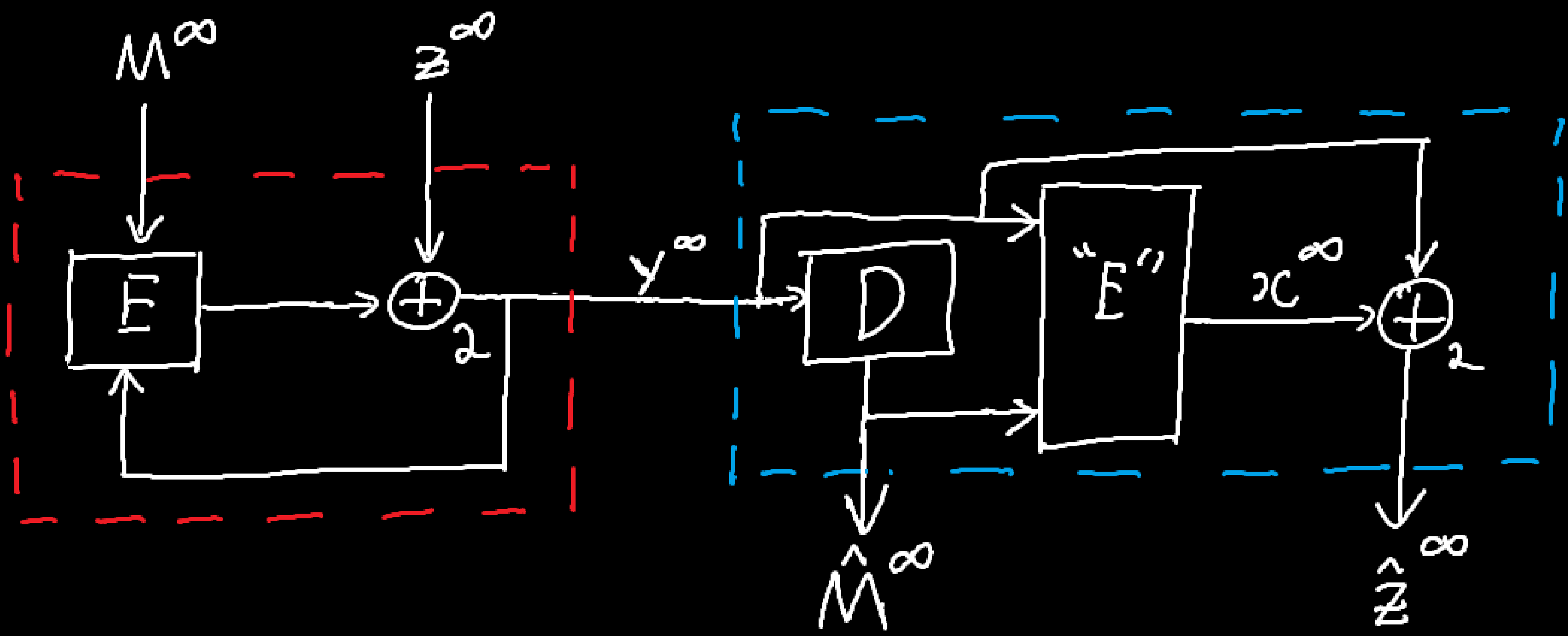
Channel Coding into Source Coding



Channel Coding into Source Coding



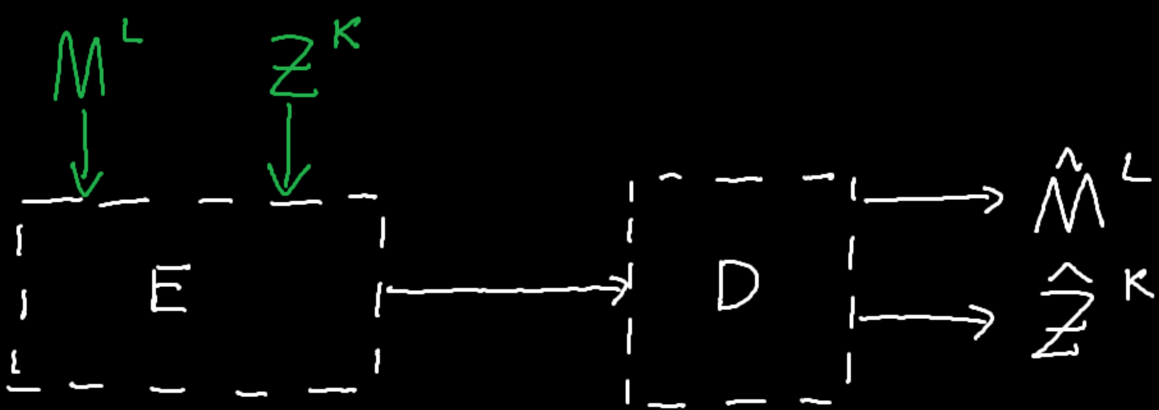
Channel Coding into Source Coding



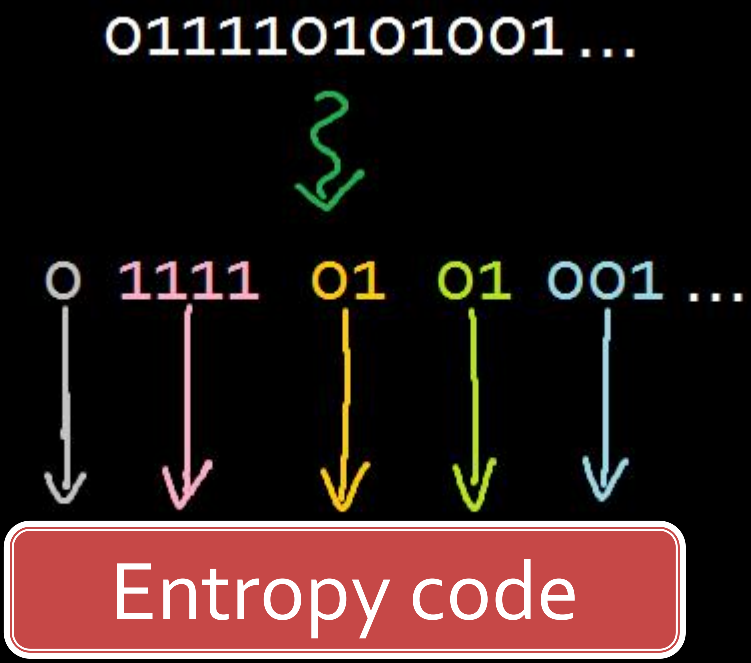
Deterministic into Stochastic

$$(M^{\infty}, z^{\infty}) \Longrightarrow \left| \begin{matrix} z_1 & z_2 & \cdots & z_K \\ M_1 & M_2 & \cdots & M_{L_1} \end{matrix} \right| \begin{matrix} z_{K+1} & z_{K+2} & \cdots & z_{2K} \\ M_{L_1+1} & \cdots & M_{L_1+L_2} & \end{matrix} \right| \cdots$$

$$\Longrightarrow \left| \begin{matrix} Z^K \\ M^L \end{matrix} \right| \sim P_{M^L Z^K}$$

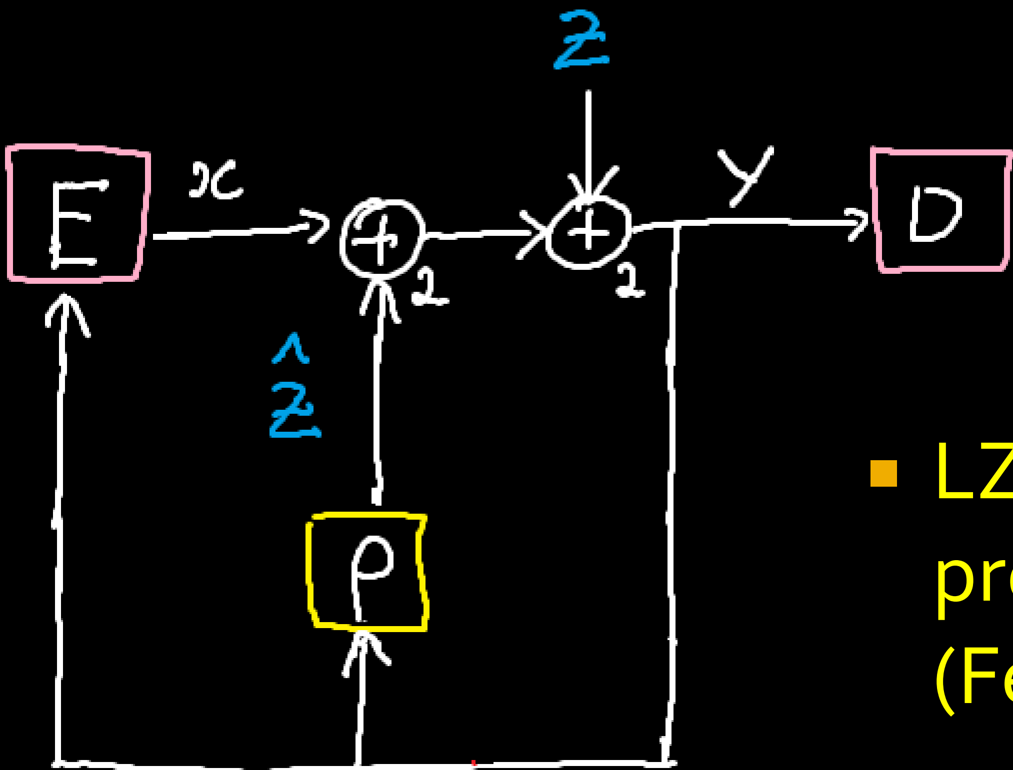


Cthulhu vs. Shannon.



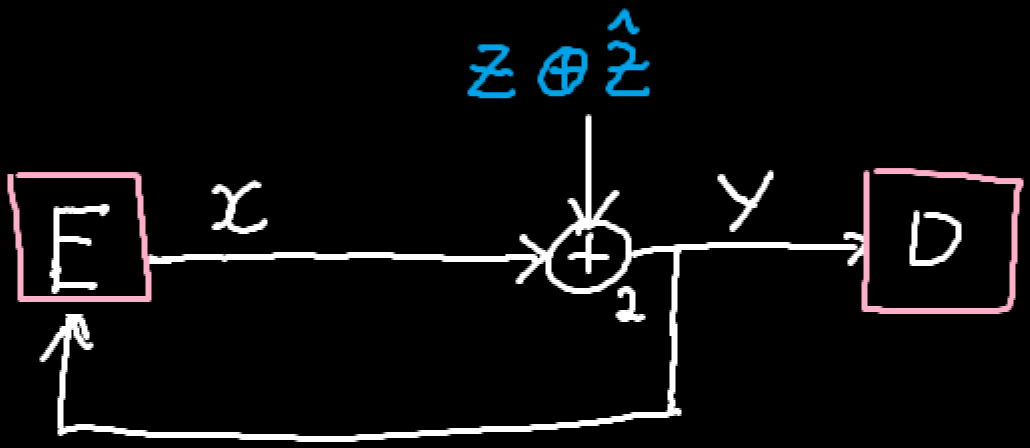
Cannot beat 1 bit per sample.

A linear-complexity alternative



- LZ-based universal predictor. (Feder et al. [92])
- **<u>1st order</u>** S/F scheme

A linear-complexity alternative



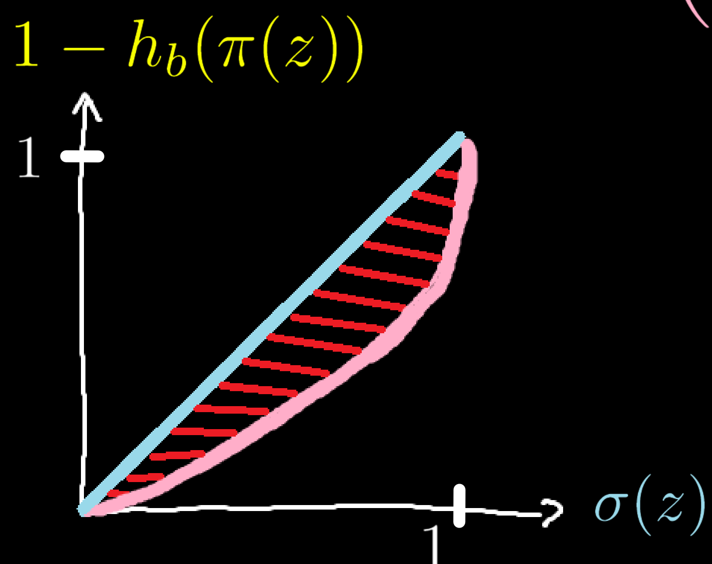
- LZ-based universal predictor.
  (Feder et al. [92])
- **<u>1st order</u>** S/F scheme

Performance?

Shayevitz/Feder rate guarantee: $1 - \widehat{H}_1(z \oplus \hat{z})$

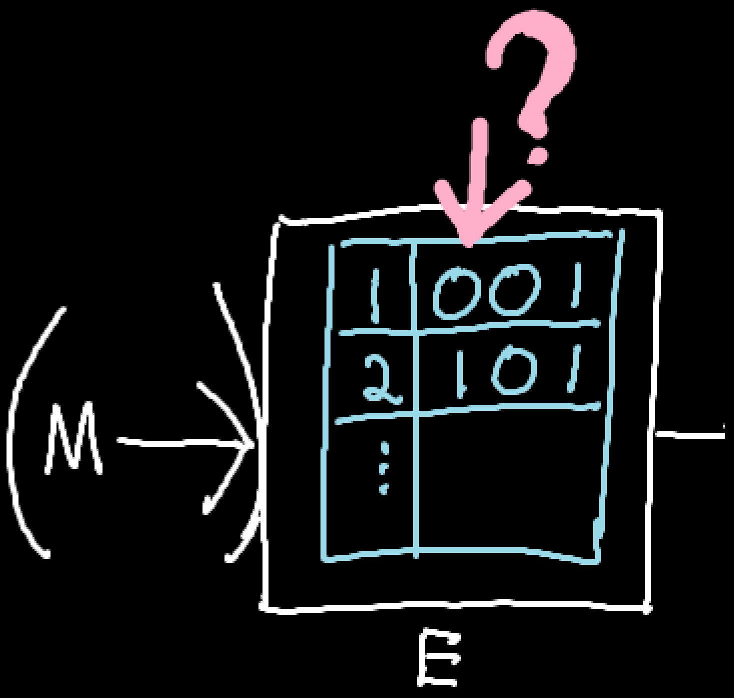
$$1 - h_b\left(\frac{1-\sigma(z)}{2}\right) \leq 1 - h_b(\pi(z)) \leq \sigma(z)$$

(predictability)



(Closing the gap?)

Universal Channel *En*Coding



- Codebook hard-wired.
- Compound channel approach: optimize for worst-case channel.
- Bayesian approach: assume distribution on possible channels.