Online Scheduling:

Basics, Paradoxes and Open Questions

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Outline

- **Introduction**
  - Problem Classes
  - Types of Models
- **General Model**
  - State-task network (STN) representation
  - State-space STN model
  - Remarks
- **Online scheduling**
  - Open-loop vs. closed-loop solutions
  - Online scheduling algorithm

Preliminaries

- Systematic scheduling practiced in manufacturing since early 20th century
- First scheduling publications in the early 1950s

  Salveson, M.E. On a quantitative method in production planning and scheduling. *Econometrica*, 20(9), 1952.
- Extensive research in 1970s
  - Closely related to developments in computing and algorithms
  - Computational Complexity: *Job Sequencing* one of 21 NP-complete problems in (Karp, 1972)
- Widespread applications

  Airlines industry (e.g., fleet, crew scheduling); sports; transportation (e.g., vehicle routing)
  Government; education (e.g., class scheduling); services (e.g., service center scheduling)
- Chemical industries
  - Batch process scheduling (e.g., pharma, food industry, fine chemicals)
  - Continuous process scheduling (e.g., polymerization)
  - Transportation and delivery of crude oil
- Scheduling in PSE
  - First publications in early 1980s; focused on sequential facilities (Rippin, Reklaitis)
  - Problems in network structures addressed in early 1990s (Pantelides et al.)
- Very challenging problem: Small problems can be very hard
  - Most *Open* problems in MIPLIB are scheduling related
    - Railway scheduling: 1,500 constraints, 1,083 variables, 794 binaries
    - Production planning: 1,307 constraints, 792 variables, 240 binaries

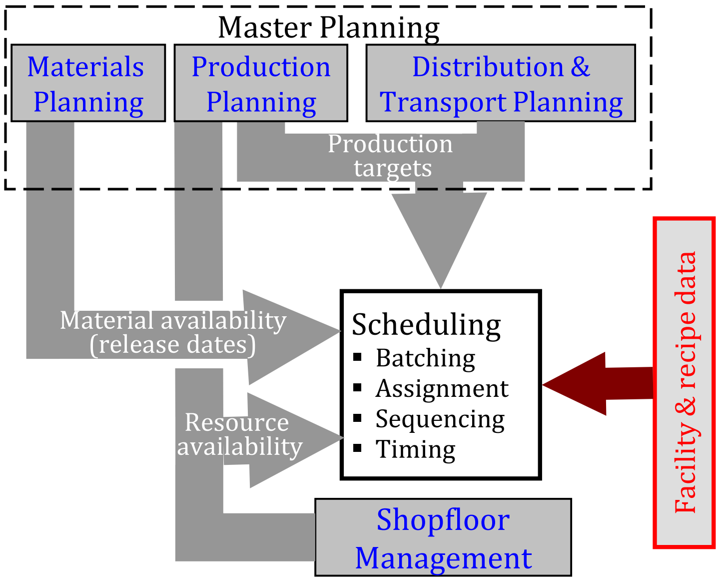
Problem Statement

Given are:

a) Production facility data; e.g., unit capacities, unit connectivity, etc.
b) Production recipes; i.e., mixing rules, processing times/rates, utility requirements, etc.
d) Production costs; e.g., raw materials, utilities, changeover, etc.
e) Material availability; e.g., deliveries (amount and date) of raw materials.
f) Resource availability; e.g., maintenance schedule, resource allocation from planning, etc.
g) Production targets or orders with due dates.

Facility and recipe data (a, b, d)

Input from other planning functions (e, f, g)



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d) **Production costs**; e.g., raw materials, utilities, changeover, etc.
e) **Material availability**; e.g., deliveries (amount and date) of raw materials.
f) **Resource availability**; e.g., maintenance schedule, resource allocation from planning, etc.
g) **Production targets or orders with due dates.**

Our goal is to find a least cost schedule that meets production targets subject to resource constraints.

Alternative objective functions are the minimization of tardiness or lateness (minimization of backlog cost) or the minimization of earliness (minimization of inventory cost) or the maximization of profit.

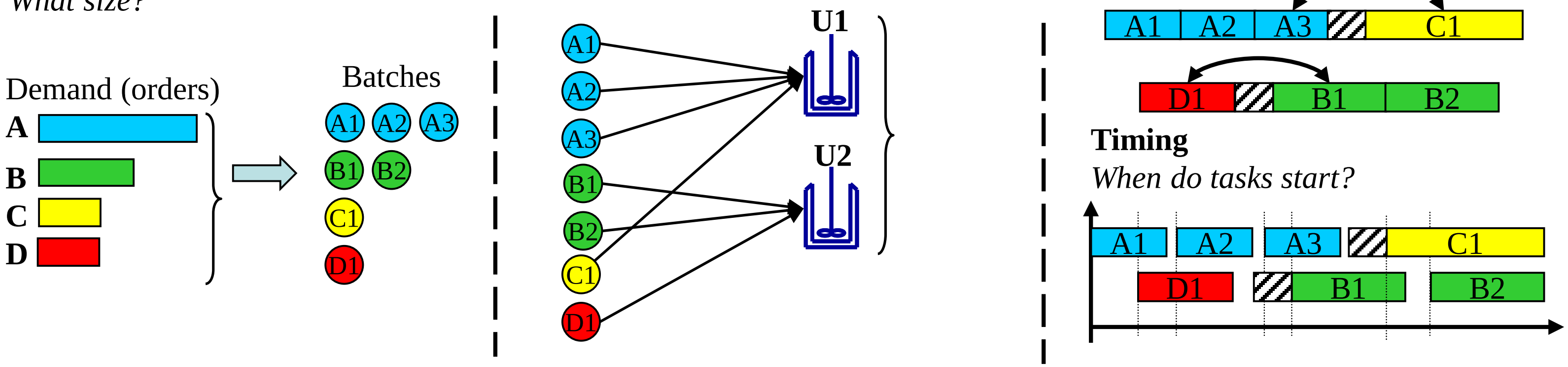
In the general problem, we seek to optimize our objective by making four types of decisions:

a) Selection and sizing of batches to be carried out (batching)
b) Assignment of batches to processing units or general resources.
c) Sequencing of batches on processing units.
d) Timing of batches.

Task selection (batching)
How many tasks/batches?
What size?

Task-resource Assignment
What resources each task requires?

Sequencing (for unary resources)
In what sequence are batches processed?



Problem Classes

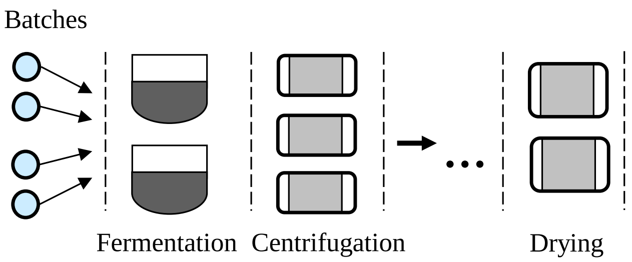
$$\alpha / \beta / \gamma$$

Machine environment
- Single machine
- Parallel machines
- …

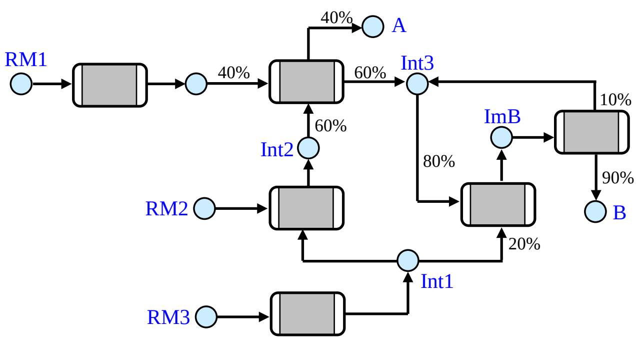
Processing characteristics
- Preemption
- Release/due times
- …

Objective
- Makespan
- Tardiness
- Cost

- ***Sequential*** **environment**
  - Problems similar to discrete manufacturing due to material handling restrictions



- ***Network*** **environment**
  - Batches and materials are split
  - Recycle streams, etc.
  - Problems different from discrete manufacturing



- ***Hybrid*** **environment**
  - Consist of different types of subsystems

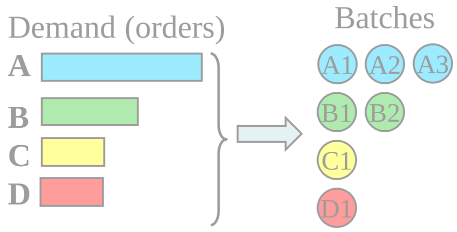
Mendez et al., (2006); Maravelias (2012)

From *Environments* to *Models*

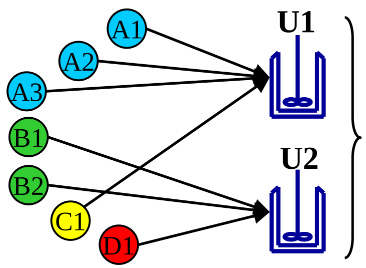
Math Programming Scheduling Models in the PSE literature (1980 – 2007):

- For *sequential* processes we developed *batch-based* approaches
  - ➔ Track batches; do not account for material amounts
  - ➔ Fixed number and size of batches: only assignment and sequencing decisions; no batching

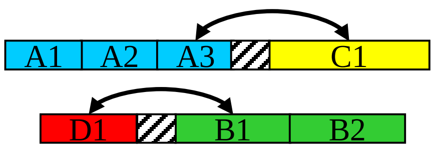
Task selection (batching)



Task-resource Assignment



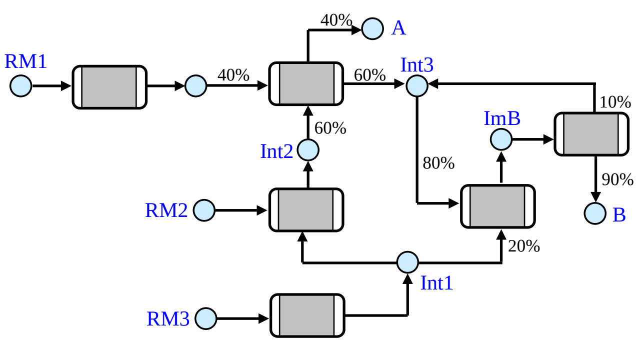
Sequencing



Other common assumptions

- No storage constraints
- No utility requirements

- For *network* processes we developed *material-based* approaches
  - ➔ We model amounts of material (material balances)
  - ➔ We make batching, assignment and sequencing/timing decisions



Also account for:

- Storage constraints
- Utility requirements
- Transfer operations
- Blending

Modeling Attributes

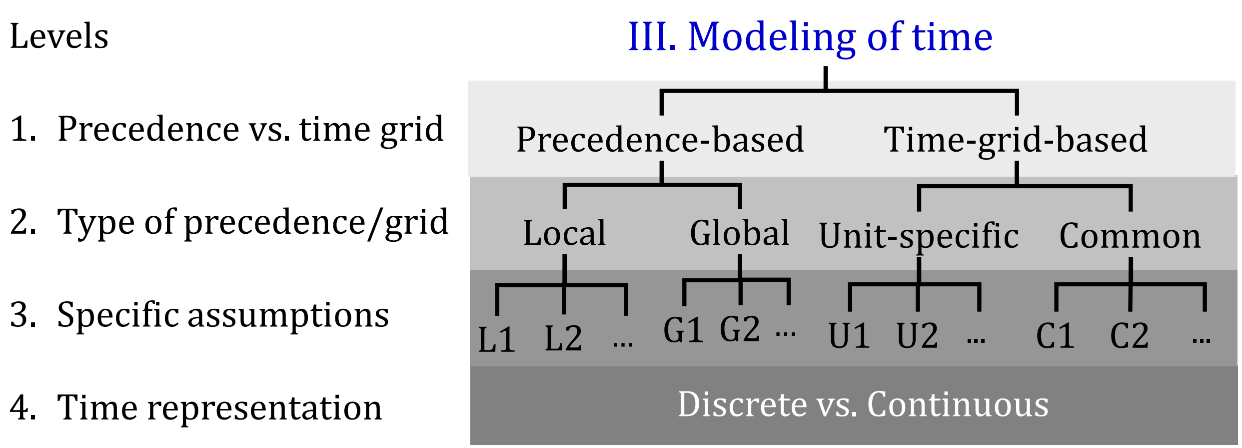
- Key modeling entities
  - Batches/tasks
  - Material amounts
  - Both
- Scheduling decisions
  - Task number & size (batching)
  - Task-unit assignment
  - Sequencing/timing

II. Scheduling Decisions

I. Key Modeling Elements	Sequencing/timing	Unit-batch assignment Sequencing/timing	Batching (no & size) Unit-batch assignment Sequencing/timing
Tasks	Sequential, no batching • Single-unit	Sequential, no batching • Single-stage, multi-stage, multi-purpose	
Materials			1. Network processes 2. Network & sequential processes
Tasks & materials			Sequential with batching

Modeling Attributes

- Key modeling entities
  - Batches/tasks
  - Material amounts
  - Both
- Scheduling decisions
  - Task number & size (batching)
  - Task-unit assignment
  - Sequencing/timing
- Modeling of time (four types of decisions)
  1. Selection between precedence based and time-grid-based approaches
  2. Selection of (i) type of precedence relationship (local vs. global)
     (ii) type of time grid (common vs. unit specific)
  3. Specific representation assumptions
  4. Selection between discrete- and continuous-time



The *Universe* of Modeling Approaches

II. Scheduling Decisions

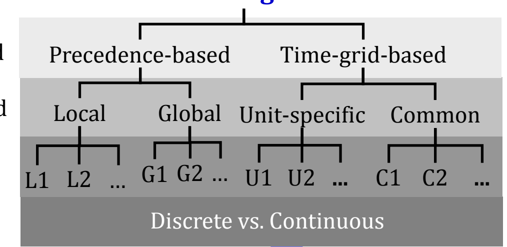
I. Key Modeling Elements

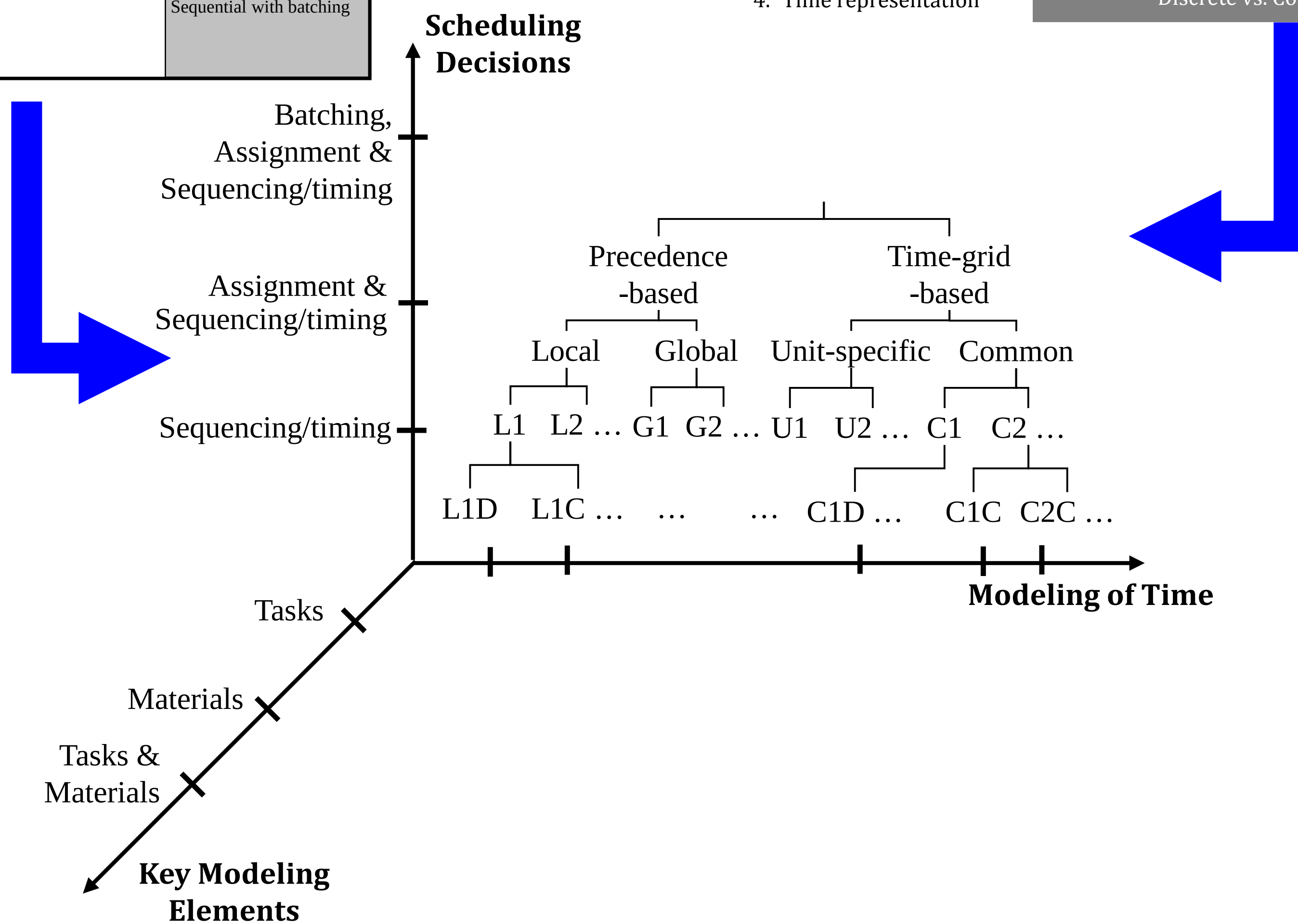
	Sequencing/timing	Unit-batch assignment Sequencing/timing	Batching (no & size) Unit-batch assignment Sequencing/timing
Tasks	Sequential, no batching • Single-unit	Sequential, no batching • Single-stage, multi-stage, multi-purpose	
Materials			1. Network processes 2. Network & sequential processes
Tasks & materials			Sequential with batching

Levels

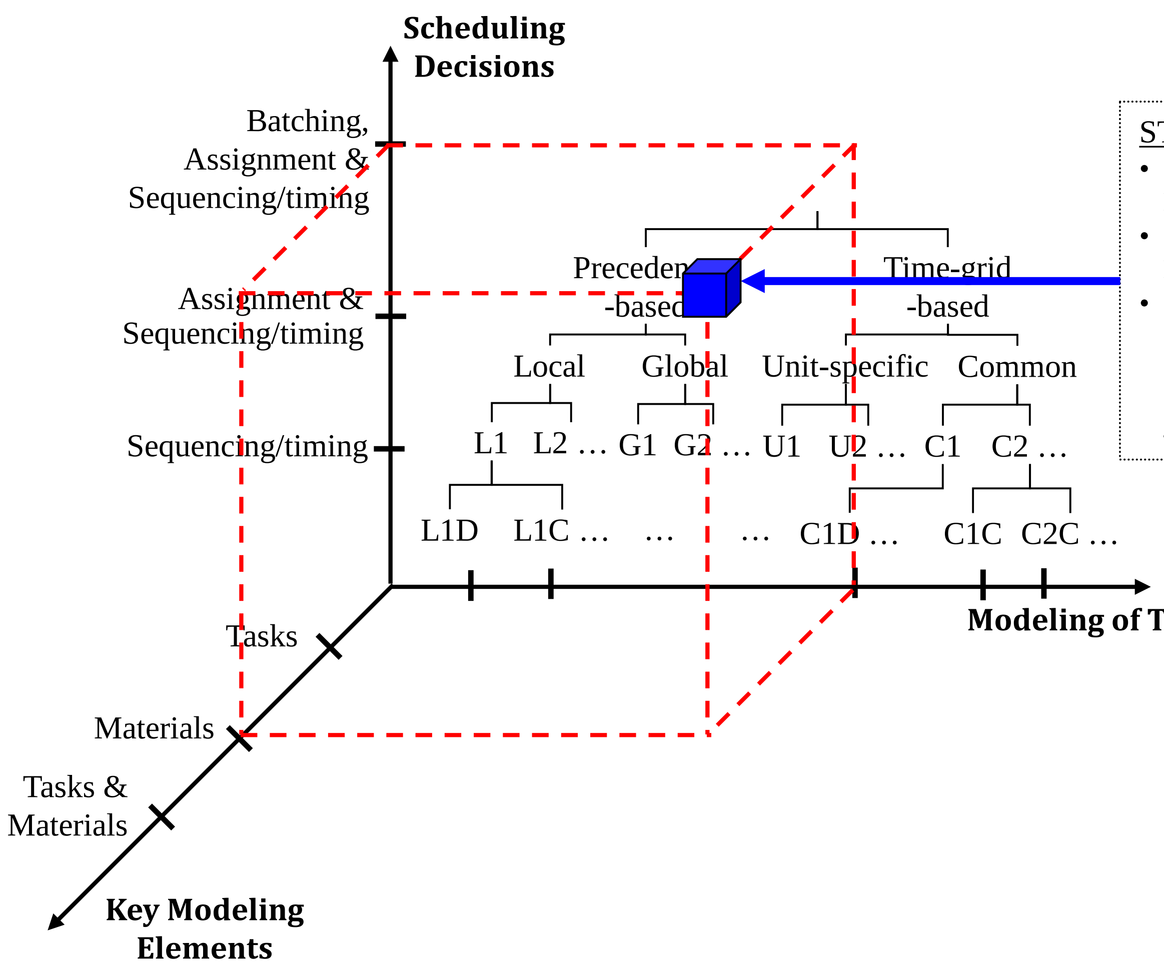
1. Precedence vs. time grid
2. Type of precedence/grid
3. Specific assumptions
4. Time representation

III. Modeling of time





The *Universe* of Modeling Approaches



STN (Kondili et al., 1993)

- Decisions:
  Batching, assignment & timing
- Modeling element:
  Materials
- Modeling of time:
  1. Time-grid-based
  2. Common grid
  3. Tasks start/finish @ time pts
  4. Discrete-time

Remarks

Major modeling advances recently

- **Sequential Environments**
  - Simultaneous batching, assignment, sequencing[1]
  - Storage policies and general resource constraints[2,3]
- **Network environments**
  - Resource-constrained material transfers and changeover activities[4]
- **Combined environments[5]**
  e.g., upstream sequential followed by downstream network, followed by continuous processing

Outstanding modeling challenges

- **Sequence-dependent changeovers**
- **Nonlinear models (blending)**

Major computational advances

- **Constraint propagation[6] and reformulation methods[7]**
- **Computational improvements of 1-2 orders of magnitude[8,9]**
- **Applicable to wide range of models and problem classes**

[1] Prasad & Maravelias, *Comp. & Chem. Eng.*, 32 (6), **2008.**
[2] Sundaramoorthy & Maravelias, *Ind. Eng. Chem. Res.*, 47 (17), **2008.**
[3] Sundaramoorthy & Maravelias, *Ind. Eng. Chem. Res.*, 48 (13), **2009.**
[4] Gimenez et al., *Comp. & Chem. Eng.*, 33 (10), **2009.**
[5] Sundaramoorthy & Maravelias, *AIChE J.*, 57(3), **2011.**
[6] Velez et al., *AIChE J.*, 59(3), **2013.**
[7] Velez & Maravelias, *Ind. Eng. Chem. Res.*, 52 (10), **2013.**
[8] Velez & Maravelias, *Ann. Rev. Chem. Biom. Eng.*, 5, 97-121, **2014.**
[9] Velez et al., *Chem. Eng. Sci.*, 136, 139-157, **2015.**

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  - Problem Classes
  - Types of Models
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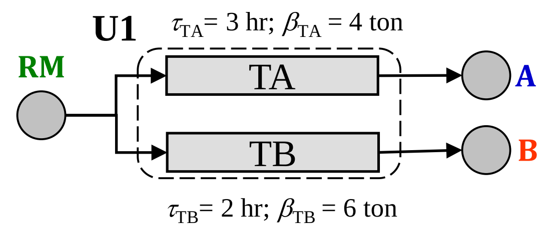
  - State-task network (STN) representation
  - State-space STN model
  - Critical insights
- Online scheduling
  - Open-loop vs. closed-loop solutions
  - Online scheduling algorithm

Unified Framework

- Unified framework
  - Material-based representation of all types of processing
  - *Sequential* processing represented using materials with special properties (constraints)

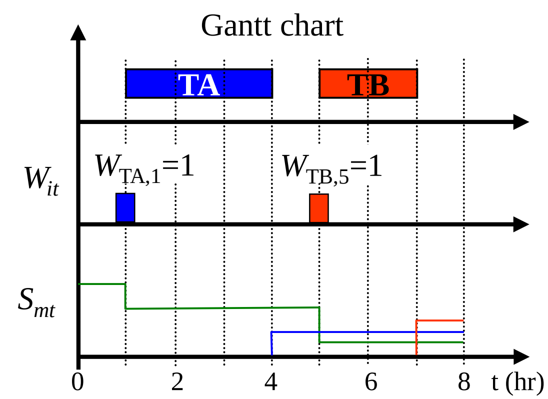
- Basic concepts (from STN; Kondili e tal., 1993)
  - Processing units, $j \in \mathbf{J}$
  - Processing tasks $i \in \mathbf{I}$ (duration, τ_i; batchsize, β_i)
  - Materials (states), $m \in \mathbf{M}$ (storage capacity, ζ_m)



- Discrete-time representation
  - Horizon, η, partitioned into T time periods, $t \in \mathbf{T}$, of length $\delta = \eta / T$

- Decision variables
  - $W_{it} \in \{0, 1\}$: = 1 if task i starts at time point t
  - $S_{mt} \in [0, \zeta_m]$: inventory level of material m during period t

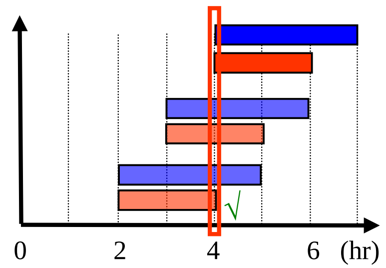
- Mixed-integer programming (MIP) model
  - Resource constraint: a unit can process at most one task at a time
  - Material balance: calculation of inventory over time



Can we express the general scheduling MIP model in state-space form?

Mixed-integer Programming Model

- Resource constraints
- Decision variable: W_{it} = 1 when a task starts



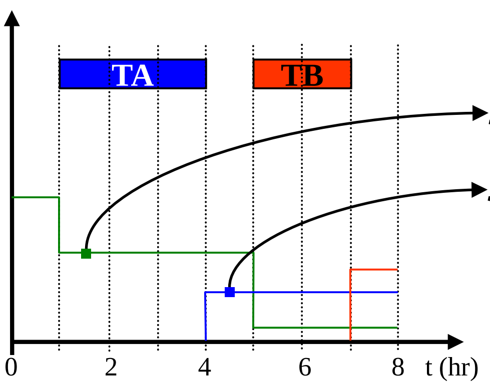
$W_{\mathrm{TA},4} + W_{\mathrm{TB},4}$

$W_{\mathrm{TA},3} + W_{\mathrm{TB},3} \leq 1$

$W_{\mathrm{TA},2}$

$$\sum_{t-3<t'\leq t} W_{\mathrm{TA},t'} + \sum_{t-2<t'\leq t} W_{\mathrm{TB},t'} \leq 1 \qquad \Rightarrow \sum_{i\in I_j} \sum_{t'=t-\tau_i+1}^{t'=t} W_{i,t'} \leq 1, \forall j, t$$

- Material balance constraints
- Inventory levels (S_{mt}) are the results of our scheduling decisions W_{it}



$S_{\mathrm{RM},2} = S_{\mathrm{RM},1} - 4W_{\mathrm{TA},1}$

$S_{\mathrm{A},5} = S_{\mathrm{A},4} + 4W_{\mathrm{TA},1}$

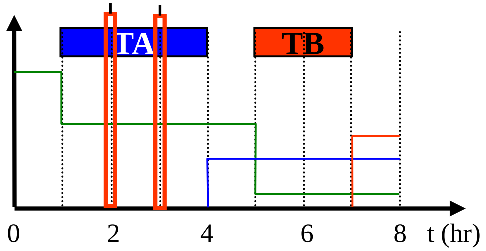
$$S_{m,t+1} = S_{m,t} + \sum \beta_i W_{i,t-\tau_i} - \sum \beta_i W_{i,t}, \quad \forall m, t$$

Scheduling State-space Model

- Inputs: task start, W_{it}
- States: material inventory level, S_{mt}

$W_{\mathrm{TA},2} = 0,\ W_{\mathrm{TB},2} = 0,\ S_{\mathrm{RM},2} = 8,\ S_{\mathrm{A},2} = S_{\mathrm{B},2} = 0$

$W_{\mathrm{TA},3} = 0,\ W_{\mathrm{TB},3} = 0,\ S_{\mathrm{RM},3} = 8,\ S_{\mathrm{A},3} = S_{\mathrm{B},3} = 0$

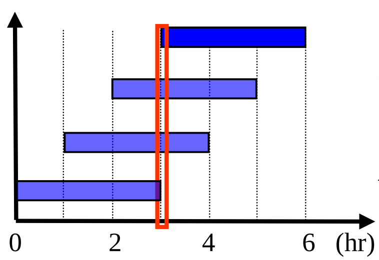


The state of the system is not fully described

$\bar{W}^1_{\mathrm{TA},2} = 1, \bar{W}^2_{\mathrm{TA},2} = 0, \bar{W}^3_{\mathrm{TA},2} = 0$

$\bar{W}^1_{\mathrm{TA},3} = 0, \bar{W}^2_{\mathrm{TA},3} = 1, \bar{W}^3_{\mathrm{TA},3} = 0$

- Lifting of task variables



$W_{\mathrm{TA},3} = 1$

$\bar{W}^1_{\mathrm{TA},3} = W_{\mathrm{TA},2} = 1$

$\bar{W}^2_{\mathrm{TA},3} = \bar{W}^1_{\mathrm{TA},2} = W_{\mathrm{TA},1} = 1$

$\bar{W}^3_{\mathrm{TA},3} = \bar{W}^2_{\mathrm{TA},2} = W_{\mathrm{TA},0} = 1$

$$\boxed{\bar{W}^n_{i,t+1} = \bar{W}^{n-1}_{i,t}, \quad \text{with} \quad \bar{W}^0_{it} = W_{it}}$$

- Material balances: $S_{m,t+1} = S_{m,t} + \sum \beta_i W_{i,t-\tau_i} - \sum \beta_i W_{i,t}$

$$\boxed{S_{m,t+1} = S_{m,t} + \sum \beta_i \bar{W}^{\tau_i}_{i,t} - \sum \beta_i W_{i,t}, \quad \forall m, t}$$

- Resource constraints: $\sum_i \sum_{t'=t-\tau_i+1}^{t'=t} W_{i,t'} \le 1$

$$\boxed{\sum_i W_{i,t} + \sum_i \sum_{n=t-\tau_i}^{n=1} \bar{W}^n_{i,t} \le 1, \forall j, t}$$

Scheduling State-space Model

- Inputs: task start $u = [W_i]$
- States: inventory and task-status $x = [S_m, \bar{W}_i^n]$
- Dynamic model: task-status $\bar{W}_i^n(t+1) = \bar{W}_i^{n-1}(t)$

  inventory $S_m(t+1) = S_m(t) + \sum \beta_i \bar{W}_i^{\tau_i}(t) - \sum \beta_i W_i(t)$

$$x(t+1) = Ax(t) + Bu(t)$$

$$\begin{bmatrix} \bar{W}_{\mathrm{TA}}^1 \\ \bar{W}_{\mathrm{TA}}^2 \\ \bar{W}_{\mathrm{TA}}^3 \\ \bar{W}_{\mathrm{TB}}^1 \\ \bar{W}_{\mathrm{TB}}^2 \\ S_{\mathrm{RM}} \\ S_{\mathrm{A}} \\ S_{\mathrm{B}} \end{bmatrix}_{k+1} = \begin{bmatrix} & & & & & & & \\ 1 & & & & & & & \\ & 1 & & & & & & \\ & & & & & & & \\ & & 1 & & & & & \\ & & & & & 1 & & \\ & & \beta_{\mathrm{TA}} & & & & 1 & \\ & & & & \beta_{\mathrm{TB}} & & & 1 \end{bmatrix} \begin{bmatrix} \bar{W}_{\mathrm{TA}}^1 \\ \bar{W}_{\mathrm{TA}}^2 \\ \bar{W}_{\mathrm{TA}}^3 \\ \bar{W}_{\mathrm{TB}}^1 \\ \bar{W}_{\mathrm{TB}}^2 \\ S_{\mathrm{RM}} \\ S_{\mathrm{A}} \\ S_{\mathrm{B}} \end{bmatrix}_k + \begin{bmatrix} 1 & \\ & \\ & \\ 1 & \\ & \\ -\beta_{\mathrm{TA}} & -\beta_{\mathrm{TB}} \\ & \\ & \end{bmatrix} \begin{bmatrix} W_{\mathrm{TA}} \\ W_{\mathrm{TB}} \end{bmatrix}_k$$

- (Resource) constraints

$$\sum_i W_i(t) + \sum_i \sum_{n=t-\tau_i}^{n=1} \bar{W}_i^n(t) \le 1$$

$$Ex(t) + Fu(t) \le b$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} W_{\mathrm{TA}} \\ W_{\mathrm{TB}} \end{bmatrix}_k + \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \bar{W}_{\mathrm{TA}}^1 \\ \bar{W}_{\mathrm{TA}}^2 \\ \bar{W}_{\mathrm{TB}}^1 \end{bmatrix}_k \le [1]$$

Disturbances

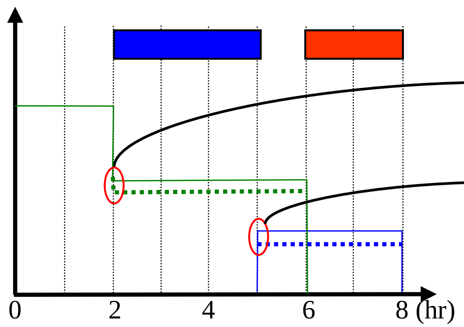
- Dynamic model with disturbances: $x(t+1) = Ax(t) + Bu(t) + B_d d(t)$

- Deliveries and orders
  - D_{mt}: net delivery of material m during period t

$$S_{m,t+1} = S_{m,t} + \sum \beta_i \bar{W}_{it}^{\tau_i} - \sum \beta_i W_{it} + \xi_{mt}$$

$$S_m(t+1) = S_m(t) + \sum \beta_i \bar{W}_i^{\tau_i}(t) - \sum \beta_i W_i(t) + \xi_m(t)$$

- Production disturbances
  - ΔC_{it}: consumption disturbance for task i starting at t
  - ΔP_{it}: production disturbance for task i finishing at t



Loading mistake (loss):

$$S_{\mathrm{RM},3} = S_{\mathrm{RM},2} - (4W_{\mathrm{TA},2} + \Delta C_{\mathrm{TA},2})$$

Lower yield:

$$S_{\mathrm{A},6} = S_{\mathrm{A},5} + (4W_{\mathrm{TA},2} + \Delta P_{\mathrm{TA},5})$$

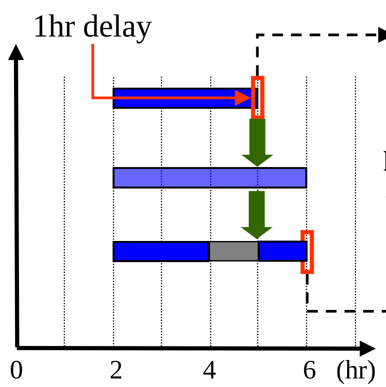
$$S_{\mathrm{A},6} = S_{\mathrm{A},5} + (4\bar{W}_{\mathrm{TA},5}^{3} + \Delta P_{\mathrm{TA},5})$$

$$S_{m,t+1} = S_{m,t} + \sum (\beta_i \bar{W}_{it}^{\tau_i} + \Delta P_{it}) - \sum (\beta_i W_{it} + \Delta C_{it}) + D_{mt}$$

Disturbances

- Dynamic model with disturbances: $x(t+1) = Ax(t) + Bu(t) + B_d d(t)$

- Delays
  - $Y_{i,t}^{n}$: 1 hr delay of task *i* after running for *n* periods



$\bar{W}_{\mathrm{TA},5}^{1} = 0$
$\bar{W}_{\mathrm{TA},5}^{2} = 0$
$\bar{W}_{\mathrm{TA},5}^{3} = 1$

$t = 5 \rightarrow 6$

Delay (at t=5)
τ_{TA} = 3hr after TA started

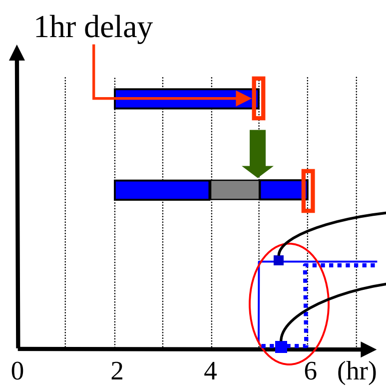
$Y_{\mathrm{TA},5}^{3}$

$\bar{W}_{\mathrm{TA},6}^{1} = W_{\mathrm{TA},5} = 0$
$\bar{W}_{\mathrm{TA},6}^{2} = \bar{W}_{\mathrm{TA},5}^{1} = 0$
$\bar{W}_{\mathrm{TA},6}^{3} = \bar{W}_{\mathrm{TA},5}^{2} + Y_{\mathrm{TA},5}^{3} = 1$

$$\bar{W}_{i,t+1}^{n} = \bar{W}_{i,t}^{n-1} + Y_{i,t}^{n} - Y_{i,t}^{n-1}$$

$\bar{W}_{\mathrm{TA},6}^{1} = 0$
$\bar{W}_{\mathrm{TA},6}^{2} = 0$
$\bar{W}_{\mathrm{TA},6}^{3} = 1$

  - Inventory correction



$S_{\mathrm{A},6} = S_{\mathrm{A},5} + 4\bar{W}_{\mathrm{TA},5}^{3} = 4$

$S_{\mathrm{A},6} = S_{\mathrm{A},5} + 4\bar{W}_{\mathrm{TA},5}^{3} - 4Y_{\mathrm{TA},5}^{3} = 0$

$$S_{m,t+1} = S_{m,t} + \sum \beta_i (\bar{W}_{it}^{\tau_i} - Y_{it}^{\tau_i}) - \sum \beta_i W_{it}$$

Scheduling State-space Model

- Input (task start) $u = [W_i]$
- States: $x = [S_m, \bar{W}_i^n]$
- Disturbances: $d = [\hat{\xi}_{mt}, \hat{\theta}_{imt}^P, \hat{\theta}_{imt}^C, \hat{Y}_{it}^n, \hat{Z}_{it}^n]$ $\quad \hat{\theta}_{imt}^P = \Delta P_{imt}, \hat{\theta}_{imt}^C = \Delta C_{imt}$
- Dynamic model $x(t+1) = Ax(t) + Bu(t) + B_d d(t)$
  - Task-status: $\bar{W}_{i,t+1}^n = \bar{W}_{i,t}^{n-1} + \hat{Y}_{it}^n - \hat{Y}_{it}^{n-1} - \hat{Z}_{it}^{n-1}, \qquad \forall i,t,n \in \{1,2,\dots,\tau_i\}$
  - Inventory: $S_{m,t+1} = S_{m,t} + \sum_i \rho_{im}\beta_i(\bar{W}_{it}^{\tau_i} + \hat{\theta}_{imt}^P - \hat{Y}_{it}^n - \hat{Z}_{it}^n) + \sum_i \rho_{im}\beta_i(W_{it} + \hat{\theta}_{imt}^C) + \hat{\xi}_{mt}, \qquad \forall m,t$
- Constraints: $Ex(t) + Fu(t) + Gd(t) \leq b$

$$\sum_{i\in I_j} W_{it} + \sum_{i\in I_j}\sum_{n=1}^{\tau_i - 1} \bar{W}_{it}^n + \sum_{i\in I_j} \hat{Y}_{it}^{\tau_i} + \sum_{i\in I_j}\sum_{n=1}^{\tau_i} \hat{Z}_{it}^n \leq 1, \qquad \forall j,t$$

Extensions

- Variable batch-sizes/rates
- General resources (and constraints)
- Additional variables (e.g., flows, setups)

Remarks - I

- **General state-space scheduling model**
  - *Inventory control* – *scheduling* integration for supply chain management
  - *Scheduling* – *control* integration
  - *MPC-friendly* scheduling model; use existing and develop new results
- ***Standardize* rescheduling**
  - Currently, unclear what rescheduling means
  - Keep the same state-space model; infer reformulation of the MIP model
  - Reverse transformation: state-space mode + disturbances → MIP model
- **Questions:**
  - What are input/output setpoint trajectories?
  - What does stability mean in scheduling?
  - What do terminal regions/penalties mean?
    How can we generate them?

Remarks - II

- **Rescheduling viewed as *Optimization Under Uncertainty* problem**
  - Demand: right-hand-side (RHS) uncertainty √√
  - Task yields: left-hand-side (LHS) √
  - Unit delay **?**
    LHS parametric uncertainty in continuous-time
    Structural uncertainty in discrete-time models
  - Unit breakdown ×
    Endogenous uncertainty

- **Treat scheduling as (deterministic) online problem**
  - Develop models and methods for deterministic problem
  - Model all *uncertainties* through disturbances (state-space model)
  - Solve deterministic problem **fast**
    - Optimal deterministic solution better than suboptimal stochastic solution
    - Reschedule more frequently to respond to disturbances
    - Consider longer-planning horizons

Outline

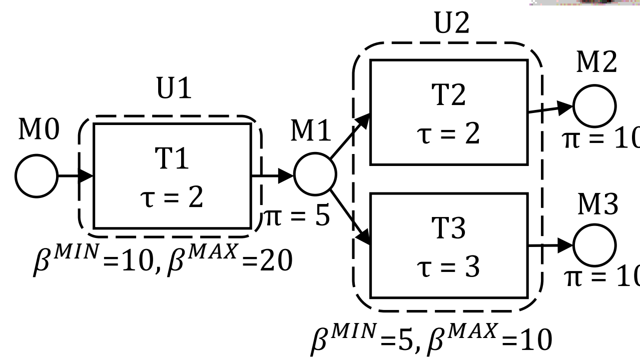
- Introduction
  - Problem Classes
  - Types of Models
- General Model
  - State-task network (STN) representation
  - State-space STN model
  - Remarks
- **Online scheduling**
  - **Open-loop vs. closed-loop solutions**
  - **Online scheduling algorithm**

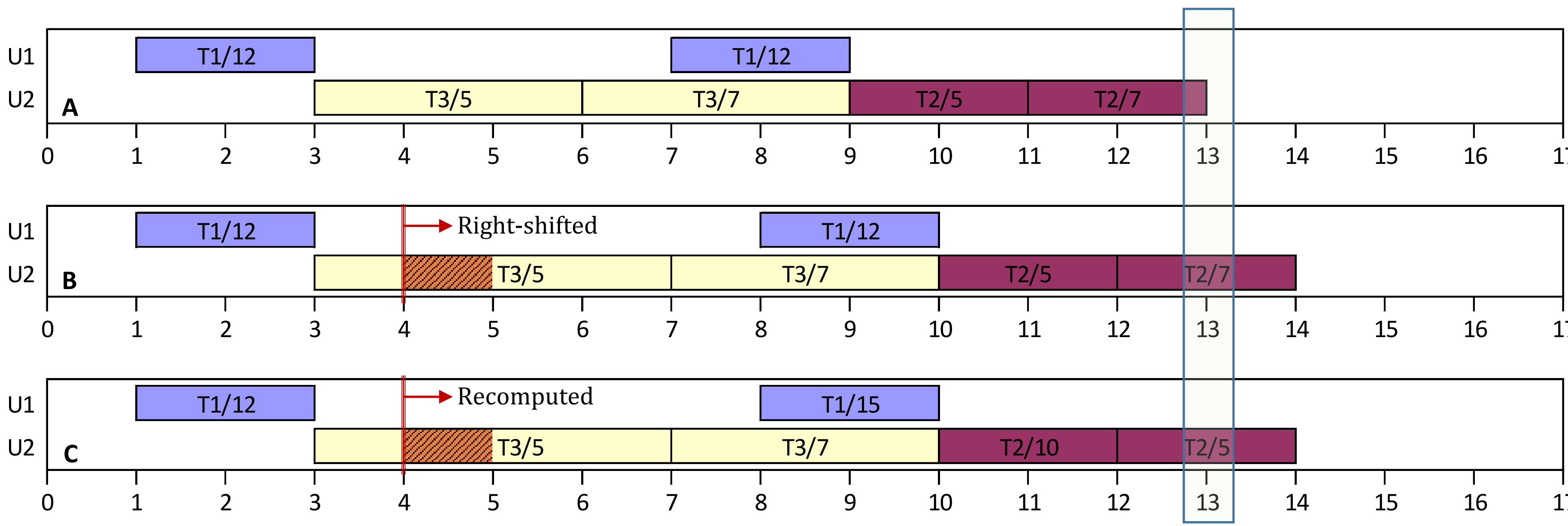
Rescheduling - Motivation

(A) Schedule to meet an order for 12 tons of M2 and M3 each due at t = 13.

(B) A delay in T3 (due to electric-power loss) between t = 3 and t = 4, requires right-shifting of the schedule. This results in part of the order (7 tons) delayed.

(C) Instead of simple right-shift, the schedule is recomputed at t = 4, thereby, reducing the part of the order delayed (to 2 tons).





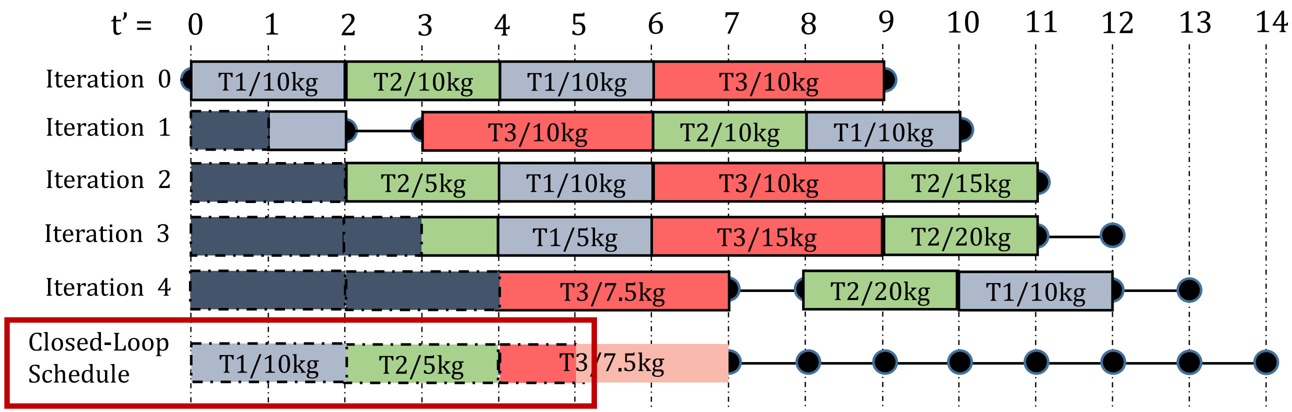
Literature:

- We need rescheduling to act upon ***trigger events***.
- By reacting, we mitigate effect of trigger events (uncertainty/disturbances)
- Even better: account for uncertainty (in trigger events?)

Open-loop *vs.* Closed Loop Schedules

- **Traditional Approach:** Solve problem the *best way possible* & react if/when necessary
- Is this really all?
- Use simulation to study what happens in *reality*

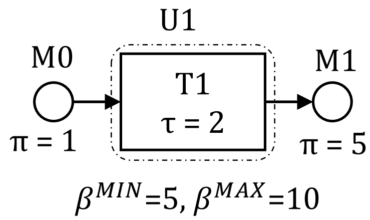
Simulation Framework

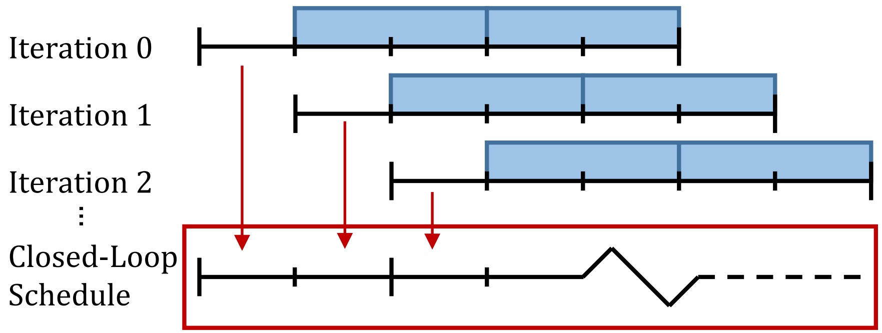


Open-loop *vs.* Closed Loop Solution

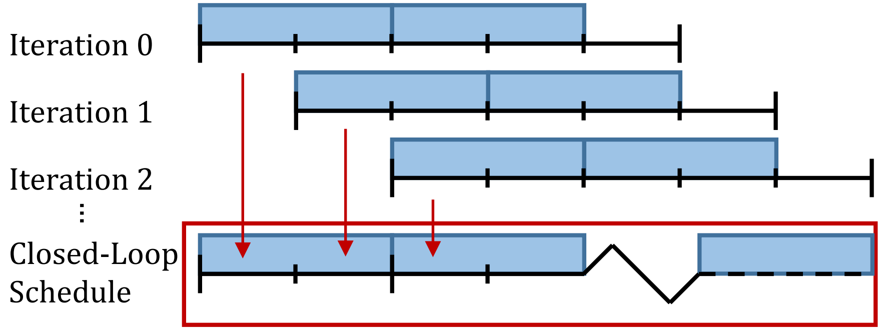
Experiment #1

- Re-optimize at every t using horizon $T = 5$ hr
- Maximize profit using Shah (1993) model [max $S_{M1,T}$ – *Inventory Costs*]
- What do we expect at $t = 0$?





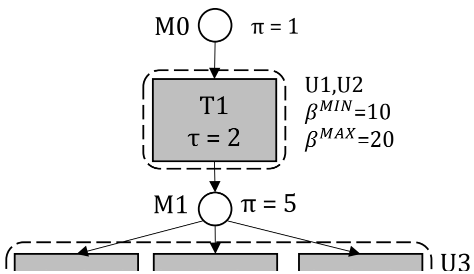
Left-shifted by **favoring early sales in objective**



Open Loop *vs.* Closed Loop Solution

Experiment #2

- Orders of size 4 tons due every 3 h for each product (M5-M7)
- The facility starts at $t = 0$ with a safety stock (inventory) of product materials sufficient to meet two orders (t = 3, 6)
- Use horizon η = 8 hr.

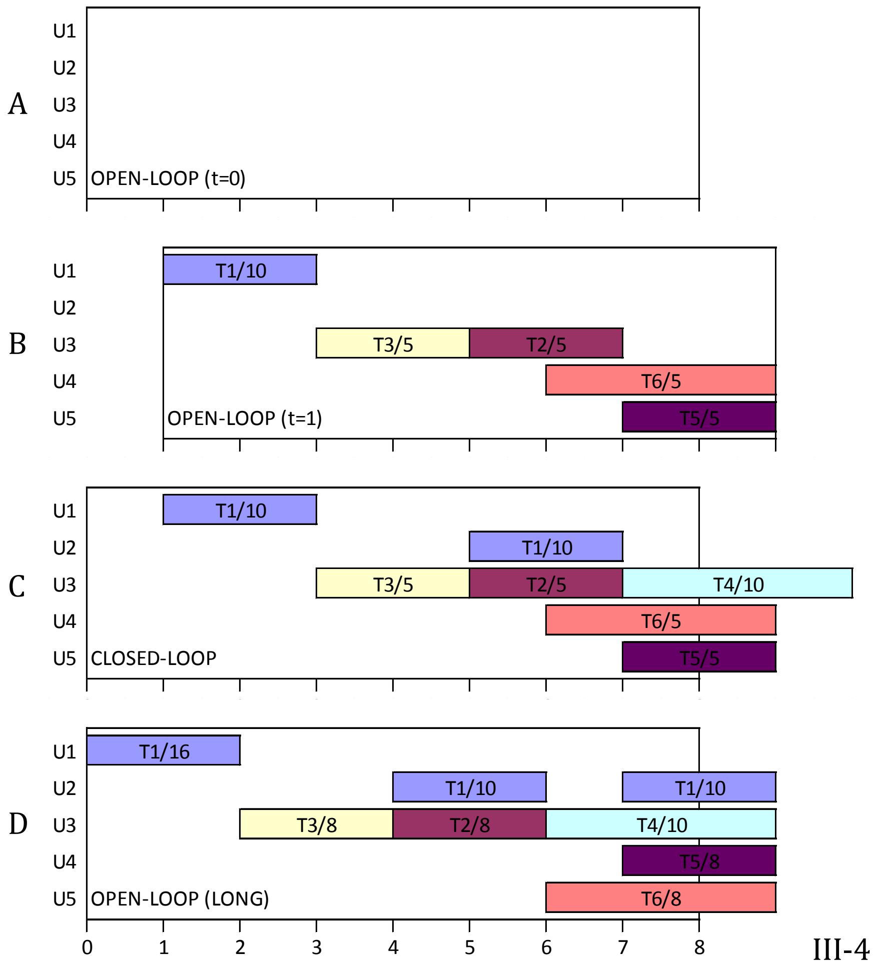


(A) Open-loop (at $t = 0$) with moving horizon length 8 h, as an empty schedule and inventory holding cost of $26,400.

(B) Production starts in open-loop (at $t = 1$) and the objective value for this open-loop solution is $31,005.

(C) Resulting closed-loop schedule from solution to 8 open-loop problems (solved at $t = 0,1,2,\ldots 7$ respectively), with an evaluated cost of $27,393.

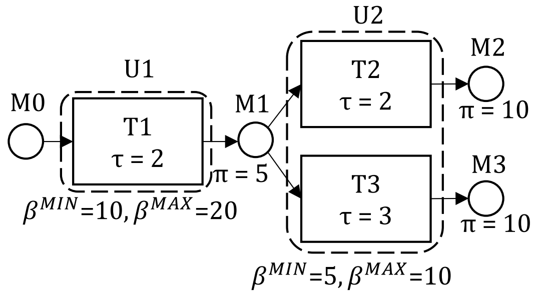
(D) Gantt chart for first 8 h, of a long open-loop problem solved spanning $t = 0$ to $t = 15$; cost for the first 8 periods is $29,592.



Open Loop vs. Closed Loop Solution

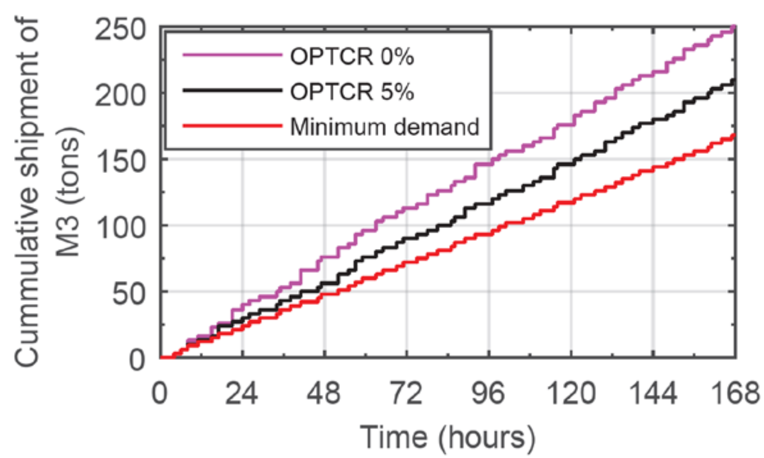
Experiment #3

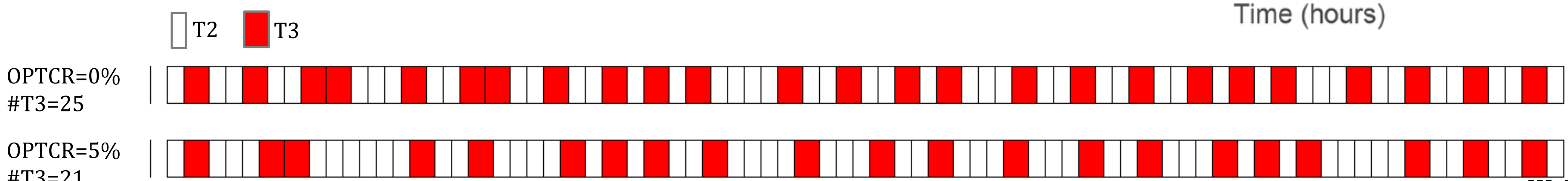
- Orders of 3 tons for each product are due every 3 ± 1 hr
- Excess sales are allowed when orders are due
- The objective is to maximize profit.
- Objective is modified to favor early sales so excess inventory is shipped as soon as possible.
- Since T2 takes less time, M2 can be produced at a faster rate; thus, executing T2 leads to more profit.
- Best schedule: T2 dominates; T3 has the minimum possible # of batches, just to meet M3 demand.



Closed Loop Schedules with 0 and 5% Optimality Gap

- Deterministic data; η = 12 hr
- Closed-loop solutions generated for 1 week.
- With OPTCR = 0%, T3 is executed 25 times
- With OPTCR = 5%, T3 is executed 21 times
- Suboptimal open-loop solutions lead to better closed loop (implemented solutions)

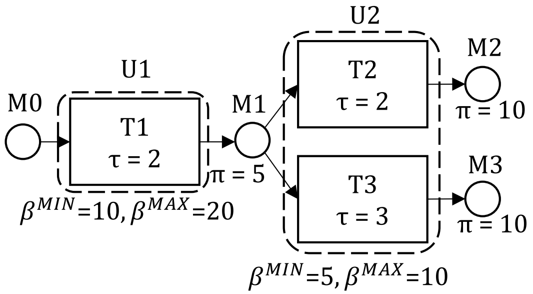




Uncertainty vs. New Information

Experiment #4

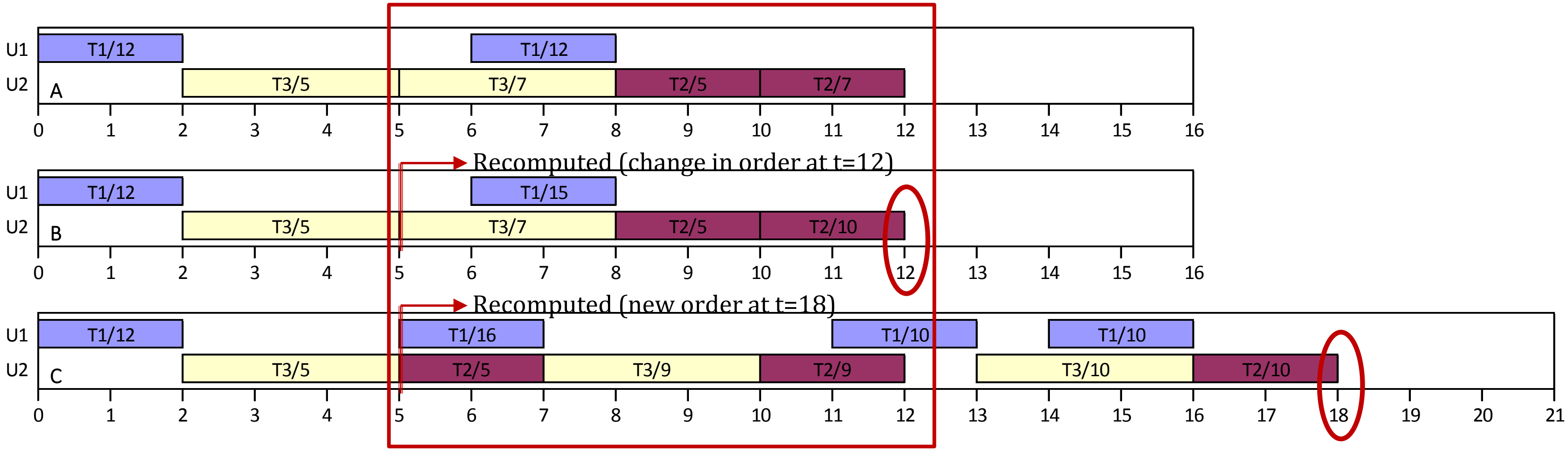
- Have to meet orders of 12 tons for M2 and M3, each due every 6 hours starting at t=12
- Use horizon η = 16 hr



(A) Schedule computed at t=0.

(B) At t=5, the uncertainty in order due at t=12 is observed; the order becomes 12 tons of M3 and 15 tons M2. Remaining schedule from t=5 onwards is recomputed and implemented.

(C) At t=5, the 16-hr horizon is advanced to span t=5 to t=21; the orders at t = 12 remain the same The new schedule is recomputed to account for a new order due at t=18.



- The difference between A and C is bigger than the difference between A and B
- Accounting for new information can be more important than accounting for uncertainty

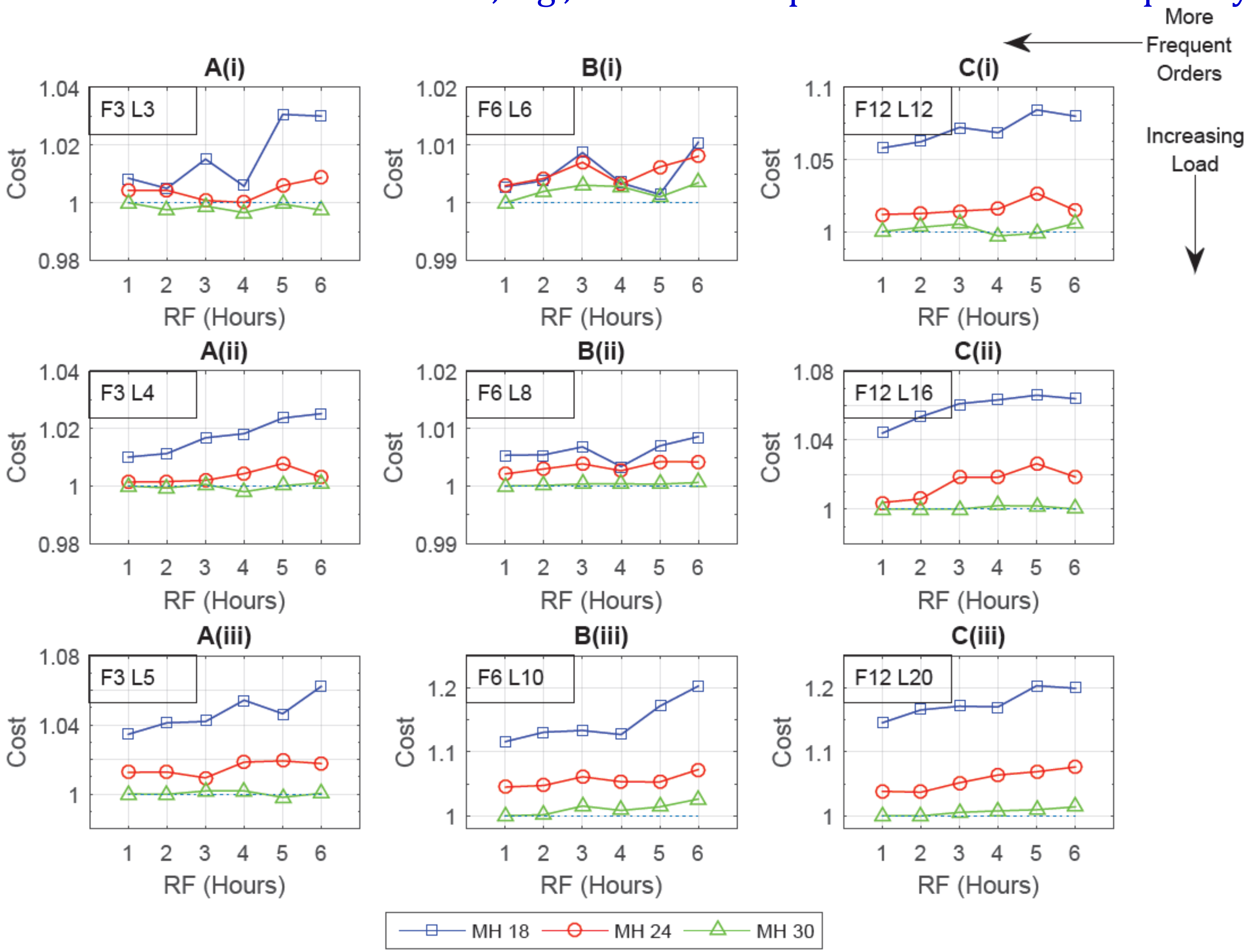
Conclusions

- **Open-loop and closed-loop scheduling are two different problems**
- **Have to re-optimize even if there are no trigger events**
- **How can we obtain good closed-loop solutions?**

1) **Online scheduling algorithm**
   - How often should we re-optimize (rescheduling frequency)?
   - How long should the horizon be?
   - What is a good optimality gap?
2) **Open-loop model**
   - What objective function should we use?
   - Anything else?

Some Analysis

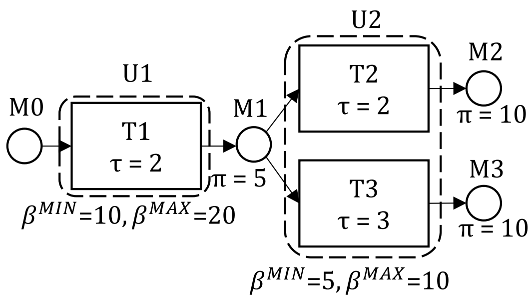
- There are threshold MH, RF, and OPTCR values which are functions of facility & demand
- Three attributes are inter-related; e.g., horizon compensates for slow frequency



... and *Some* Answers

Experiment #3 revisited

- Orders of 3 tons for each product are due every 3 ± 1 hr
- Max Profit with excess sales when orders are due
- Favoring early sales is bad idea: excess cheap product is sold, so new batches should be started to meet future demand.
- It is better to NOT sell early on.
- Model modification: no excess sales allowed during the first 6 hr



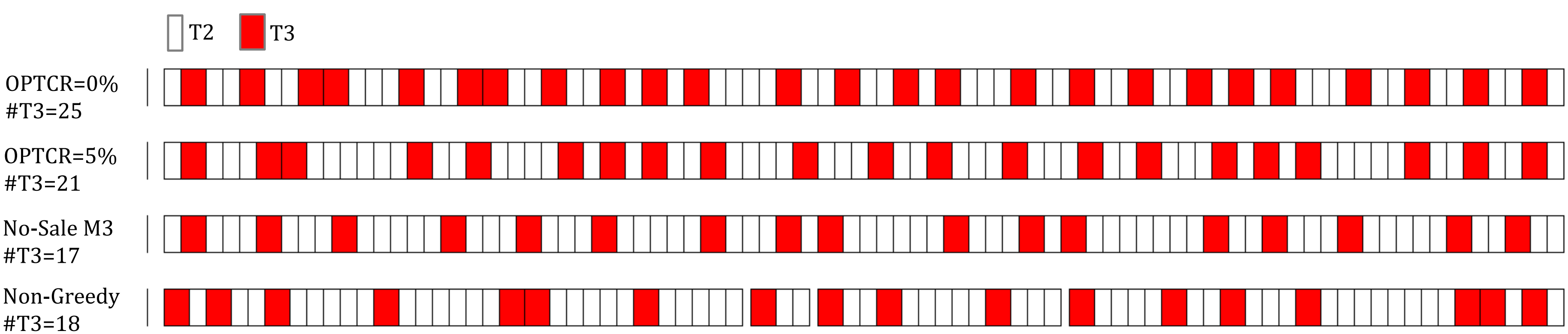
Closed loop schedules for unit 2:

(A) MH = 12, OPTCR = 0%; T3 is executed 25 times

(B) MH = 12, OPTCR = 5%; T3 is executed 21 times

(C) MH = 12, OPTCR = 5%, plus no sales constraint;.T3 is executed 17 times (best).

(D) MH = 12, OPTCR = 0%, no obj function modification: T3 is executed 18 times but T2 is executed fewer times (lower profit)



Final Thoughts

- **Chemical production scheduling**
  - ***Most* modeling challenges addressed**
  - **Major computational advances**
  - **Unsolved problems**
    Sequence-dependent changeovers, nonlinear models
- **From rescheduling to online scheduling**
  - **State-space STN model facilitates representation**
  - **Deterministic problem**
    - Consider larger problem
    - Re-optimize faster
- **Open questions**
  - **Open-loop problem modifications**
  - **Online scheduling algorithm attributes**