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# The Theory of Fast and Robust Adaptation

## $L_1$ Adaptive Control

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**A. M. Lyapunov**  
1857-1918

LCCC workshop April 21-23, 2010  
Lund University, Sweden



**G. Zames**  
1934-1997

# Outline

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- **Historical Overview**
- **V&V Challenge of Adaptive Control**
- **Certification of Advanced FCS**
- **Speed of Adaptation, Performance, Robustness**
- **Separation between Adaptation and Robustness**
- **Aerospace Applications**
  - **NPS flight tests**
  - **AirSTAR flight tests (IRAC, NASA)**
- **Networked control systems**
  - **NPS flight tests**
- **Conclusions, summary, and future work**

# Motivation

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- **Early 1950s – design of autopilots operating at a wide range of altitudes and speeds**
  - Fixed gain controller did not suffice for all conditions
    - Gain scheduling for various conditions
  - Several schemes for self-adjustment of controller parameters
    - Sensitivity rule, MIT rule
  - **1958, R. Kalman, self-tuning controller**
    - **Optimal LQR with explicit identification of parameters**
- **1950-1960 – flight tests X-15 (NASA, USAF, US Navy)**
  - bridge the gap between manned flight in the atmosphere and space flight
  - Mach 4 - 6, at altitudes above 30,500 meters (100,000 feet)
  - 199 flights beginning June 8, 1959 and ending October 24, 1968
  - Nov. 15, 1967, X-15A-3

# First Flight Test in 1967

- The crash of the X-15A-3 (November 15, 1967)

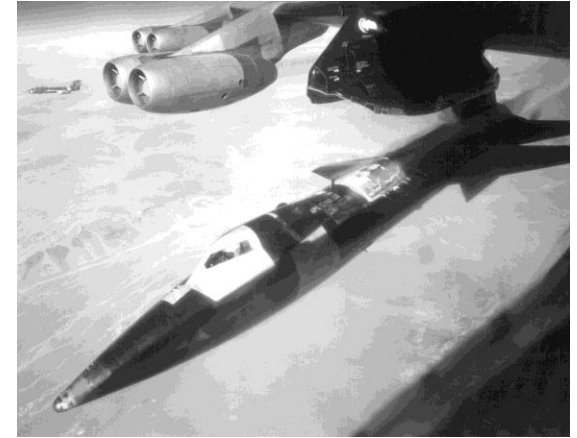


X-15 in Flight Air Force Photo Date Unknown



NASA Dryden Flight Research Center Photo Collection  
<http://www.dfrc.nasa.gov/gallery/photo/index.html>  
NASA Photo: E-4935 Date: 1959 Photo by: NASA

X-15 Mated to B-52 Captive Flight



Crash site of the X-15A-3

**Crash due to stable, albeit non-robust  
adaptive controller!**

# Historical Background

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- Sensitivity Method, MIT Rule, Limited Stability Analysis (1960s)
    - ❖ Whitaker, Kalman, Parks, et al.
  - Lyapunov based, Passivity based (1970s)
    - ❖ Morse, Narendra, Landau, et al.
  - Global stability proofs (1970-1980s)
    - ❖ Astrom, Wittenmark, Morse, Narendra, Landau, Goodwin, Keisselmeier, Anderson, et al.
  - Robustness issues, instability (early 1980s)
    - ❖ Egardt, Ioannou, Stein, Athans, Valavani, Rohrs, Anderson, Sastry, et al.
  - Robust Adaptive Control (1980s)
    - ❖ Ioannou, Praly, Tsakalis, Sun, Tao, Datta, Middleton, Basar, et al.
  - Nonlinear Adaptive Control (1990s)
    - Adaptive Backstepping, Neuro, Fuzzy Adaptive Control
      - ❖ Krstic, Kanelakopoulos, Kokotovic, Zhang, Ioannou, Narendra, Ioannou, Lewis, et al.
- 
- Search methods, multiple models, switching techniques (1990s)
    - ❖ Martenson, Miller, Barmish, Morse, Narendra, Anderson, Safonov, Hespanha, et al.

# Landmark Achievement: Adaptive Control in Transition

Air Force programs: **RESTORE** (X-36 unstable tailless aircraft 1997), **JDAM** (late 1990s, early 2000s)

- Demonstrated that there is no need for wind tunnel testing for determination of aerodynamic coefficients
  - ✓ an estimate for the wind tunnel tests is \$8-10mln at Boeing

Lessons Learned: limited to slowly-varying uncertainties, lack of transient characterization

- Fast adaptation leads to high-frequency oscillations in control signal, reduces the tolerance to time-delay in input/output channels
- Determination of the “best rate of adaptation” heavily relies on “expensive” Monte-Carlo runs



**Boeing question:** How fast to adapt to be robust?

# Main Features of $L_1$ Adaptive Control

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- Separation (decoupling) between adaptation & robustness
- Performance limitations consistent with hardware limitations
- Guaranteed **fast adaptation**
- Guaranteed **transient response** for system's **input and output**
  - **NOT** achieved via **persistence of excitation** or **gain-scheduling**
- Guaranteed (bounded away from zero) time-delay margin
- Uniform **scaled transient response** dependent on changes in initial conditions, unknown parameters, and reference input
- Suitable for development of **theoretically justified Verification & Validation tools** for feedback systems

# Key Feature: Feasibility of the Control Objective

- System:  $\dot{x}(t) = A_m x(t) + b(u + \theta^\top(t)x(t))$ ,  $x(0) = x_0$

- Nominal controller in MRAC:  $u_{\text{MRAC}}(t) = -\theta^\top(t)x(t) + k_g r(t)$

- Desired Reference System:

$$\dot{x}_{\text{des}}(t) = A_m x_{\text{des}}(t) + b k_g r(t)$$

Overly  
ambitious goal

- Nominal controller in  $L_1$ :  $u_{\mathcal{L}_1}(t) = C(s) \{-\theta^\top(t)x(t) + k_g r(t)\}$

- Achievable reference system:

$$\dot{x}_{\text{ref}}(t) = A_m x_{\text{ref}}(t) + b \left( (1 - C(s)) \{\theta^\top(t)x_{\text{ref}}(t)\} + C(s) \{k_g r(t)\} \right)$$

- Sufficient condition for stability:

$$\left\| (1 - C(s)) (s\mathbb{I} - A_m)^{-1} b \right\|_{\mathcal{L}_1} < \frac{1}{L} \quad \Rightarrow \quad \|x_{\text{ref}}\|_{\mathcal{L}_\infty} < \infty$$

**Result:** Fast and robust adaptation with continuous feedback!



# Red Flags Raised in Literature

- **Brian Anderson's quote\*:**

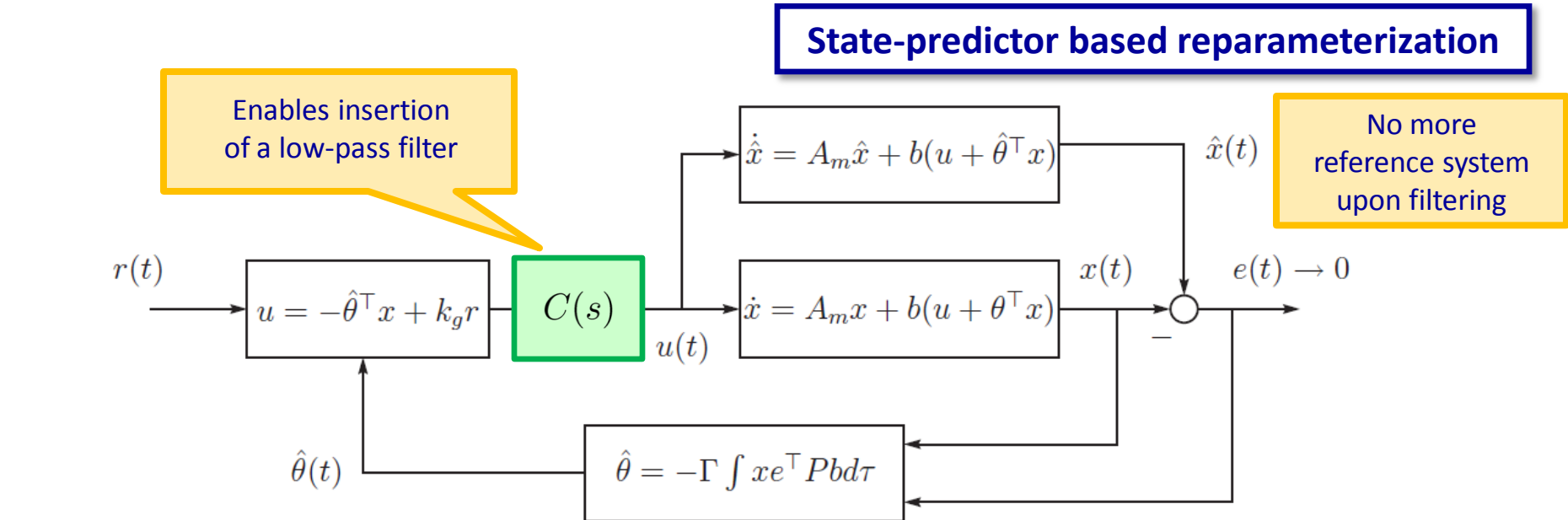
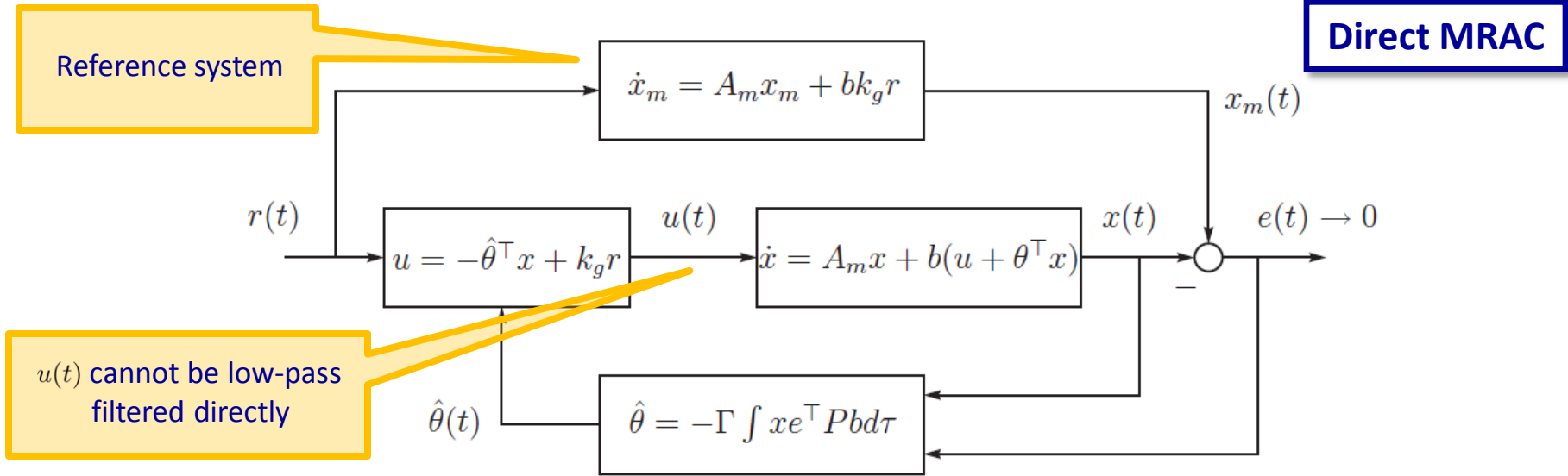
*“The notion of having a flag in an adaptive control algorithm to indicate the inappropriateness of an originally posed objective is practically important, and missing from older adaptive control literature. Logic really demands it. If a plant is initially unknown or only partially unknown, a designer may not know a priori that a proposed design objective is or is not practically obtainable for the plant.”*

\* **“Failures of Adaptive Control Theory”**,  
COMMUNICATIONS IN INFORMATION AND SYSTEMS, Vol. 5, No. 1, pp. 1-20, 2005

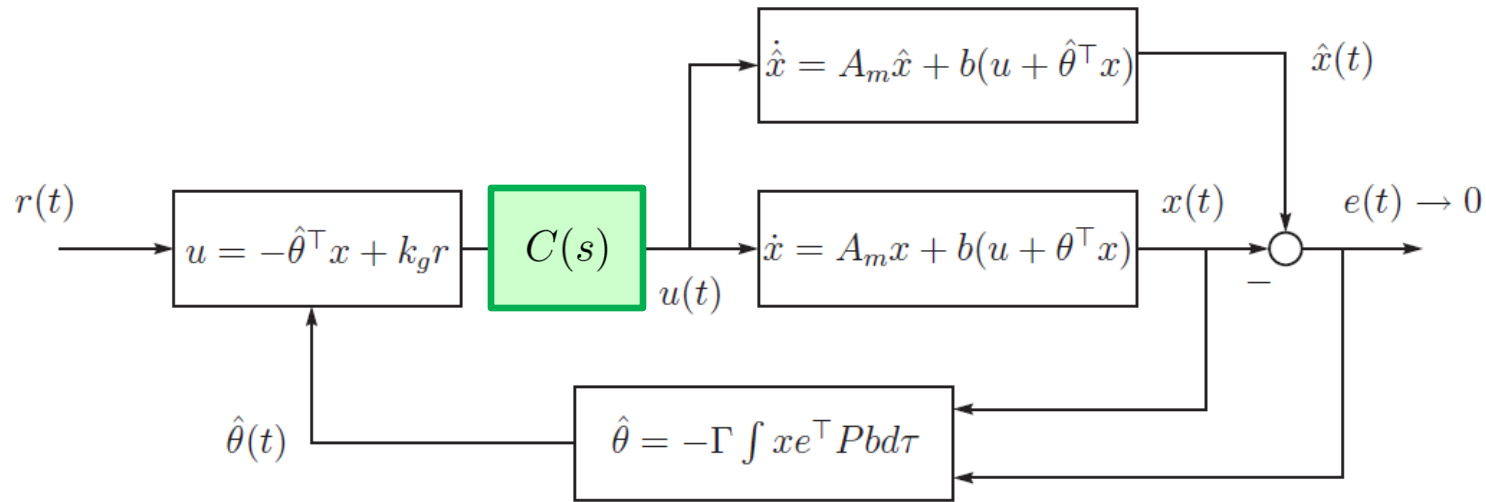
**Dedicated to Prof. Thomas Kailath on his 70<sup>th</sup> Birthday**

1. Fekri, Athans, and Pascoal, “Issues, Progress and New Results in Robust Adaptive Control”, International Journal on Adaptive Control and Signal Processing, March 2006
2. B. Anderson, Challenges of adaptive control: past, permanent and future, Annual Reviews in Control, pages 123-125, December, 2008

# Two Equivalent Architectures of Adaptive Control



# Stability and Asymptotic Convergence



- Closed-loop:  $s\hat{x}(s) = A_m \hat{x}(s) + b \left( (1 - C(s)) \left\{ \hat{\theta}^\top(t) x(t) \right\} + C(s) k_g r(s) \right)$

- Solving:  $\hat{x}(s) = (s\mathbb{I} - A_m)^{-1} b \left( (1 - C(s)) \left\{ \hat{\theta}^\top(t) (\hat{x}(t) - e(t)) \right\} + C(s) k_g r(s) \right)$

bounded

- Sufficient condition for stability:

$$\left\| (1 - C(s)) (s\mathbb{I} - A_m)^{-1} b \right\|_{\mathcal{L}_1} L < 1$$



$$\lim_{t \rightarrow \infty} (\hat{x}(t) - x(t)) = 0$$

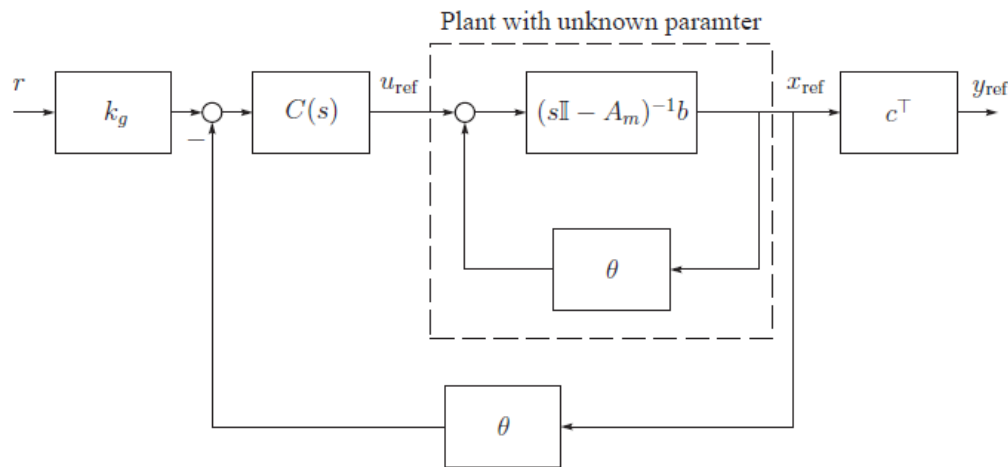
(Barbalat's Lemma)

# Closed-Loop Reference System

- Filtered ideal controller:  $u_{\text{ref}}(s) = C(s)u^*(s), \quad u^*(t) = -\theta^\top x_{\text{ref}}(t) + k_g r(t)$
- Closed-loop: 
$$s x_{\text{ref}}(s) = A_m x_{\text{ref}}(s) + b(u_{\text{ref}}(s) + \theta^\top x_{\text{ref}}(s))$$

$$= A_m x_{\text{ref}}(s) + b((1 - C(s))\theta^\top x_{\text{ref}}(s) + C(s)k_g r(s))$$

$$x_{\text{ref}}(s) = (s\mathbb{I} - A_m)^{-1} b((1 - C(s))\theta^\top x_{\text{ref}}(s) + C(s)k_g r(s))$$



- Sufficient condition for stability:

$$\left\| (1 - C(s)) (s\mathbb{I} - A_m)^{-1} b \right\|_{\mathcal{L}_1} L < 1$$

$C(s) = 1$

**Reference System  
of MRAC**

**Closed-loop Reference System**

# Guaranteed Adaptation Bounds: SCALING

- System state:  $\|x - x_{\text{ref}}\|_{\mathcal{L}_\infty} \leq \frac{\gamma_1}{\sqrt{\Gamma}}$   $\lim_{\Gamma \rightarrow \infty} \|x - x_{\text{ref}}\|_{\mathcal{L}_\infty} = 0$
- System input:  $\|u - u_{\text{ref}}\|_{\mathcal{L}_\infty} \leq \frac{\gamma_2}{\sqrt{\Gamma}}$   $\lim_{\Gamma \rightarrow \infty} \|u - u_{\text{ref}}\|_{\mathcal{L}_\infty} = 0$

$$\gamma_2 = \left\| \left\| C(s) \frac{1}{c_0^\top (s\mathbb{I} - A_m)^{-1} b} c_0^\top \right\|_{\mathcal{L}_1} \right\| \sqrt{\frac{\theta_m}{\lambda_{\min}(P)} + \|C(s)\|_{\mathcal{L}_1} L \gamma_1}$$

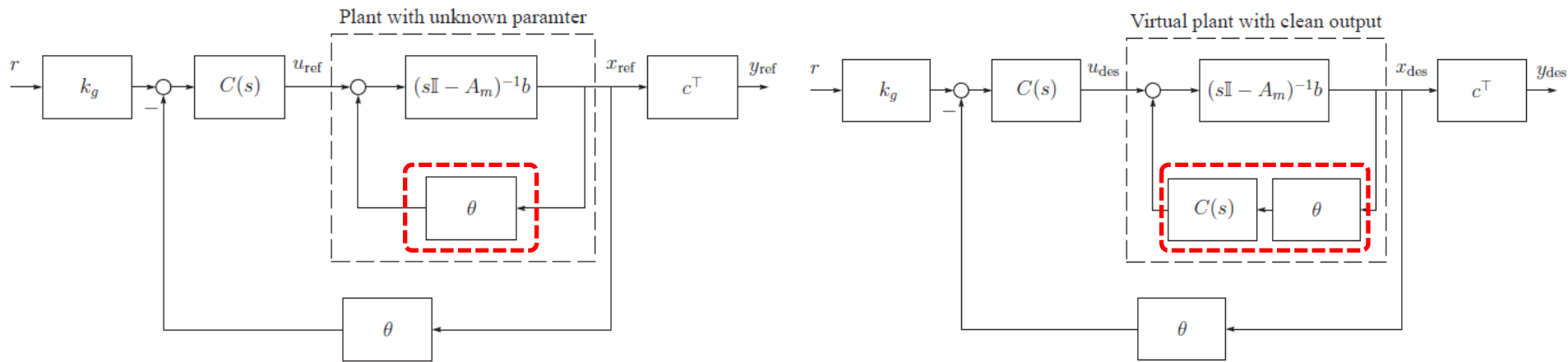
**MRAC:**

$$C(s) = 1 \Rightarrow$$

$$\gamma_2 \rightarrow \infty$$

- \* **Remark:** Non-zero trajectory initialization errors lead to additional additive exponentially decaying terms in the performance bounds

# LTI System for Control Specifications



Closed-loop Reference System  
achieved via fast adaptation

Design System  
for defining control specs

- Closed-loop reference system:

$$y_{\text{ref}}(t) = c^T \left[ \mathbb{I} - (1 - C(s)) (s\mathbb{I} - A_m)^{-1} b \theta^T \right]^{-1} (s\mathbb{I} - A_m)^{-1} b C(s) \{k_g r(t)\}$$

- Design system:

$$y_{\text{des}}(t) = \underbrace{c^T (s\mathbb{I} - A_m)^{-1} b C(s) k_g}_{M(s)} \{r(t)\}$$

Independent of the unknown parameters

# Guaranteed Robustness Bounds

- Achieving desired specifications:

- System output: 
$$\|y_{\text{ref}} - y_{\text{des}}\|_{\mathcal{L}_\infty} \leq \frac{\lambda}{1 - \lambda} \|c^\top\|_{\mathcal{L}_1} \left\| k_g (s\mathbb{I} - A_m)^{-1} b C(s) \right\|_{\mathcal{L}_1} \|r\|_{\mathcal{L}_\infty}$$

- System input: 
$$\|u_{\text{ref}} - u_{\text{des}}\|_{\mathcal{L}_\infty} \leq \frac{\lambda}{1 - \lambda} \|C(s)\theta^\top\|_{\mathcal{L}_1} \left\| k_g (s\mathbb{I} - A_m)^{-1} b C(s) \right\|_{\mathcal{L}_1} \|r\|_{\mathcal{L}_\infty}$$

- Sufficient condition for stability:

$$\lambda = \left\| (1 - C(s))(s\mathbb{I} - A_m)^{-1} b \right\|_{\mathcal{L}_1} L < 1$$

- Performance improvement:

$$\lambda \rightarrow \min$$

# Guaranteed (*Uniform and Decoupled*) Performance Bounds

- Use large adaptive gain ( $\Gamma$ -tuning):

$$\|y - y_{\text{ref}}\|_{\mathcal{L}_\infty} \leq \mathcal{O}\left(\frac{1}{\sqrt{\Gamma}}\right) \quad \|u - u_{\text{ref}}\|_{\mathcal{L}_\infty} \leq \mathcal{O}\left(\frac{1}{\sqrt{\Gamma}}\right)$$

- Design  $C(s)$  to render  $\lambda$  sufficiently small (trade-off robustness for performance):

$$\|y_{\text{ref}} - y_{\text{des}}\|_{\mathcal{L}_\infty} \leq \mathcal{O}(\lambda) \quad \|u_{\text{ref}} - u_{\text{des}}\|_{\mathcal{L}_\infty} \leq \mathcal{O}(\lambda)$$

## Decoupling of Adaptation from Robustness

$$\|y - y_{\text{des}}\|_{\mathcal{L}_\infty} \leq \mathcal{O}\left(\frac{1}{\sqrt{\Gamma}}\right) + \mathcal{O}(\lambda) \quad \|u - u_{\text{des}}\|_{\mathcal{L}_\infty} \leq \mathcal{O}\left(\frac{1}{\sqrt{\Gamma}}\right) + \mathcal{O}(\lambda)$$

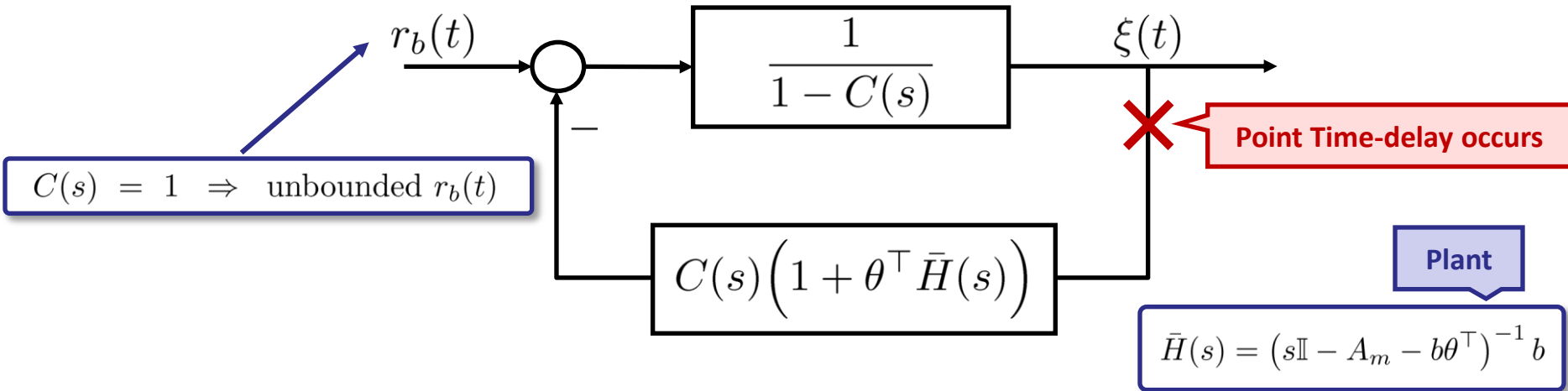
Large adaptive gain  $\longrightarrow$  Smaller step-size  $\longrightarrow$  Faster CPU

\* **Remark:** Sensor and control sampling can be done at a lower rate



# Time-Delay Margin and Gain Margin

- Time-delay margin:



$$\exists \Gamma_0, \Gamma_0 < \Gamma \Rightarrow \tau_{\text{margin}} \geq \tau_m(H(s))$$

Lower bound for the time-delay margin

where

$$H(s) = \frac{C(s)(1 + \theta^\top \bar{H}(s))}{1 - C(s)}$$

- Gain margin:

$$\mathcal{G}_m = [\omega_l, \omega_u]$$

Projection defines the gain margin

# Main Result

- If  $\|(1 - C(s))(s\mathbb{I} - A_m)^{-1}b\|_{\mathcal{L}_1} L < 1$ , then the  $L_1$  adaptive controller ensures **uniform transient and steady-state performance bounds**:

$$\|x - x_{\text{ref}}\|_{\mathcal{L}_\infty} \leq \mathcal{O}\left(\frac{1}{\sqrt{\Gamma}}\right) \quad \|u - u_{\text{ref}}\|_{\mathcal{L}_\infty} \leq \mathcal{O}\left(\frac{1}{\sqrt{\Gamma}}\right)$$

- Moreover, there exists  $\Gamma_0$  such that if  $\Gamma_0 < \Gamma$ , then the **time-delay margin** is guaranteed to stay **bounded away from zero**:

$$\tau_{\text{margin}} \geq \tau_m(H(s)) > 0$$

where  $\tau_m$  is the time-delay margin of an LTI system. The **gain margin** can be arbitrarily improved by increasing the domain of projection.

# Design Philosophy

- Adaptive gain: as large as CPU and sensors permit (**fast adaptation**)
  - ✓ Fast adaptation ensures arbitrarily close tracking of the auxiliary closed-loop reference system with **bounded away from zero time-delay margin**.

**Fast adaptation leads to improved performance and improved robustness**

- Low-pass filter:
  - ✓ Defines the trade-off between **performance** and **robustness**
  - ✓ Increase the **bandwidth of the filter**:
    - The auxiliary closed-loop reference system can approximate arbitrarily closely the ideal desired reference system
    - Leads to reduced time-delay margin

**Tracking vs Robustness can be analyzed analytically**

**Performance can be predicted a priori**

# Time-Delay Margin: MRAC and $L_1$ for a PI Controller

MRAC

$$\dot{x}(t) = -x(t) + u(t) + \theta$$

$L_1$

- Loop transfer functions in the presence of time-delay:

$$L(s) = \frac{\Gamma}{s(s+1)} e^{-\tau s}$$

$$L(s) = \frac{C(s)\Gamma}{s^2 + s + (1 - C(s))\Gamma} e^{-\tau s}$$

- Time-delay margin  $\tau^*$ :  $\exists \omega^*$

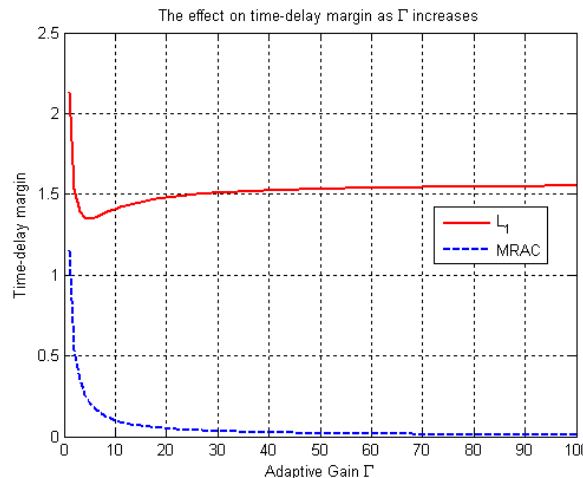
$$L(j\omega^*) = \frac{\Gamma}{j\omega^*(j\omega^* + 1)} e^{-j\tau^*\omega^*} = -1$$

$$\tau^*(\Gamma) = \frac{\angle L(j\omega^*)}{\omega^*} \rightarrow 0 \text{ as } \Gamma \rightarrow \infty$$

- Time-delay margin  $\tau^*$ :  $\exists \omega^*$

$$L(j\omega^*) = \frac{\Gamma}{j\omega^*(j\omega^* + 1)^2 + j\omega^*\Gamma} e^{-j\tau^*\omega^*} = -1$$

$$\tau^*(\Gamma) = \frac{\angle L(j\omega^*)}{\omega^*} \rightarrow \frac{\pi}{2} \text{ as } \Gamma \rightarrow \infty$$



- Application of nonlinear  $L_1$  theory:

$$C(s) = \frac{1}{s+1}, \quad \tau_m = \frac{\pi}{2}$$

$$\lim_{\Gamma \rightarrow \infty} \tau^* = \tau_m$$

# Extensions of the Theory

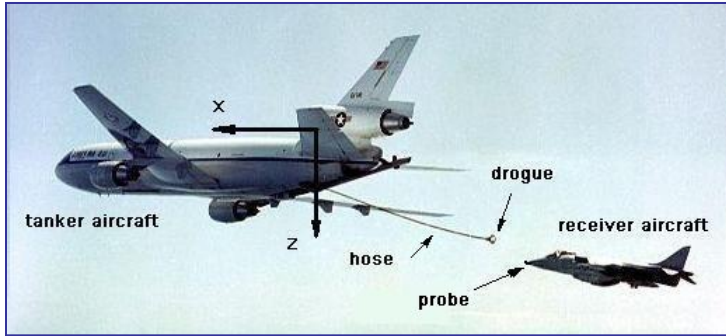
## ▪ State-Feedback:

- $L_1$  Adaptive Control for Systems with **TV Parametric Uncertainty and TV Disturbances**
- $L_1$  Adaptive Control for Systems with **Unknown System Input Gain**
- $L_1$  Adaptive Control for a class of Systems with **Unknown Nonlinearities**
- $L_1$  Adaptive Control for **Nonlinear** Systems in the presence of **Unmodeled Dynamics**
- $L_1$  Adaptive Control for Systems in the presence of **Unmodeled Actuator Dynamics**
- $L_1$  Adaptive Control for **Time-Varying Reference Systems**
- $L_1$  Adaptive Control for **Nonlinear Strict Feedback** Systems in the presence of **Unmodeled Dynamics**
- $L_1$  Adaptive Control for Systems with **Hysteresis**
- $L_1$  Adaptive Control for a Class of Systems with **Unknown Nonaffine-in-Control Nonlinearities**
- $L_1$  Adaptive Control for MIMO Systems in the Presence of **Unmatched Nonlinear Uncertainties**
- $L_1$  Adaptive Control in the Presence of **Input Quantization**
- ...

## ▪ Output-Feedback:

- $L_1$  Adaptive Output-Feedback Control for Systems of **Unknown Dimension (SPR ref. system)**
- $L_1$  Adaptive Output-Feedback Control for **Non-Strictly Positive Real Reference Systems**

# Aerospace Applications



NASA Dryden Flight Research Center Photo Collection  
<http://www.dfrc.nasa.gov/Gallery/Photo/index.html>  
NASA Photo: ED02-0295-5 Date: December 19, 2002 Photo By: Jim Ross

NASA Dryden Flight Research Center Photo Collection  
<http://www.dfrc.nasa.gov/gallery/photo/index.html>  
NASA Photo: EC87-0182 Date: July 24, 1987 Photo by: NASA

X-29 in Banked Flight



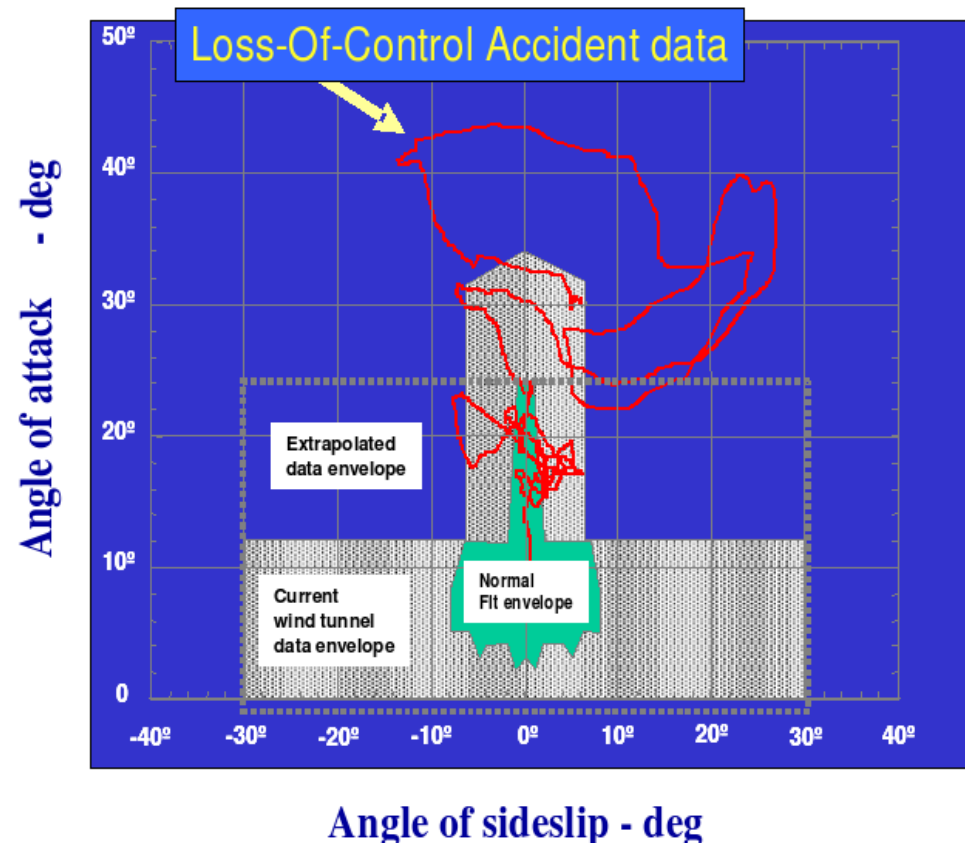
# Integrated Resilient Aircraft Control (IRAC)

*IRAC research is focused on loss-of-control, failure and damage scenarios, and their mitigation through the application of adaptive control.*

## **Control law objectives:**

- Keep aircraft in the Extended flight envelope
- Return to Normal Flight Envelope

- Control actions within 2-4 seconds of failure onset are **critical**:
  - Need for **transient performance guarantees**
    - ✓ Predictable response
  - Need for **fast adaptation**



# Generic Transport Model

*High-risk flight conditions, some unable to be tested in target application environment.*



- 5.5 % geometrically and dynamically scaled model
  - 82in wingspan, 96 in length, 49.6 lbs (54 lbs full), 53 mph stall speed
  - Model angular response is 4.26 faster than full scale
  - Model velocity is 4.26 times slower than regular scale



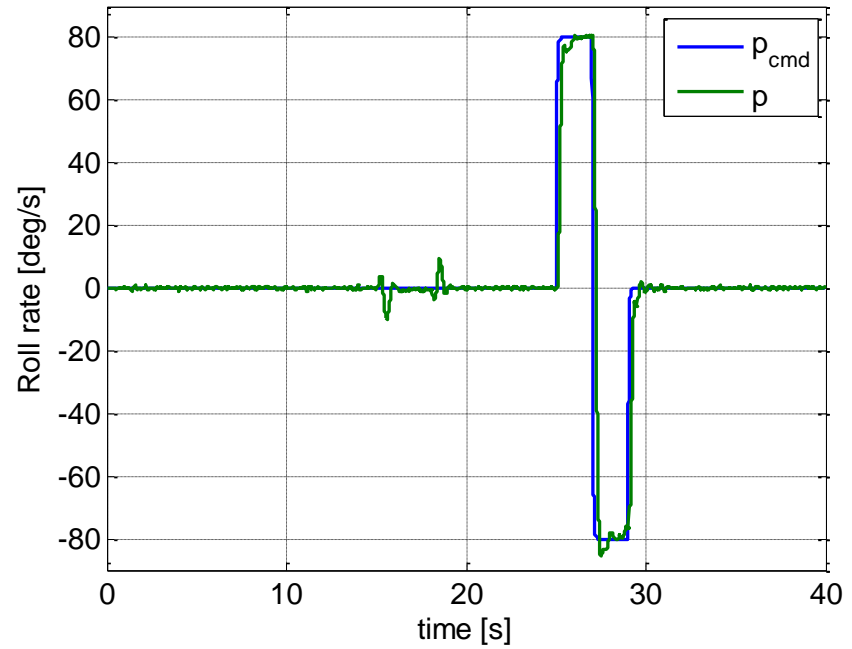
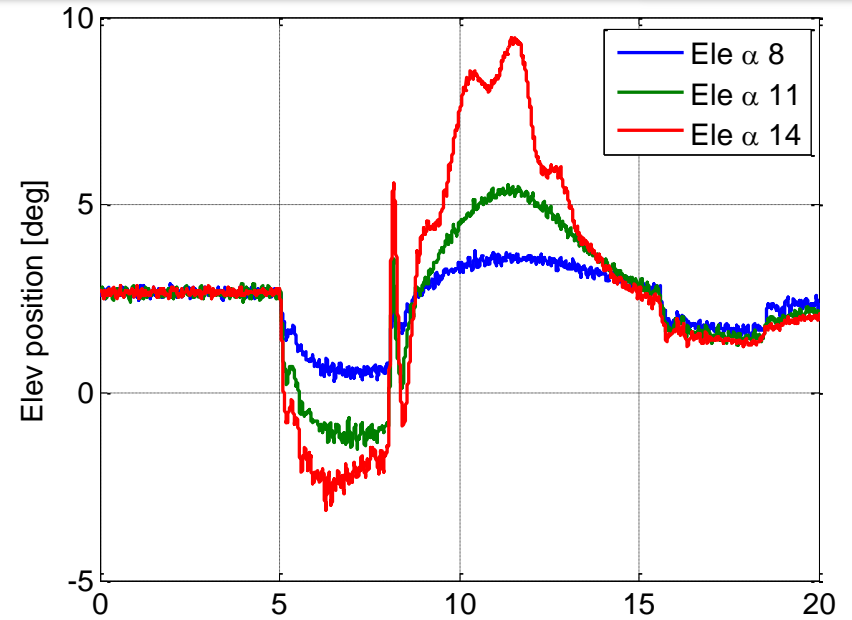
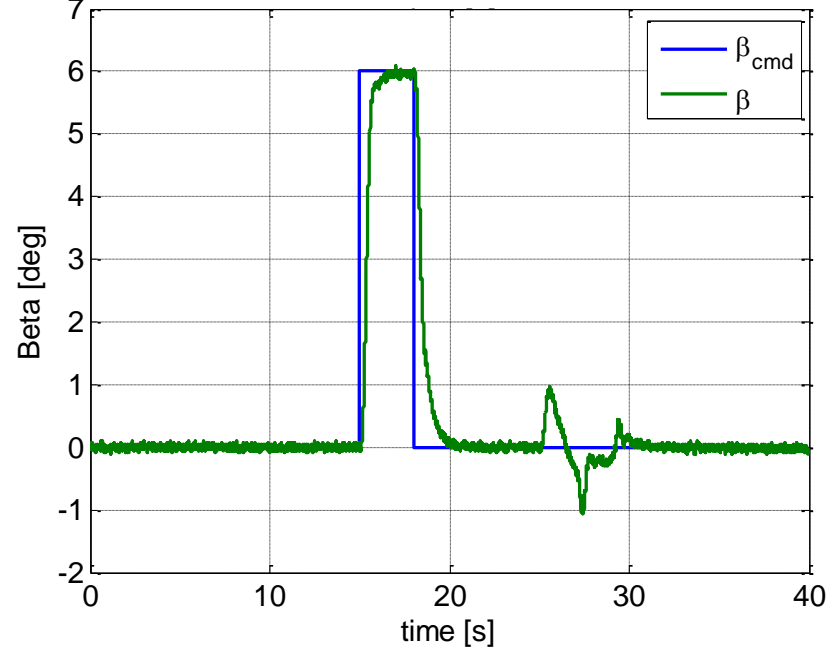
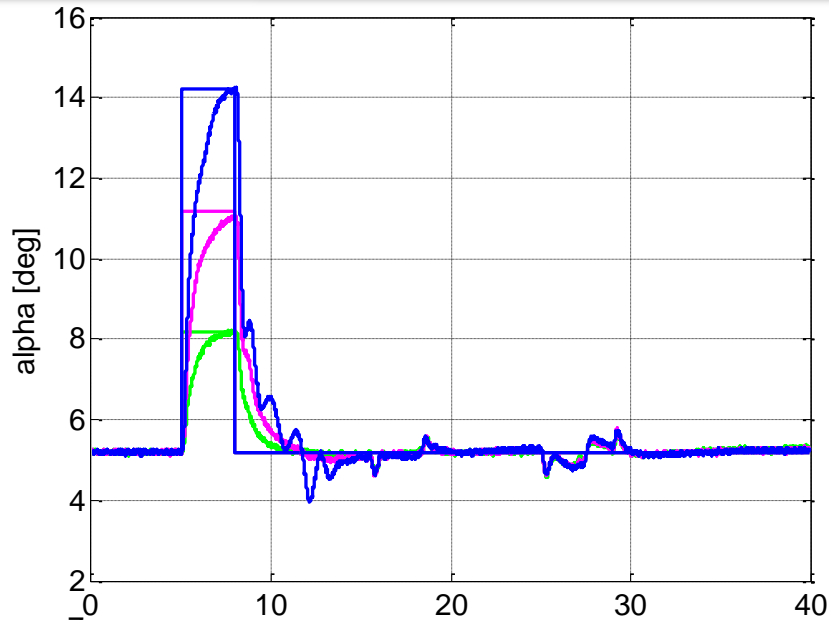
# Controlling High- $\alpha$ Regimes...

**Stall...  
Nose-slice roll-off...  
Recovery...**



**Control aircraft in  
stall and post-stall  
regimes!**

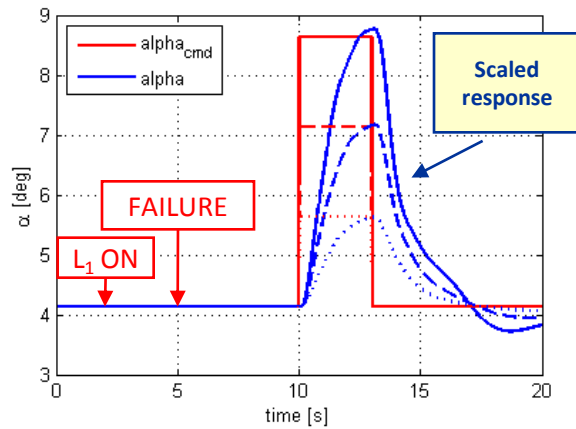
# AirSTAR :: Batch Sims (*Healthy UAV*)



# AirSTAR :: Batch Sims (*Impaired UAV*)

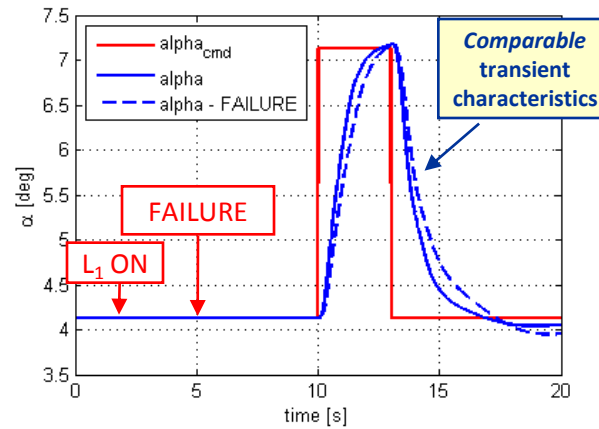
Outboard elevators stuck at trim

Impaired UAV



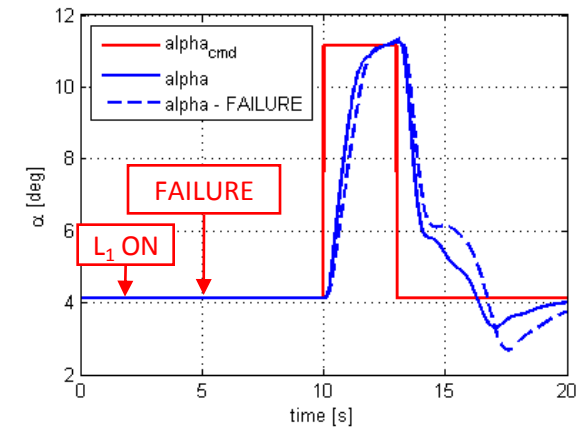
Angle of attack

Healthy vs Impaired UAV  
(normal flight conditions)

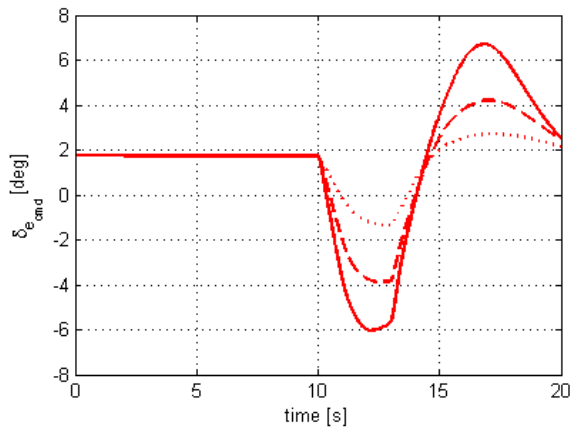


Angle of attack

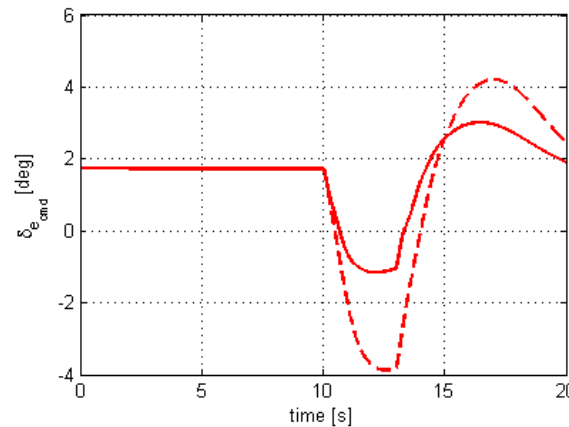
Healthy vs Impaired UAV  
(stall regimes: <40knots)



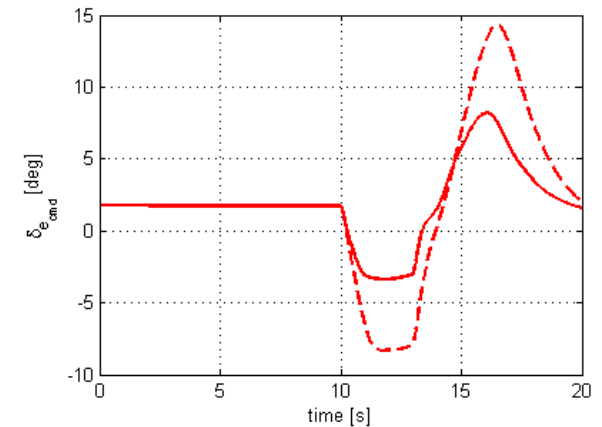
Angle of attack



Elevator deflection



Elevator deflection

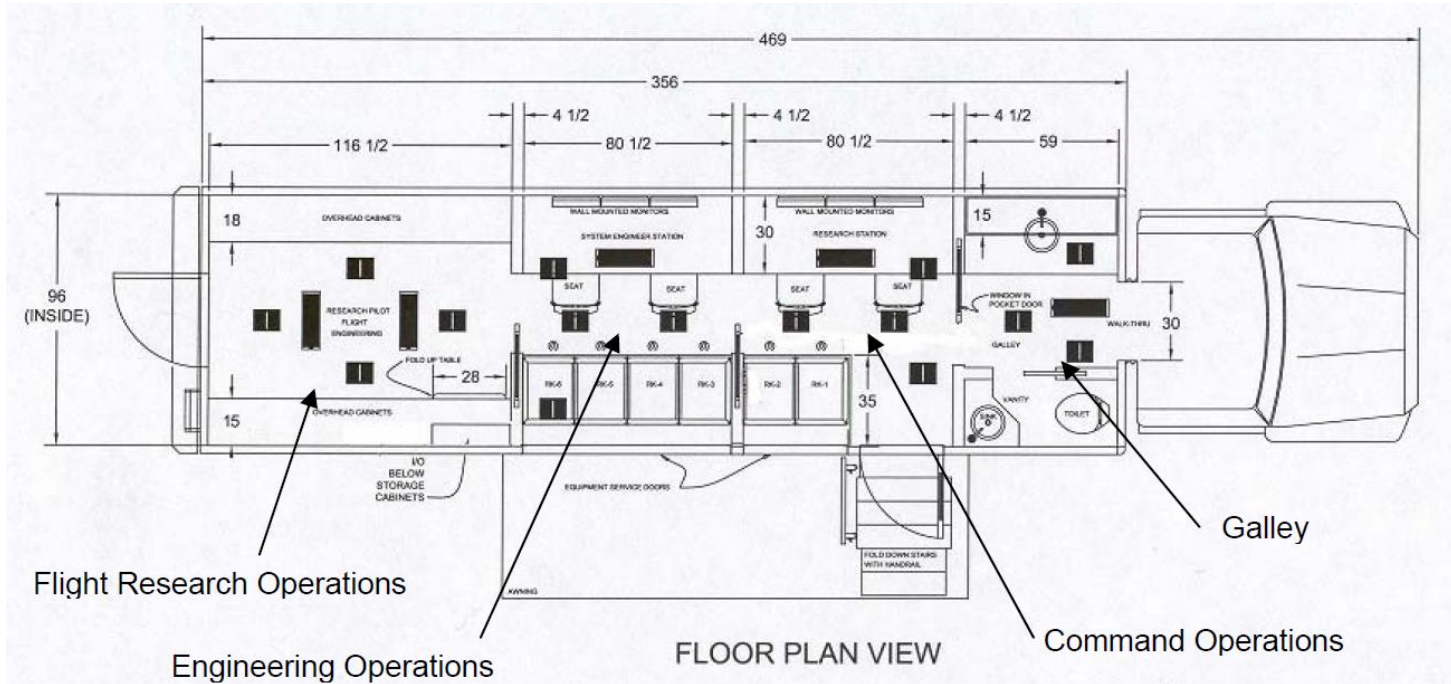


Elevator deflection

# Pre-Flight and Check-List



# Mobile Operations Station



Flight Research Operations

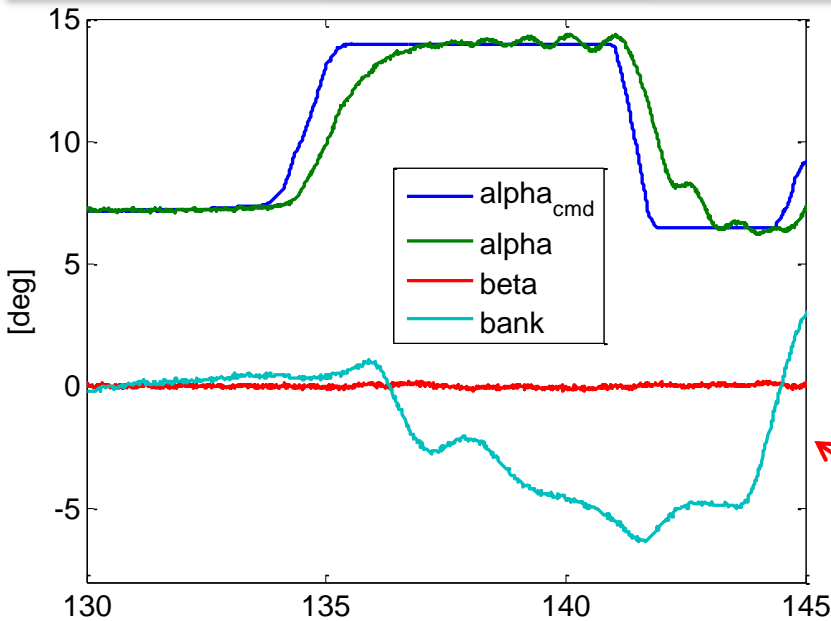
Engineering Operations

FLOOR PLAN VIEW

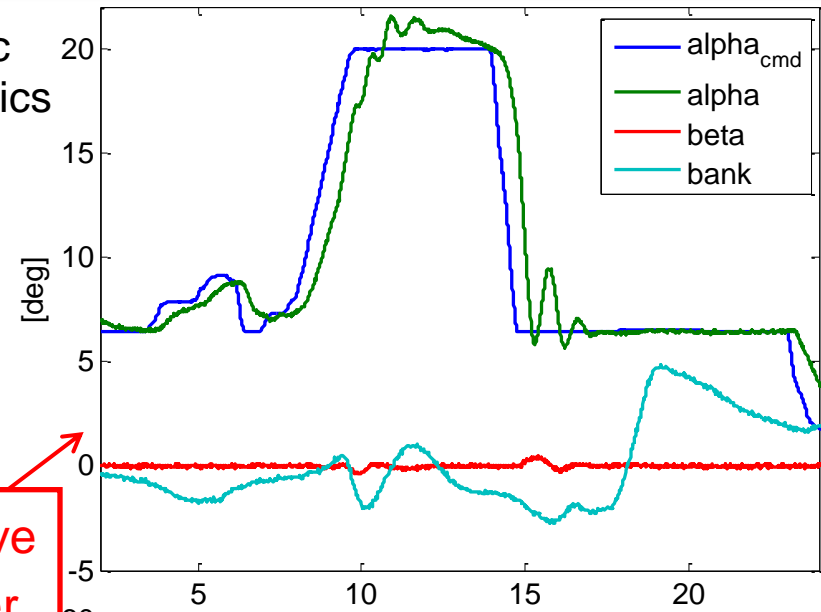
Command Operations



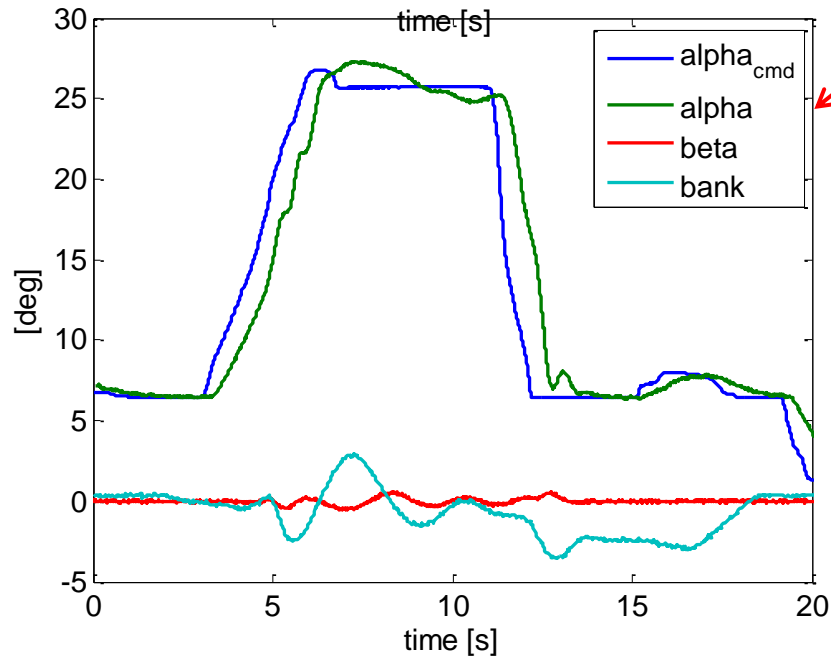
# AirSTAR :: Piloted Task (AoA capture – high $\alpha$ )



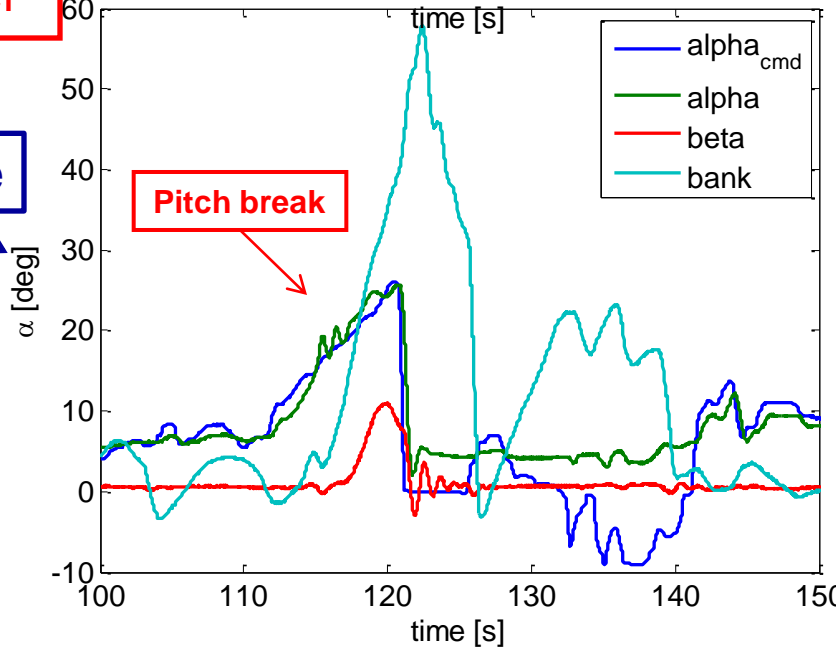
- Asymmetric aerodynamics
- Low  $C_{lp}$
- Roll off
- Noseslice



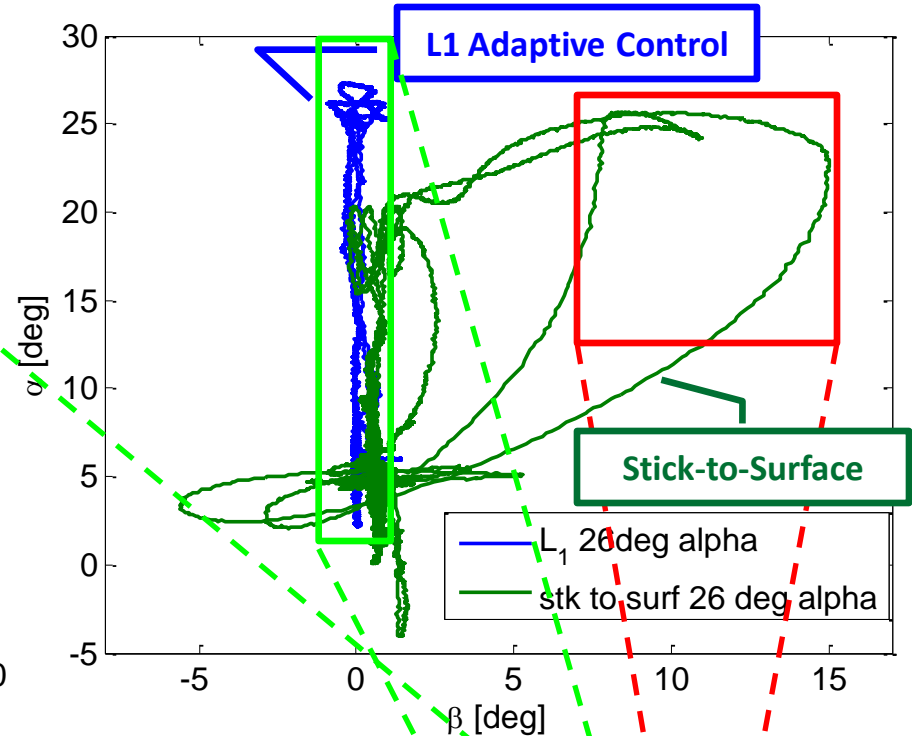
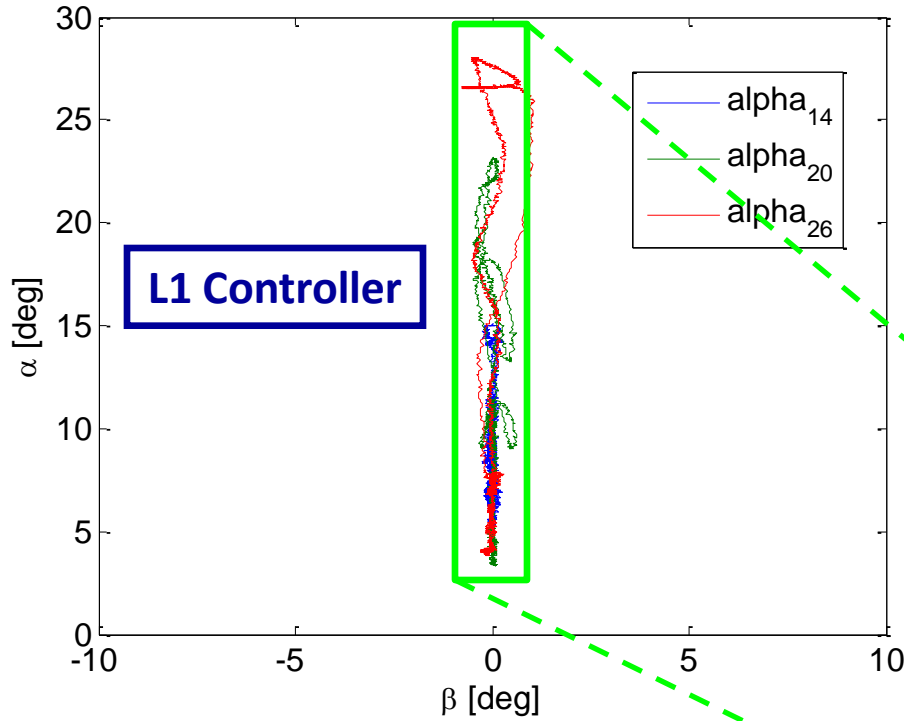
**L<sub>1</sub> Adaptive Controller**



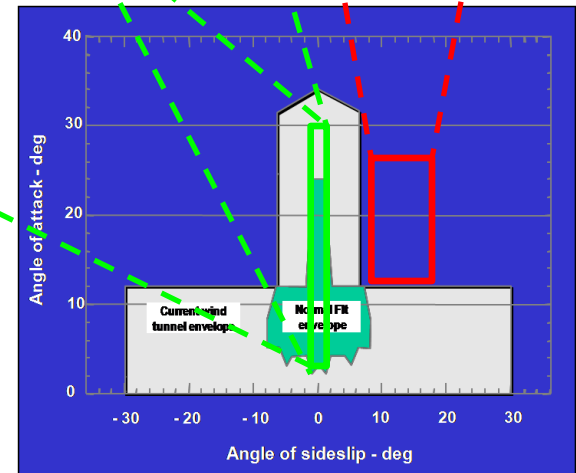
**Baseline**



# AirSTAR :: Piloted Task (AoA capture – high $\alpha$ )



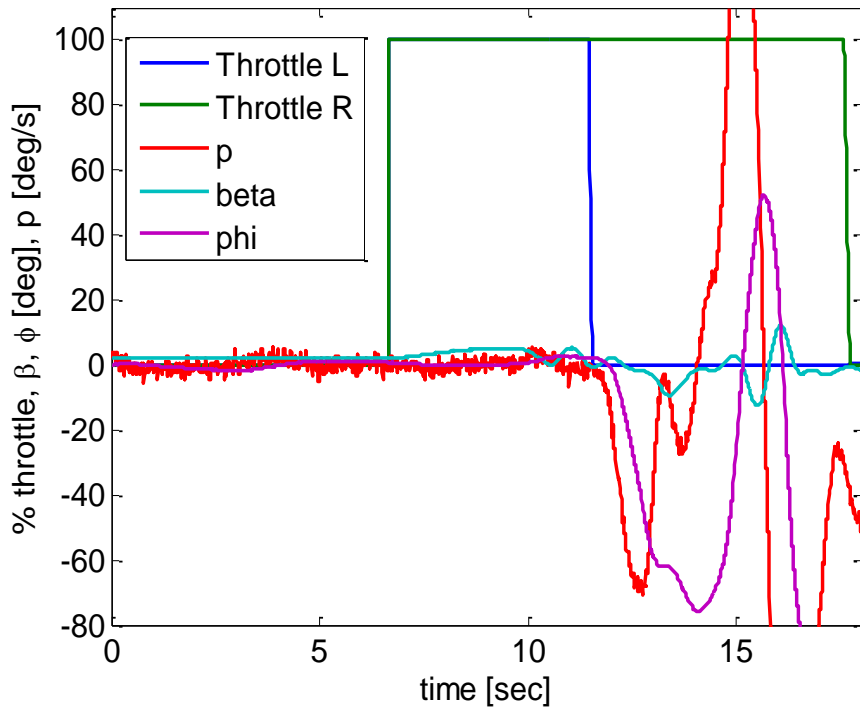
- $\alpha$ - $\beta$  coupling for
  - L<sub>1</sub> adaptive controller at different AoA
  - L<sub>1</sub> adaptive vs. stick to surface control at 26 deg AoA



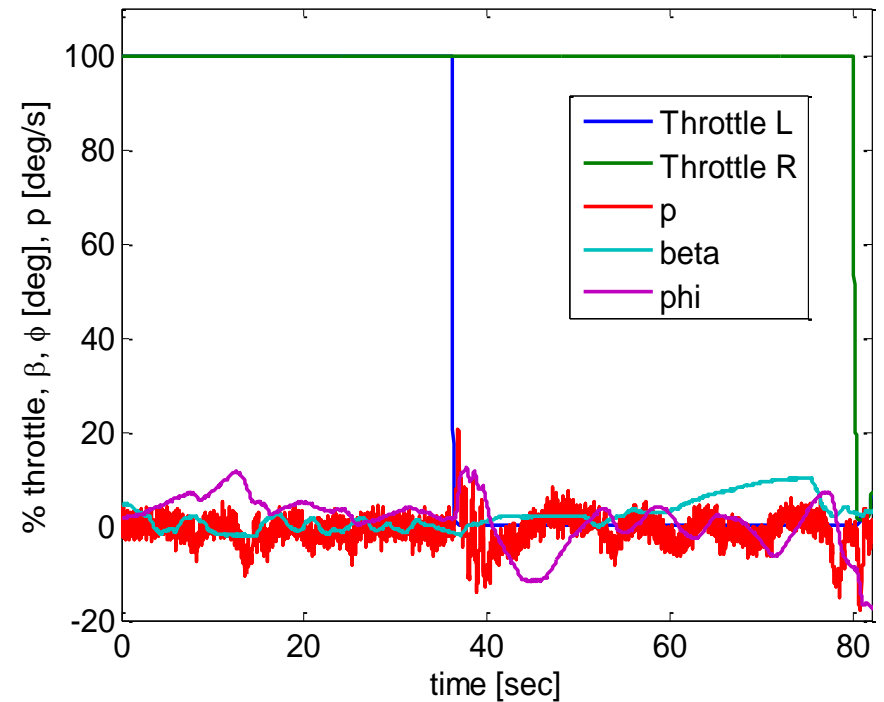
# AirSTAR :: Piloted Task (*full throttle* → *asymmetric thrust*)

1. Full throttle (100%)
2. Climb at 25-30 deg pitch
3. Left Throttle cut to 0% in <0.5sec

**Stick-to-Surface**

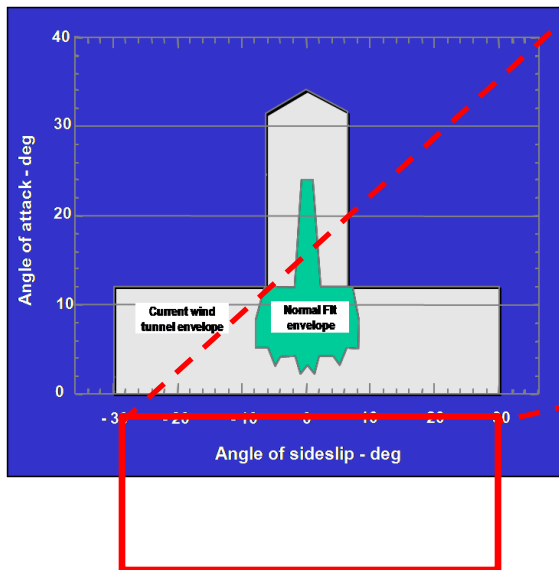
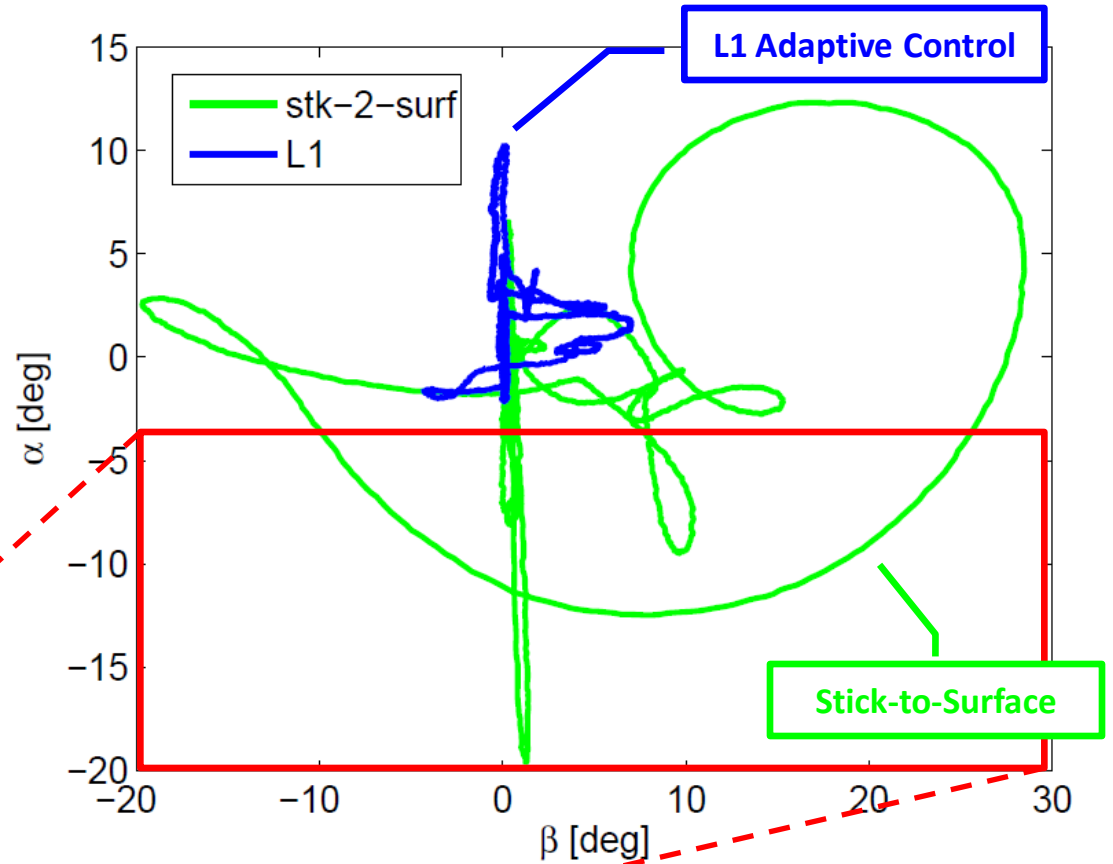


**L<sub>1</sub> Controller**





# AirSTAR :: Piloted Task (*full throttle* → *asymmetric thrust*)



# GTM T2 :: March 2010 Deployment

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*“...this is the first successful flight of an all-adaptive control law that deals with aircraft stability degradation as well as actuator failures...”*

*“...it is the first flight of a direct all-adaptive controller with a pilot in the loop...”*

*NASA RTD weekly key activities report*

*Dr. I. Gregory*

# Networked Control Systems

## Challenges:

- Cyber challenges
- Modeling challenges
  - Need to predict the performance
  - Need robustness assessment
- Military use
  - Time-critical missions in constrained airspace
- Commercial use
  - Air-traffic control
  - Hospitals
  - Power grids, etc.



# Networked Control Systems: Main Result

Consider the ideal system and the closed-loop real system. Assume that  $f_i(t, x)$  is locally Lipschitz and  $\frac{\partial f_i}{\partial t}$  and  $\frac{\partial f_i}{\partial x}$  are linearly bounded in a compact set. Given arbitrary  $\gamma \in \mathcal{R}^+$ , if

$$\|G(s)\|_{\mathcal{L}_1} \alpha_{\max} + \frac{a}{\sqrt{\Gamma_{\min}}} + b\epsilon_{\max} < \gamma,$$

where  $G(s) = H(s)B(I - C(s))$ ,  $\Gamma_{\min} = \min_i \Gamma_i$ , and  $\epsilon_{\max} = \max_i \epsilon_i$ , then

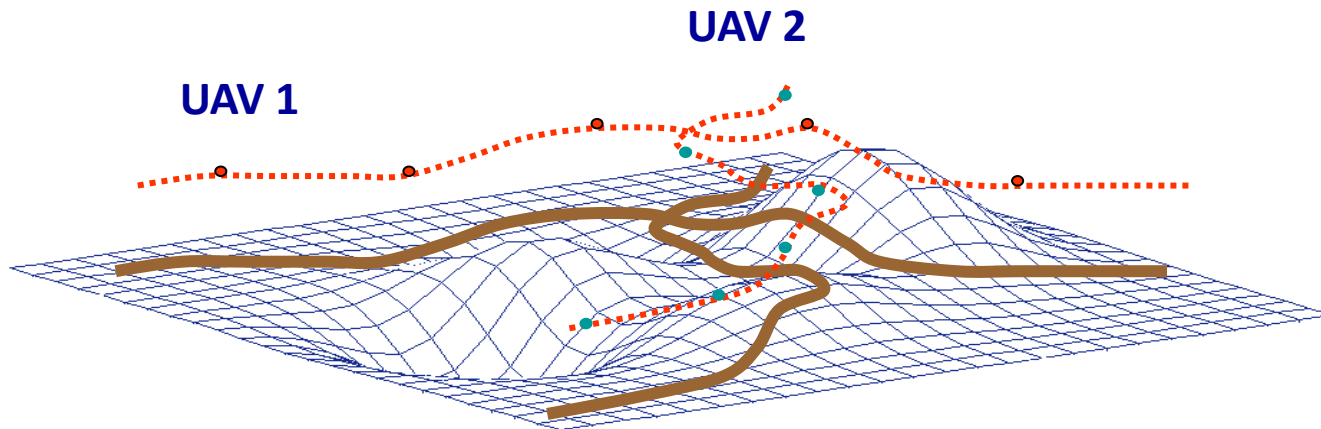
$$\begin{aligned} \|x - x^{\text{ideal}}\|_{\mathcal{L}_\infty} &< \gamma \\ \|u - u^{\text{ideal}}\|_{\mathcal{L}_\infty} &< \gamma u. \end{aligned}$$

Moreover, the broadcast periods are always greater than a positive constant.

**It suggests a tradeoff among the robustness, the adaptation, and communication.**

# Motivation in Applications of Homeland Security

- Time-critical applications for multiple UAVs with spatial constraints:
  - Sequential autoland
  - Coordinated reconnaissance – synchronized high-resolution pictures
  - Coordinated road search



- Coordinate on the **arrival** of the leader subject to deconfliction, network, and spatial **constraints**

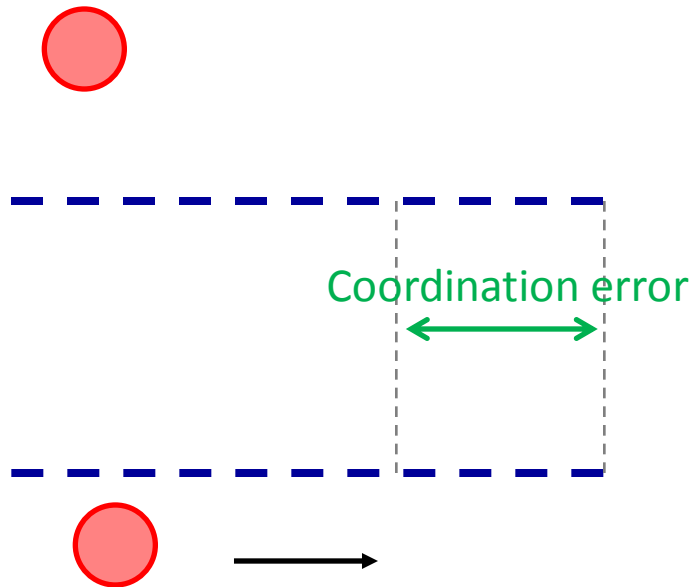
# Overall Approach

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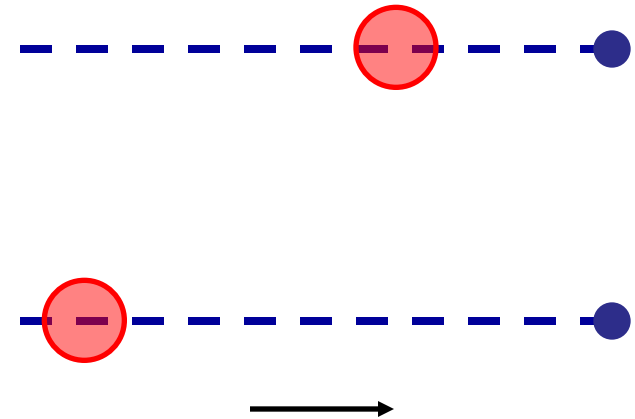
- Integrated solution to **time-critical coordination problems** that includes:
  - 1) **Real-time (RT) path generation** accounting for
    - ✓ Vehicle dynamics
    - ✓ Spatial and temporal constraints;
  - 2) **Nonlinear path following** that relies on UAV **attitude** to follow the given path and leaving speed along the path as a degree of freedom;
  - 3) **Time-critical coordination** adjusting the **speed** of each vehicle over a time-varying faulty network to provide robustness – account for the uncertainties and/or unavoidable deviations from the plan that cannot be addressed in the path generation step;
  - 4)  **$L_1$  adaptive control** to augment the off-the-shelf autopilots and improve path following performance and ensure coordination in time.

# Key Idea

- Decoupling of **space** and **time**:
  - ✓ In the path generation phase, reduces drastically the number of optimization parameters;
  - ✓ Makes the speed profile an extra-degree of freedom to be exploited in the time-coordination step.



*Each vehicle runs locally its own  
PATH FOLLOWING  
controller to steer itself to the path*

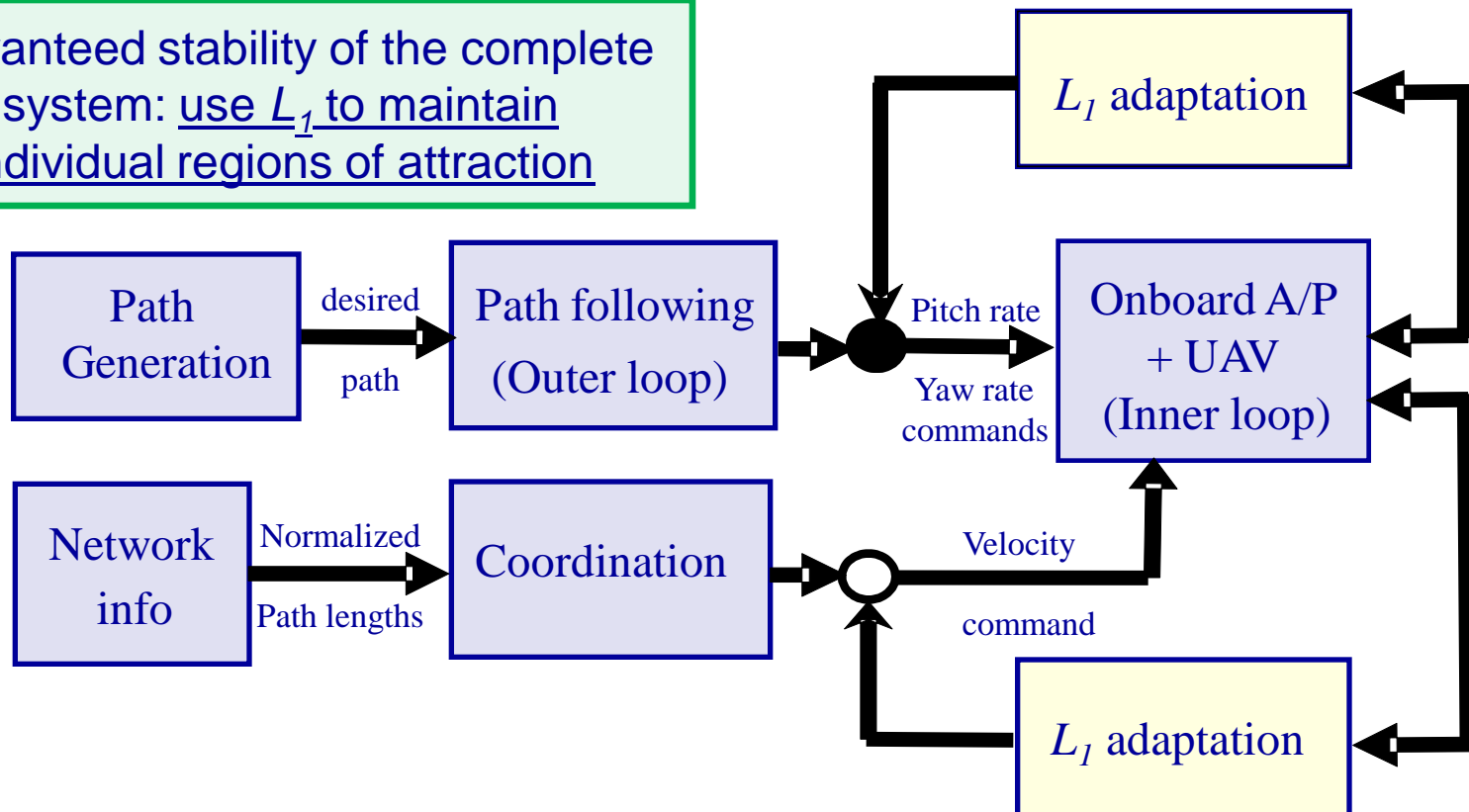


*Vehicles TALK each other and  
adjust their SPEEDS in order  
to COORDINATE themselves*

# Architecture for Coordination with Limited Information

- **Objective:** successfully accomplish the missions given communication constraints from (wireless) network (limited bandwidth, package dropouts, time-delay, ...)
- **Questions:** Lower bound on channel capacity for mission accomplishment? What are the effective communication schemes (coder, decoder)?
- The **solution** depends upon the **performance bounds** guaranteed by inner-loop controllers, the communication constraints and the given boundary conditions.

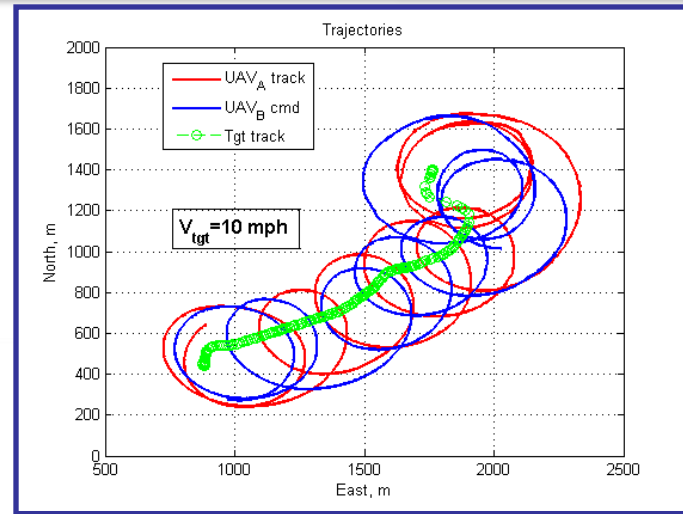
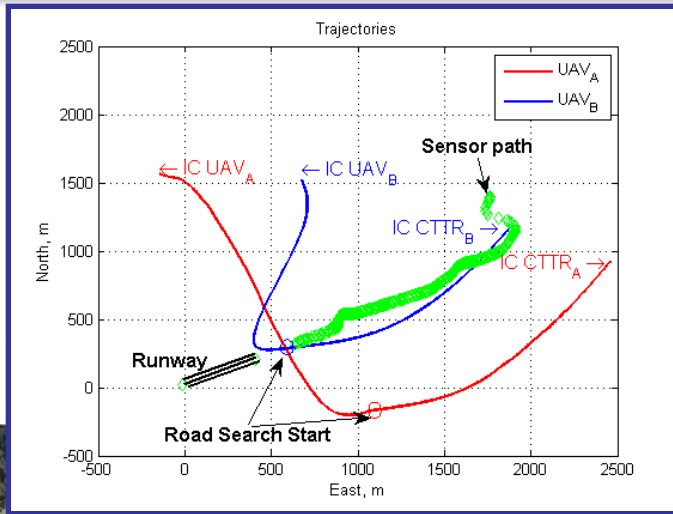
Guaranteed stability of the complete system: use  $L_1$  to maintain individual regions of attraction





# Flight Tests :: CPF - Coordinated Road Search

2DoF P/T gimbal with video camera



Virtual target

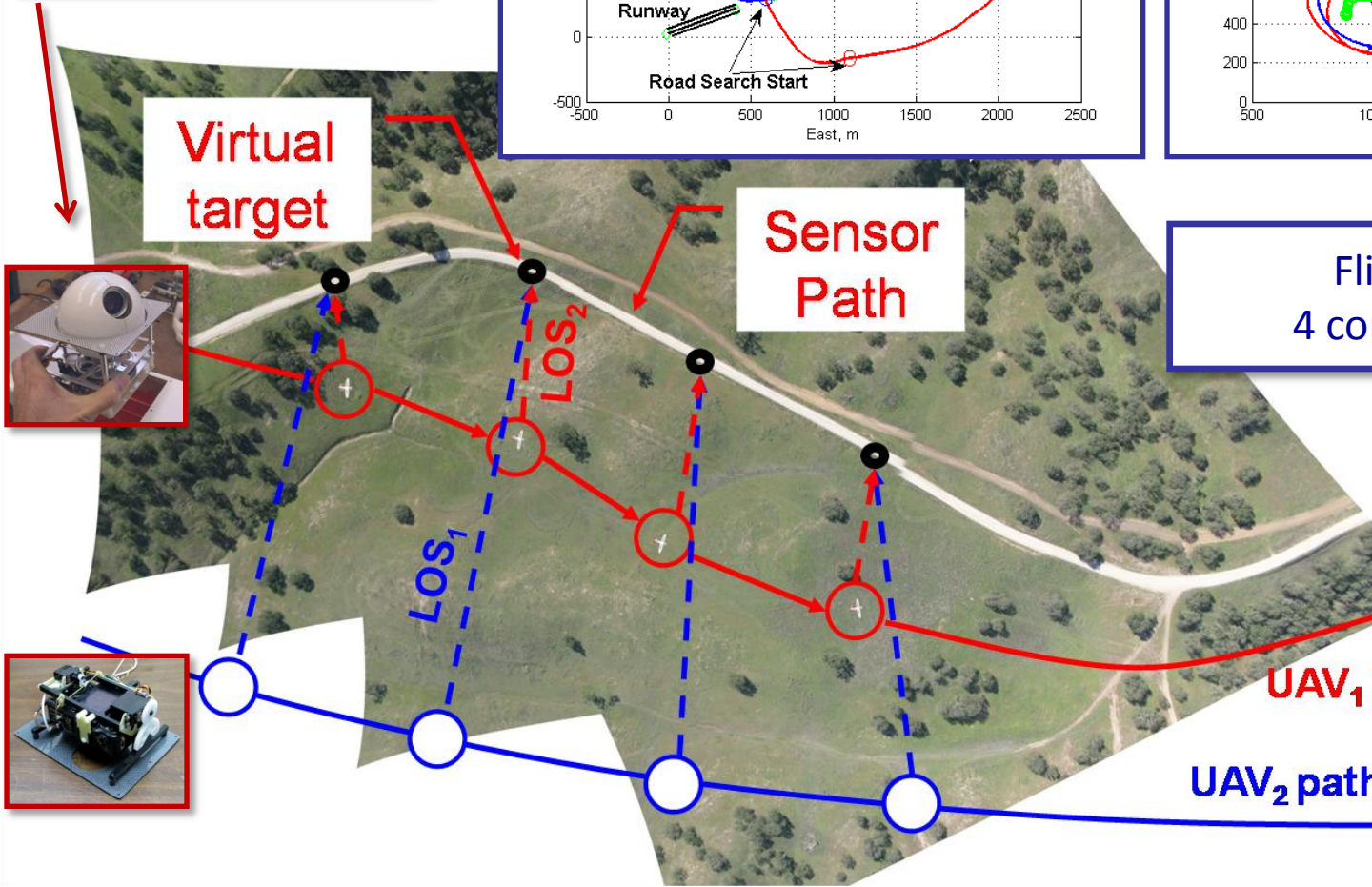
Sensor Path

Flight imagery of 4 consecutive frames

Single DOF gimbal with high resolution camera

UAV<sub>1</sub> path

UAV<sub>2</sub> path



# $L_1$ in Applications of Other Groups

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- $L_1$  control of anesthesia (Carolyn Beck, UIUC)
- $L_1$  control of viruses (Tamer Basar, UIUC)
- $L_1$  control of smart materials with hysteresis (Ralph Smith, SUNC)
- $L_1$  control of drilling pressure (StatOilHydro, Norway)
- $L_1$  control of engines (Chengyu Cao, UConn, P&W, UTRC)
- $L_1$  control of micro UAVs (Randy Beard, BYU)
- $L_1$  control of rotorcraft (Jon How, MIT)
- $L_1$  control of helicopters (Carlos Silvestre, ISR, IST, Lisbon, Portugal)
- $L_1$  control of .....

# Major Lesson Learned

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**Also in Adaptive Control...**

**... *robustness* has to be a part of the problem formulation,  
and not just the “responsibility of analysis”**

# Conclusions

- What do we need to know?
  - Boundaries of uncertainties → sets the filter bandwidth
  - CPU and sensors (hardware) → sets the adaptive gain

**Performance limitations reduced to hardware limitations!**

- Achieves clear **separation** between **adaptation** and **robustness**
  - performance can be predicted *a priori*
  - robustness/stability margins can be quantified analytically
  - performance scales similar to linear systems
- Theoretically justified Verification & Validation tools for feedback systems...

**...at reduced costs!**

**with very short proofs!**



# Group, Collaborations, and Sponsors

## Prof. Hovakimyan group:

- Chengyu Cao (University of Connecticut)
- Xiaofeng Wang (Post-doctoral Fellow)
- Ronald Choe (PhD student - AE)
- Evgeny Kharisov (PhD student - AE)
- Kwang-Ki Kim (PhD student - AE)
- Dapeng Li (PhD student - ME)
- Zhiyuan Li (PhD student - ME)
- Hui Sun (PhD student - ECE)
- Enric Xargay (PhD student - AE)

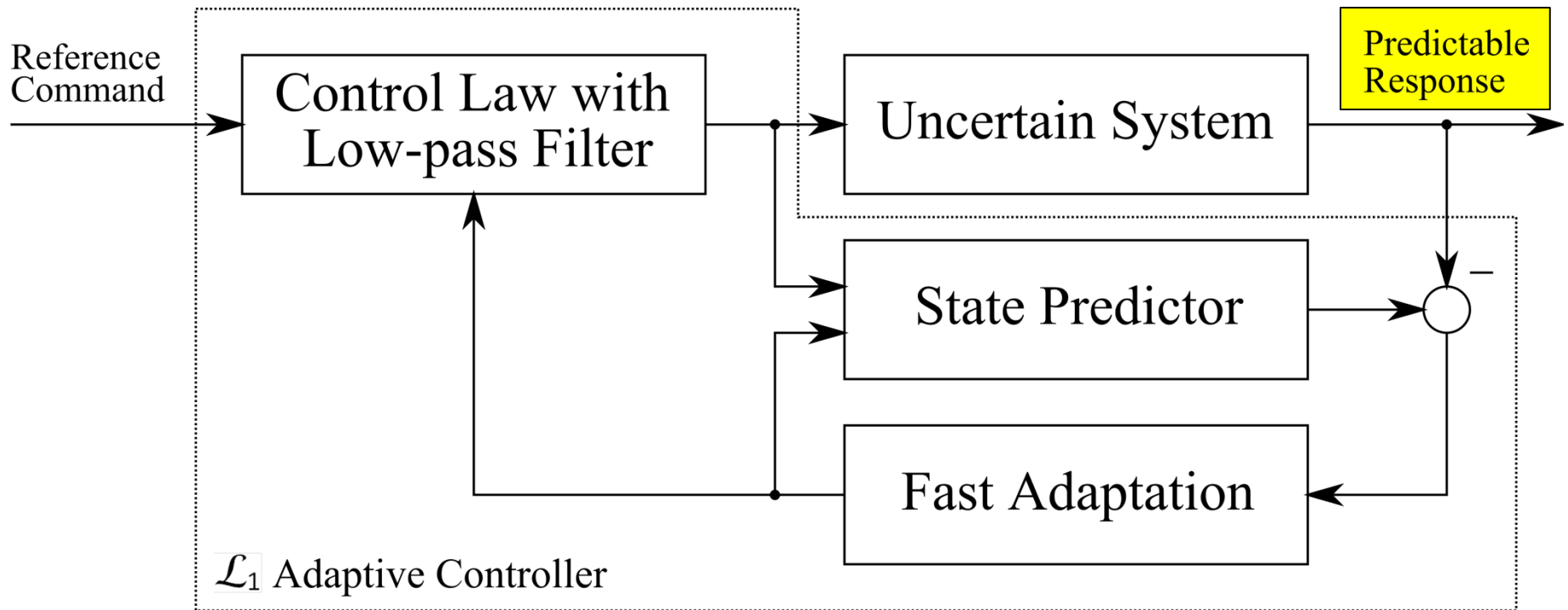


## Collaborations:

- The Boeing Co. (E. Lavretsky, K. Wise)
  - Wright Patterson AFRL (D. Doman, M. Bolender)
  - WP AFRL CerTA FCS program/Boeing (T. Wendell)
  - Raytheon Co. (R. Hindman, B. Ridgely)
  - NASA LaRC and Ames (I. Gregory, N. Nguyen, K. Krishnakumar)
  - NASA Dryden (J. Burken, B. Griffin)
  - Eglin AFRL (J. Evers)
- Randy Beard (BYU), Isaac Kaminer (NPS), Jon How (MIT), Ralph Smith (NCSU)

**Sponsors: AFOSR, AFRL (WP and EGLIN), ARO, ONR, Boeing, NASA**

# Architecture



Questions?