Control Design Based on Limited Plant Model Information with Applications to Power Systems

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Joint work with Farhad Farokhi and Cédric Langbort

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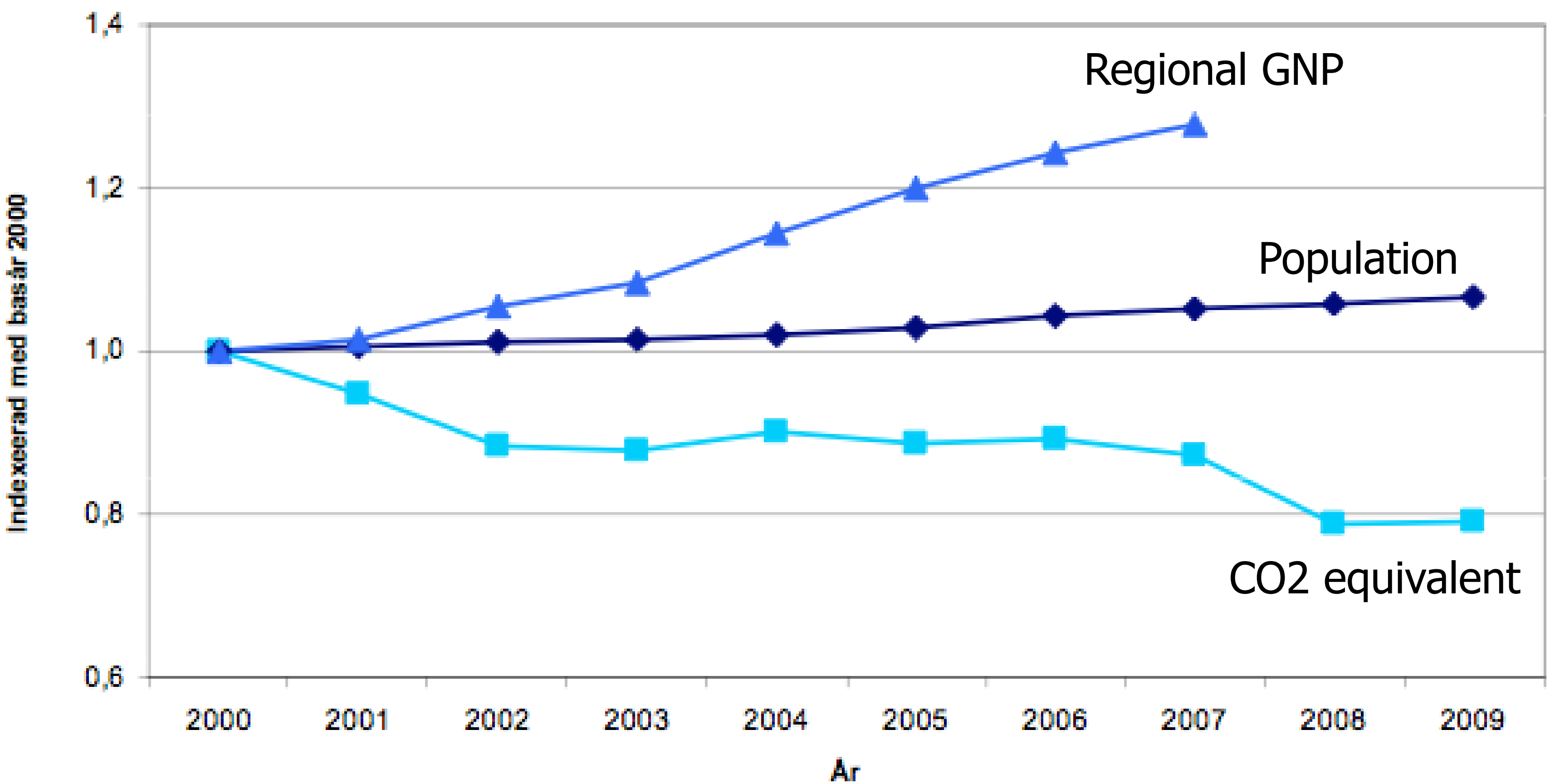
Stockholm Challenge



Stockholm Royal Seaport

2010

- Oil depot
- Container terminal
- Ports
- Gas plant

2030

- 10,000 new homes
- 30,000 new work spaces
- 600,000 m2 commercial space
- Modern port and cruise terminal
- 236 hectares sustainable urban district
- Walking distance to city centre

From a brown field area to a sustainable city district

Stockholm Royal Seaport

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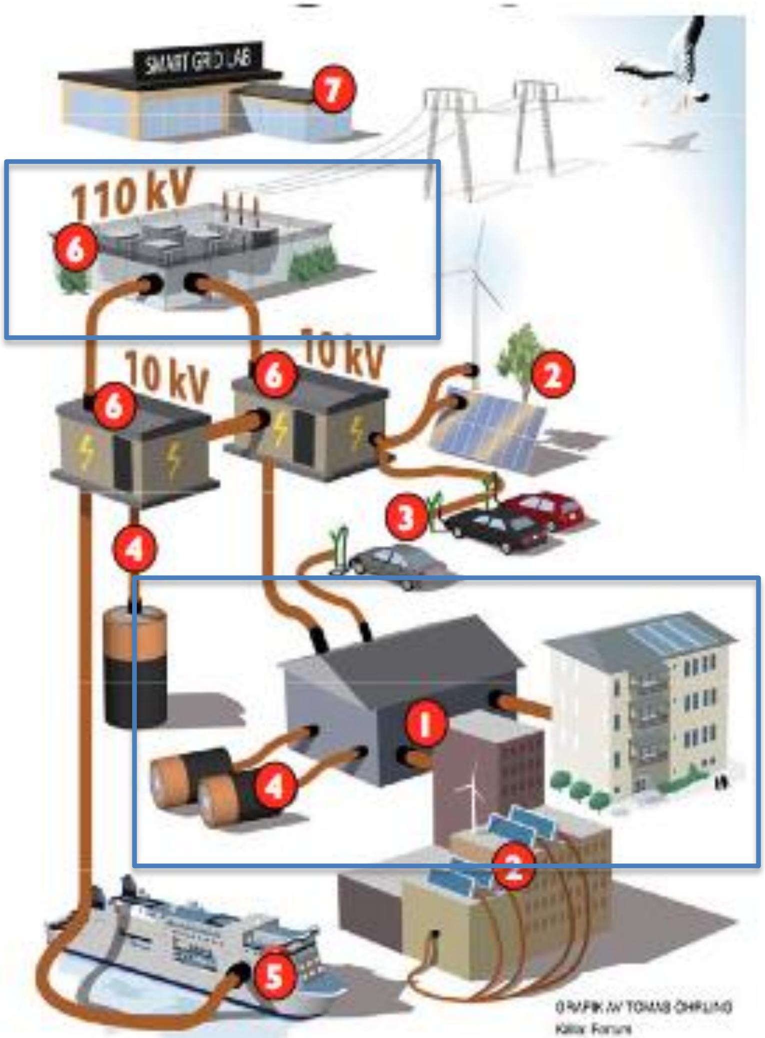
Smart Grid in the Stockholm Royal Seaport

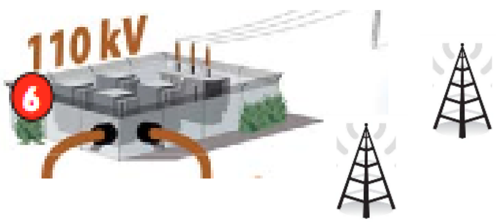


1. Smart homes/Buildings and Demand Response
2. Distributed Energy Systems
3. Integration and Use of electric vehicles
4. Energy Storage for customers and the grid
5. Smart electrified harbour
6. Smart Primary Substations
7. Smart Grid Lab (part of an innovation Center)

Fortum · KTH · ABB · NCC · JM · Interactive Institute · Ericsson · Electrolux · ByggVesta · Stockholms Stad · Stockholm European Green Capital · C40 Cities · Clinton Climate Initiative

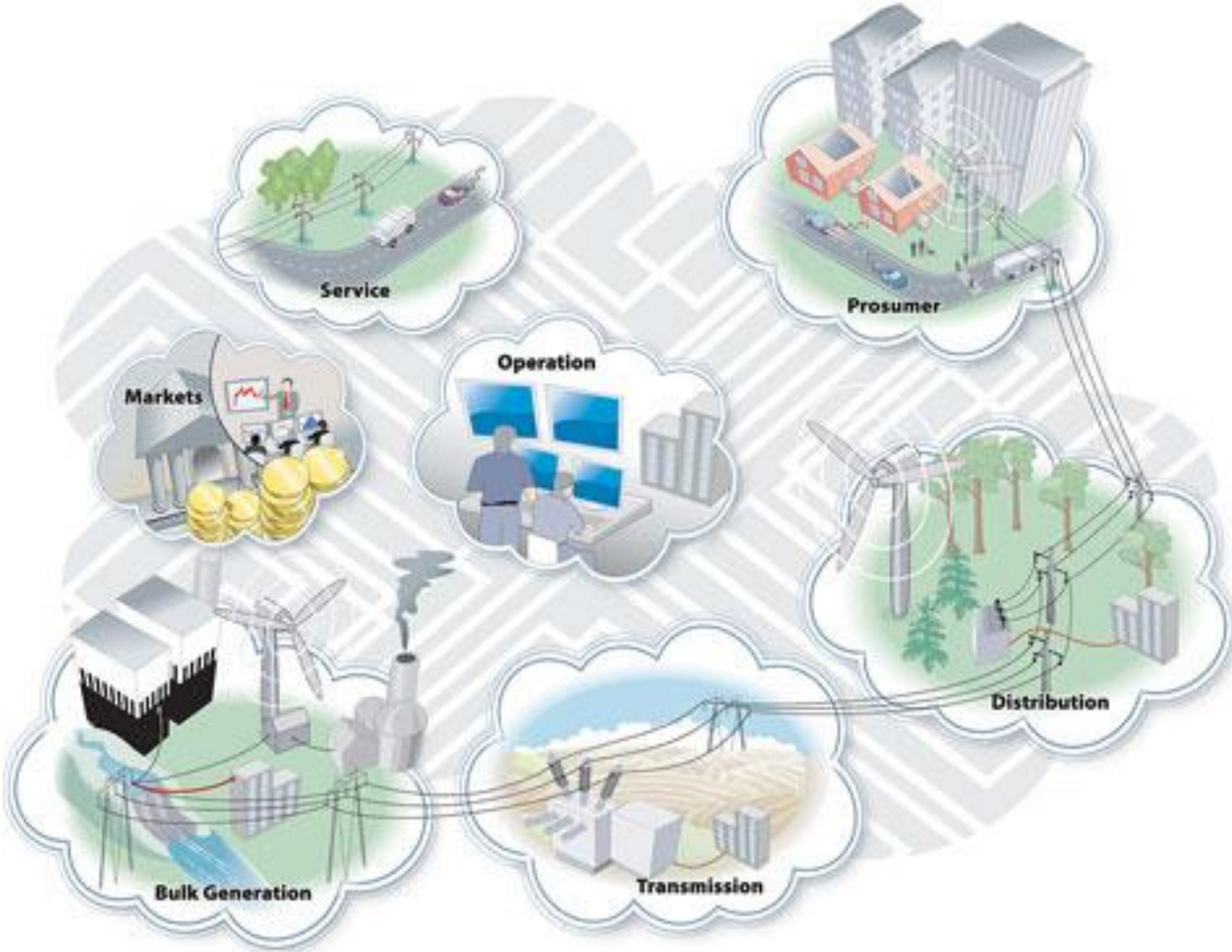
Two Royal Seaport Projects

- Smart grid communications over 4G LTE
  - Can power grid control loops be closed over the mobile communication infrastructure?



- Appliance scheduling and storage in smart buildings
  - Control architecture and mechanisms for demand-response

From Centralized to Distributed Control



Example

$$x_1(k+1) = a_{11}x_1(k) + a_{12}x_2(k) + u_1(k)$$
$$x_2(k+1) = a_{21}x_1(k) + a_{22}x_2(k) + u_2(k)$$

Keep J small, when

- Controller 1 knows a_{11} and a_{12}
- Controller 2 knows a_{21} and a_{22}

$$J = \sum_{k=1}^{\infty} \|x(k)\|^2 + \|u(k)\|^2$$

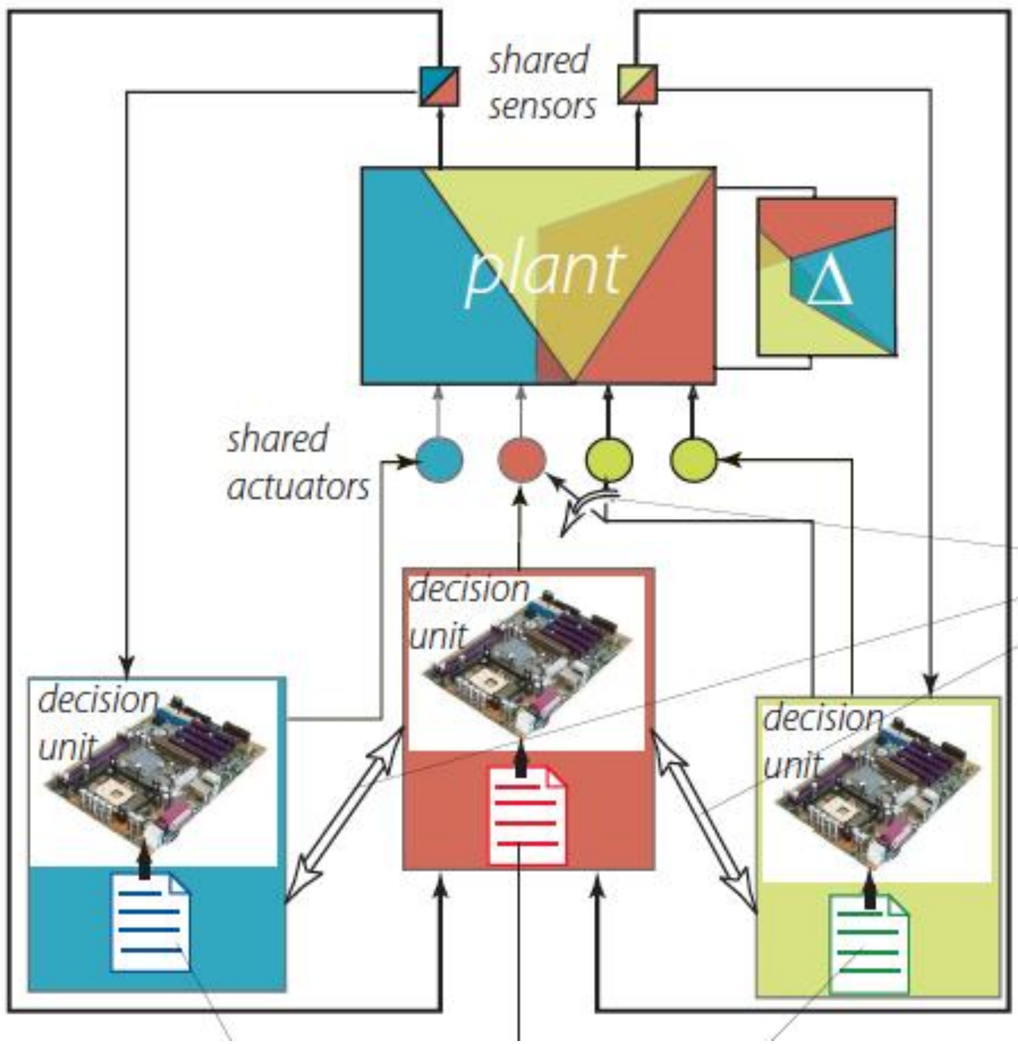
$$u_1(k) = -a_{11}x_1(k) - a_{12}x_2(k)$$
$$u_2(k) = -a_{21}x_1(k) - a_{22}x_2(k)$$

achieves $J \leq 2J^*$

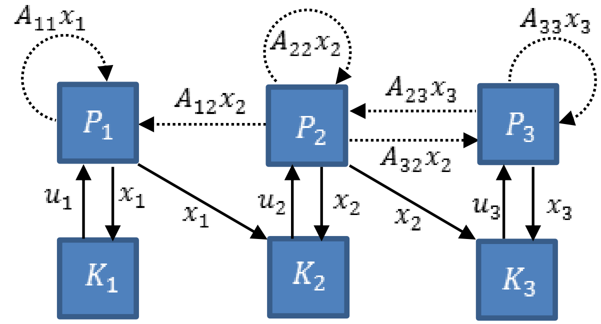
No limited plant model information strategy can do better.

Langbort & Delvenne, 2010

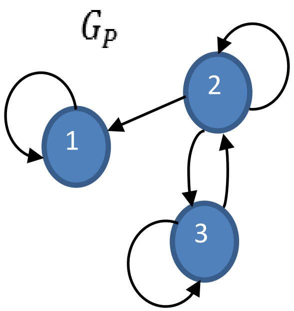
Distributed Control with Limited Model Information



Plant



Plant graph



Adjacency matrix

$$S_P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

System matrix

$$A = \begin{bmatrix} A_{11} & A_{12} & 0_{n_1 \times n_3} \\ 0_{n_2 \times n_1} & A_{22} & A_{23} \\ 0_{n_3 \times n_1} & A_{32} & A_{33} \end{bmatrix}$$

Plant

$$P = (A, B, x_0) \in \mathcal{A} \times \mathcal{B} \times \mathcal{X}$$

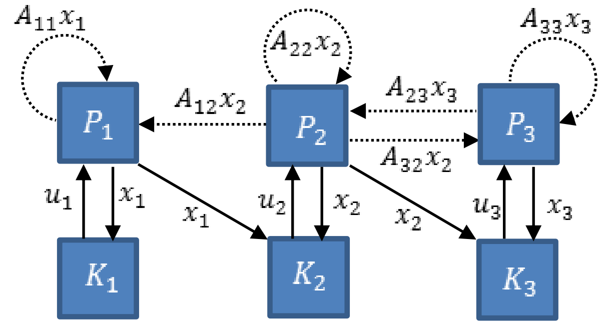
$$\mathcal{A} = \{ A \in R^{n \times n} | A_{ij} = 0 \in R^{n_i \times n_j} \text{ for all } 1 \leq i, j \leq q \text{ such that } (s_P)_{ij} = 0\}$$

$$\mathcal{B} = \{B \in R^{n \times n} \mid \underline{\sigma}(B) \geq \epsilon, B_{ij} = 0 \in R^{n_i \times n_j} \text{ for all } 1 \leq i \neq j \leq q\}$$

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j \neq i} A_{ij}x_j(k) + B_{ii}u_i(k)$$

$$x_i \in R^{n_i} \text{ and } u_i \in R^{n_i}$$

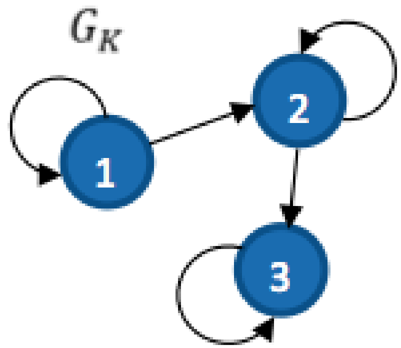
Controller



Control graph

Adjacency matrix

State feedback gain



$$S_K = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

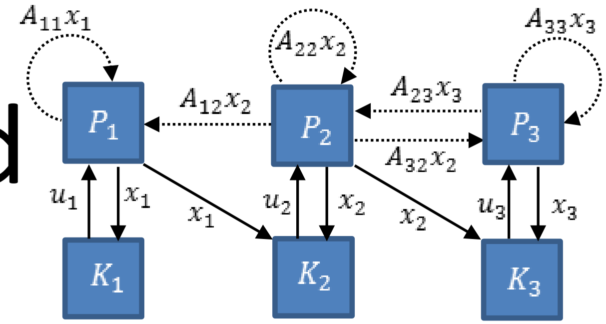
$$K = \begin{bmatrix} K_{11} & 0_{n_1 \times n_2} & 0_{n_1 \times n_3} \\ K_{21} & K_{22} & 0_{n_2 \times n_3} \\ 0_{n_3 \times n_1} & K_{32} & K_{33} \end{bmatrix}$$

Controller

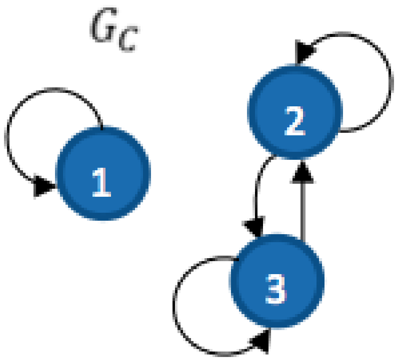
$$u(k) = Kx(k)$$

$$\mathcal{K} = \{ K \in R^{n \times n} | K_{ij} = 0 \in R^{n_i \times n_j} \text{ for all } 1 \leq i,j \leq q \text{ such that } (s_K)_{ij} = 0 \}$$

Control Design Method



Control design graph



Adjacency matrix

$$S_C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Control design method

$$\Gamma : \mathcal{A} \times \mathcal{B} \rightarrow \mathcal{K}$$

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \cdots & \Gamma_{1q} \\ \vdots & \ddots & \vdots \\ \Gamma_{q1} & \cdots & \Gamma_{qq} \end{bmatrix}$$

$$\begin{bmatrix} \Gamma_{i1} & \cdots & \Gamma_{iq} \end{bmatrix} \text{ is a function of } \left\{ \begin{bmatrix} A_{j1} & \cdots & A_{jq} \end{bmatrix}, B_{jj} \mid (s_C)_{ij} \neq 0 \right\}$$

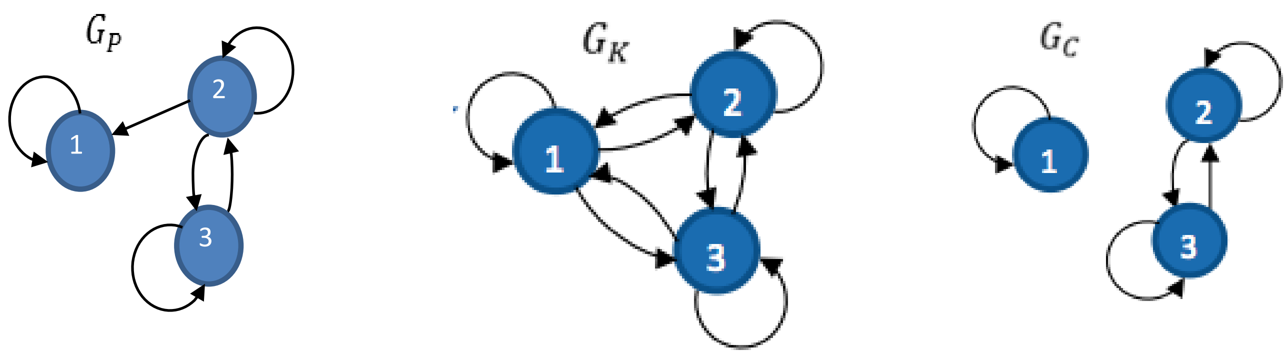
$$K = \Gamma(A, B)$$

$$\mathcal{K} = \{ K \in R^{n \times n} | K_{ij} = 0 \in R^{n_i \times n_j} \text{ for all } 1 \leq i, j \leq q \text{ such that } (s_K)_{ij} = 0 \}$$

Performance metric

$$J_P(K) = \sum_{k=1}^{\infty} x(k)^T Q x(k) + \sum_{k=0}^{\infty} u(k)^T R u(k)$$

given



Assumptions

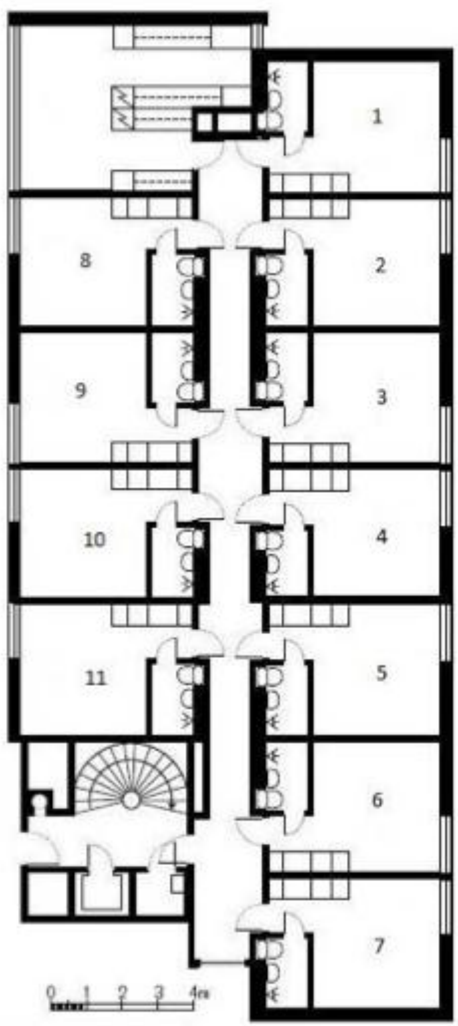
- $\mathcal{B} = \{B \in R^{n \times n} \mid \underline{\sigma}(B) \geq \epsilon, B_{ij} = 0 \in R^{n_i \times n_j} \text{ for all } 1 \leq i \neq j \leq q\}$

  is a set of diagonal matrices
- G_K is a complete graph
- $Q = R = I.$
- for every plant $P = (A, B, x_0) \in \mathcal{P}$, there

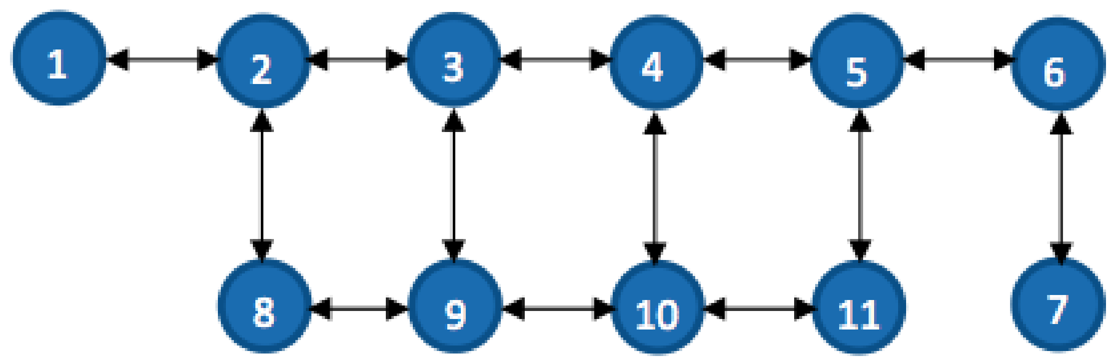
  exists an optimal controller $K^*(A, B) \in \mathcal{K}$ such that

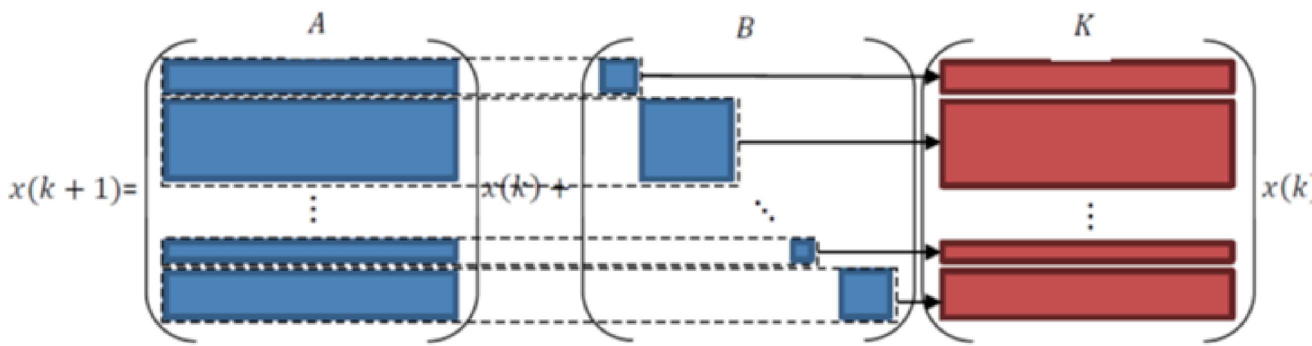
  $$J_P\,(K^*(A, B)) \leq J_P\,(K), \qquad \text{for all } K \in \mathcal{K}$$

Motivating HVAC Example



Plant graph





Competitive Ratio and Dominance

The **competitive ratio** of a control design method $\Gamma : \mathcal{A} \times \mathcal{B} \rightarrow \mathcal{K}$

$$r_{\mathcal{P}}(\Gamma) = \sup_{P=(A,B,x_0)\in\mathcal{P}} \frac{J_P(\Gamma(A,B))}{J_P(K^*(P))}$$

Γ **dominates** another control design method Γ' if

$$J_P(\Gamma(A,B)) \leq J_P(\Gamma'(A,B)), \quad \forall\, P = (A,B,x_0) \in \mathcal{P};$$

Langbort & Delvenne, 2011

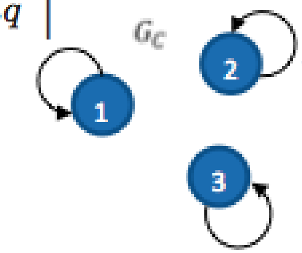
Deadbeat Control

The deadbeat control design method $\Gamma^{\Delta} : \mathcal{A}(S_{\mathcal{P}}) \times \mathcal{B}(\epsilon) \to \mathcal{K}$ is

$$\Gamma^{\Delta}(A, B) = -B^{-1}A, \; \textit{for all } P = (A, B, x_0) \in \mathcal{P}$$

The control design for subsystem i depends only on subsystem i's controller gains:

$$\begin{bmatrix} \Gamma_{i1}(A,B) & \Gamma_{i2}(A,B) & \cdots & \Gamma_{iq}(A,B) \end{bmatrix} = B_{ii}^{-1} \begin{bmatrix} A_{i1} & A_{i2} & \cdots & A_{iq} \end{bmatrix}$$



Lemma Suppose $G_{\mathcal{P}}$ contains no isolated node. Then,

$$r_{\mathcal{P}}(\Gamma^{\Delta}) = 1 + 1/\epsilon^2$$

The performance of the deadbeat control design method is at most $1 + 1/\epsilon^2$ times the performance of the optimal control design method as

$$r_{\mathcal{P}}(\Gamma) = \sup_{P=(A,B,x_0)\in\mathcal{P}} \frac{J_P(\Gamma(A,B))}{J_P(K^*(P))}$$

Cheap Control

For the performance metric

$$J_P(K) = \sum_{k=1}^{\infty} x(k)^T Q x(k) + \sum_{k=0}^{\infty} u(k)^T R u(k)$$

the competitive ratio

$$r_{\mathcal{P}}(\Gamma) = \sup_{P=(A,B,x_0)\in\mathcal{P}} \frac{J_P(\Gamma(A,B))}{J_P(K^*(P))}$$

is

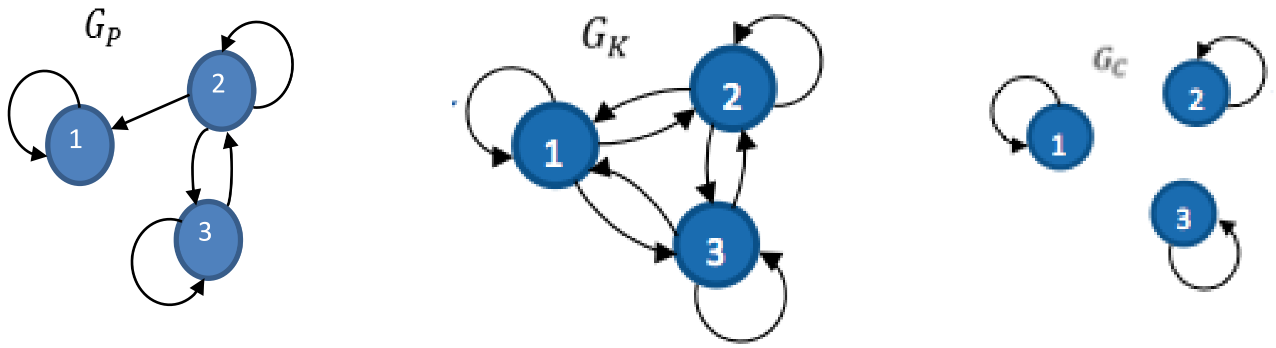
$$r_P(\Gamma^\Delta) = 1 + \overline{\sigma}(R)/(\underline{\sigma}(Q)\epsilon^2)$$

Hence, as $\overline{\sigma}(R)/\underline{\sigma}(Q)$ goes to zero, the LQ controller converges to deadbeat.

Competitive Ratio

Theorem Suppose $G_{\mathcal{P}}$ has no isolated node, G_K is a complete graph, and and G_C is totally disconnected. Then, the competitive ratio of any control design method $\Gamma \in \mathcal{C}$ satisfies

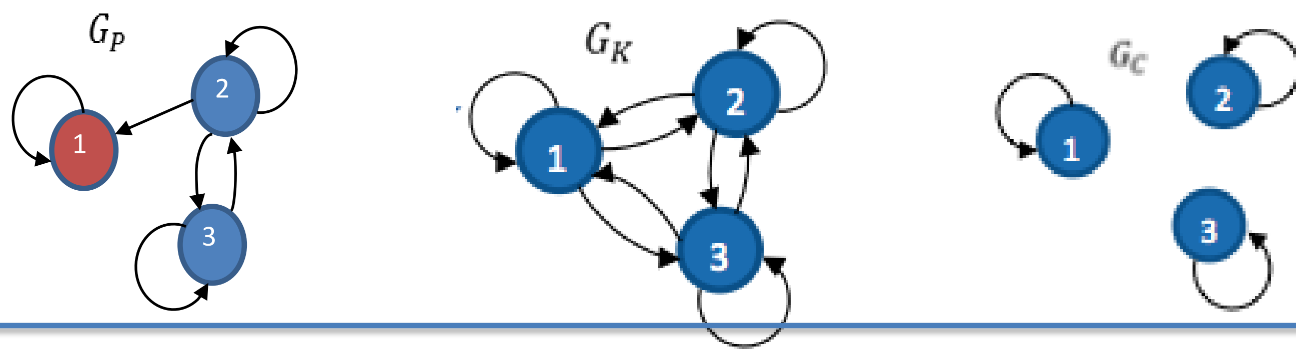
$$r_{\mathcal{P}}(\Gamma) \geq 1 + 1/\epsilon^2.$$



$$r_{\mathcal{P}}(\Gamma) = \sup_{P=(A,B,x_0)\in\mathcal{P}} \frac{J_P(\Gamma(A,B))}{J_P(K^*(P))}$$

Deadbeat is Undominated

Theorem Suppose $G_{\mathcal{P}}$ has no isolated node, G_K is a complete graph, and and G_C is totally disconnected. Then, deadbeat is undominated if and only if $G_{\mathcal{P}}$ has no sink

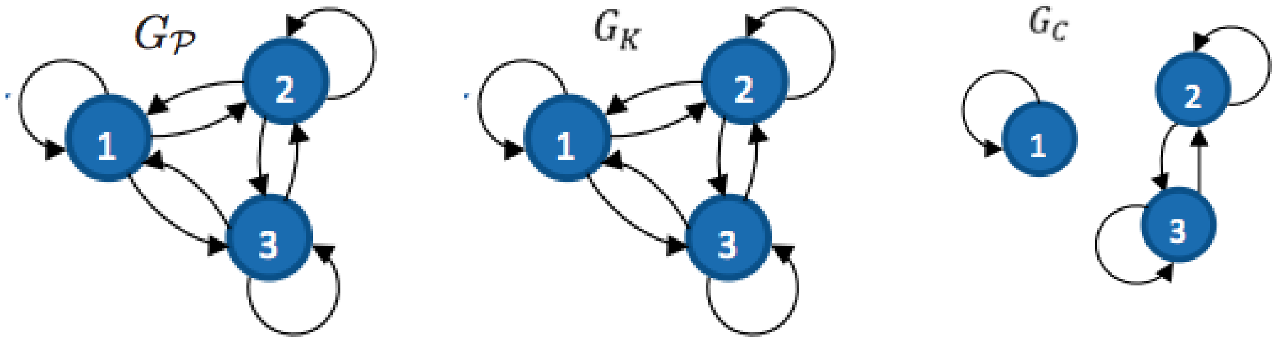


If $G_{\mathcal{P}}$ has one or more sinks, then the control method should be modified to do "local optimal control" for each sink and deadbeat for the other nodes.

Influence of Design Information

Theorem Suppose $G_{\mathcal{P}}$ and G_K are complete graphs. If $G_{\mathcal{C}} \neq G_{\mathcal{P}}$ then

$$r_{\mathcal{P}}(\Gamma) \geq 1 + 1/\epsilon^2.$$



Achieving a better competitive ratio than the deadbeat design strategy requires each subsystem to have full knowledge of the plant model in the design of each subcontroller.

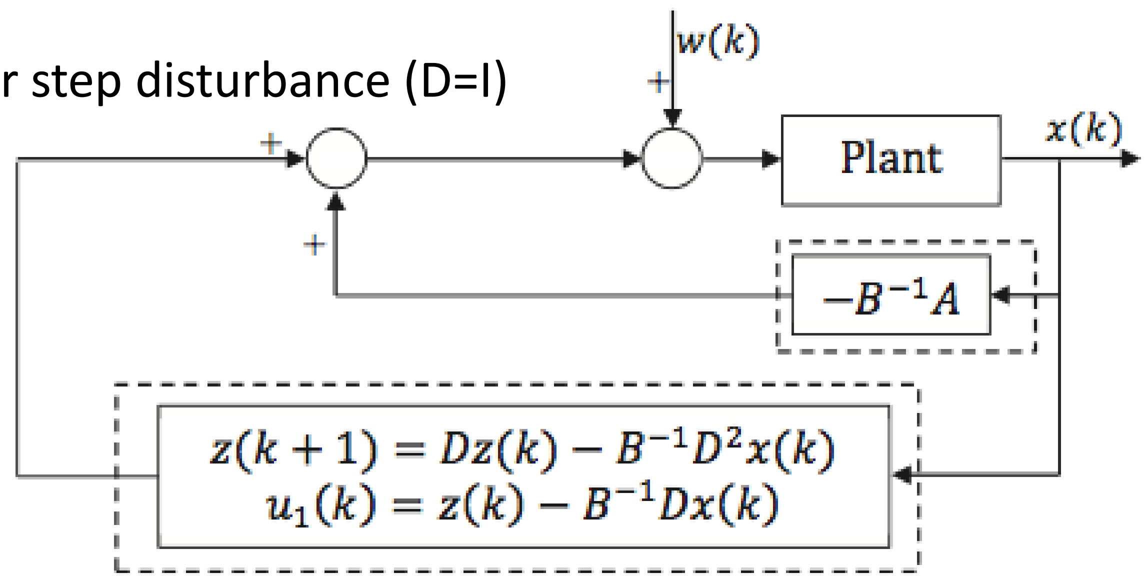
Servomechanism Design

Extensions to plant with disturbance:

$$x(k+1) = Ax(k) + B(u(k) + w(k))\ ;\ x(0) = x_0,$$
$$w(k+1) = Dw(k)\ ;\ w(0) = w_0$$

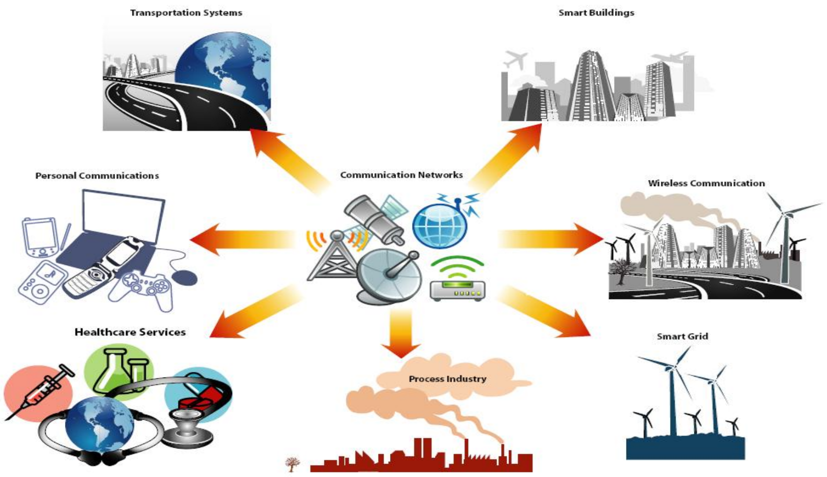
Deadbeat controller with deadbeat observer

Corresponds to PI control for step disturbance (D=I)



Conclusions

- Smart energy systems often lead to distributed control problems with limited information exchange
- Considered the role of plant model information
- Achievable performance in terms of competitive ratio can be derived for certain cases
- Provides insights on control information topologies (not necessarily design)



http://www.ee.kth.se/~kallej

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- F. Farokhi, C. Langbort, and K. H. Johansson, “Control design with limited model information,” American Control Conference, 2011.

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the future