Performance Improvements in Extremum Seeking Control

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Introduction

- Extremum seeking is a real-time optimization technique.

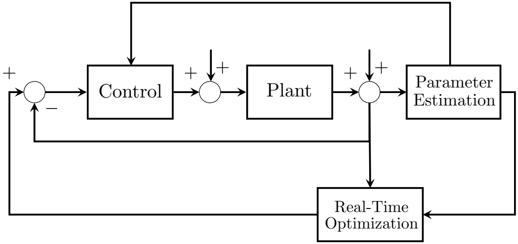


Figure : Basic RTO loop.

- RTO is a supervisory system designed to monitor and improve process performance.
- It uses process data to move the process to operating points that are optimal wrt a meaningful user-defined metric

Introduction

- In most applications, RTO exploits process models and optimization techniques to compute optimal steady-state operating conditions
  - Control objectives vs. Optimization objectives
- Success of RTO relies on
  - the accuracy of the (steady-state) model
  - robustness of the RTO approach
  - flexibility of the control system
- In the absence of accurate process descriptions (model-based) RTO yields erratic results

Successful RTO requires integrated solutions.

Introduction

- Extremum Seeking Control (ESC) is a model free technique that relies on minimal assumptions concerning:
  - the process model
  - the objective function
  - the constraints
- ESC only requires the measurement of the objective function and the constraints
- Considerable appeal in practice
  - Achieves RTO objectives without the need for complex model-based formulations.

Introduction

- Extremum-seeking control (ESC) has been the subject of considerable research effort over the last decade.
- Mechanism dates back to the 1920s [Leblanc, 1922]
  - Objective is to drive a system to the optimum of a measured variable of interest [Tan et al., 2010]
- Revived interest in the field was primarily sparked by Krstic and co-workers [Krstic and Wang, 2000]
  - Provided an elegant proof of the convergence of a standard perturbation based ESC for a general class of nonlinear systems

Introduction

Basic ESC objectives:

- Given an (unknown) nonlinear dynamical system and (unknown) measured cost function:

$$\begin{aligned} \dot{x} &= f(x, u) && (1) \\ y &= h(x) && (2) \end{aligned}$$

- The objective is to steer the system to the equilibrium x^* and u^* that achieves the minimum value of $y(= h(x^*))$.

Problem Definition

- The objective is to steer the system to the equilibrium x^* and u^* that achieves the minimum value of $y(= h(x^*))$.
  - The equilibrium (or steady-state) map is the n dimensional vector $\pi(u)$ which is such that:

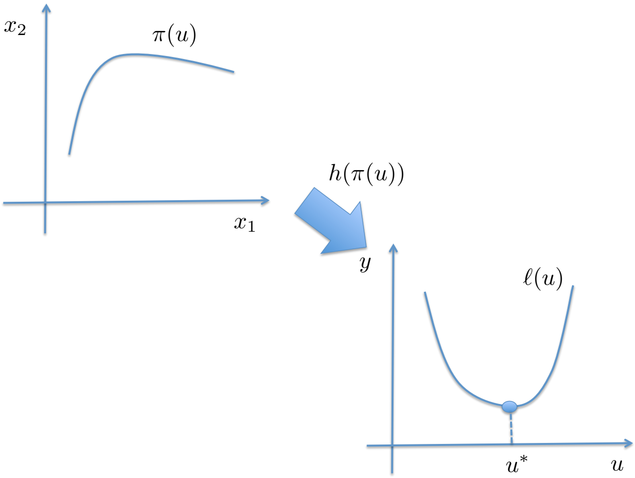
$$f(\pi(u), u) = 0.$$

  - The equilibrium cost function is given by:

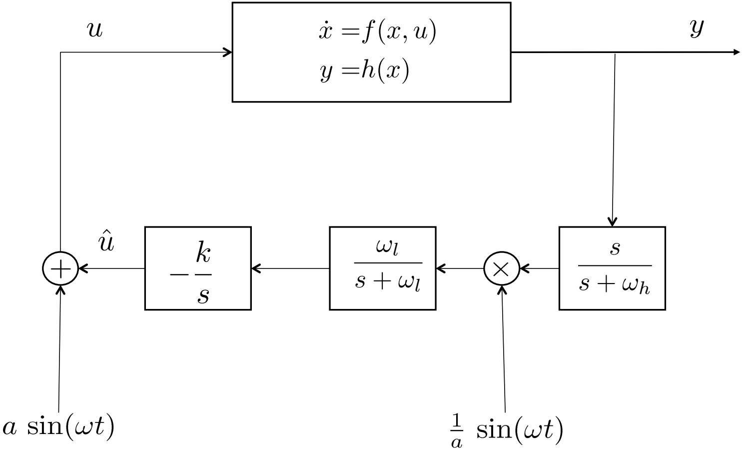
$$y = h(\pi(u)) = \ell(u) \tag{3}$$

  - The problem is to find the minimizer u^* of $y = \ell(u^*)$.

Problem Definition



Basic ESC Loop



Basic ESC Loop

- Closed-loop dynamics are:

$$
\begin{aligned}
\dot{x} &= f(x, \widehat{u}(t) + a\sin(\omega t)) \\
\dot{\widehat{u}} &= -\omega k \xi \\
\dot{\xi} &= -\omega\omega_l \xi + \omega \frac{\omega_l}{a} (h(x) - \eta) \sin(\omega t) \\
\dot{\eta} &= -\omega\omega_h \eta + \omega\omega_h h(x).
\end{aligned}
$$

- Tuning parameters are:
  - k the adaptation gain
  - a the dither signal amplitude
  - ω the dither signal frequency
  - ω_l and ω_h the low-pass and high-pass filter parameters

Basic ESC loop

- The stability analysis [Krstic and Wang, 2000] relies on two components:
    1. an averaging analysis of the persistently perturbed ESC loop
    2. a time-scale separation of ESC closed-loop dynamics between the system dynamics and the quasi steady-state extremum-seeking task.
- This is a very powerful and very general result.
- Analysis confirms properties: *small* a, *small* ω, *small* k.
- Convergence is slow with limited robustness.

Proportional Integral ESC

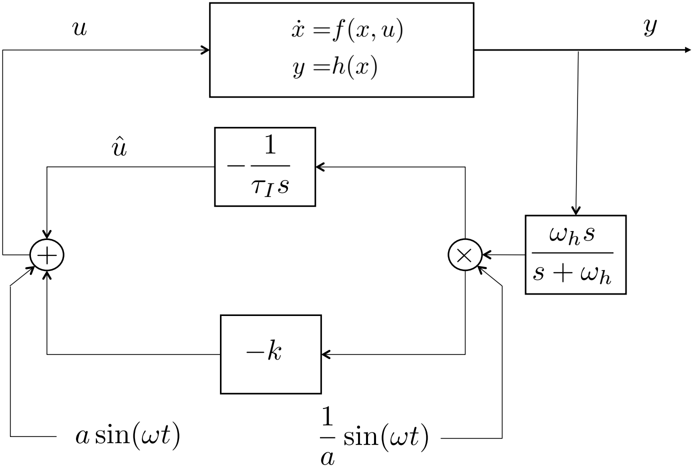
Limitations associated with the two time-scale approach to ESC remains problematic.

- Two (or more) time-scale assumption is required to ensure that optimization operates at a quasi steady-state time-scale
- Convergence is very slow.
- Limits applicability in practice.

Improvement in transient performance are possible:

- Standard ESC is an integral controller $\rightarrow$ Performance limitation
- Add proportional action.

Proportional Integral ESC



Proportional Integral ESC

- Proposed PI-ESC algorithm:

$$\begin{aligned}
\dot{x} &= f(x) + g(x)u \\
\dot{v} &= -\omega_h v + y \\
\dot{\widehat{u}} &= -\frac{1}{\tau_I}(-\omega_h^2 v + \omega_h y)\sin(\omega t) \\
u &= -\frac{k}{a}(-\omega_h^2 v + \omega_h y)\sin(\omega t) + \widehat{u} + a\sin(\omega t).
\end{aligned}$$

- Tuning parameters:
  - k and τ_I are the proportional and integral gain
  - a and ω are the dither amplitude and frequency
  - $\omega_h (>> \omega)$ is the high-pass filter parameter.

Proportional Integral ESC

Theorem 1

Consider the nonlinear closed-loop PIESC system with cost function $y = h(x)$. Let Assumptions 1, 2, 3 and 4 hold. Then

1. there exists a τ_I^* such that for all $\tau_I > \tau_I^*$ the trajectories of the nonlinear system converge to an $\mathcal{O}(1/\omega)$ neighbourhood of the unknown optimum equilibrium, $x^* = \pi(u^*)$,
2. there exists $\omega^* > 0$ such that, for any $\omega > \omega^*$, the unknown optimum is a practically stable equilibrium of the PIESC system with a region of attraction whose size grows with the ratio $\frac{a}{k}$,
3. $\|x - x^*\|$ enters an $\mathcal{O}(\frac{1}{\omega}) + \mathcal{O}(\frac{k}{\omega a}) + \mathcal{O}(\frac{a}{\omega})$ neighbourhood of the origin and $\|\widehat{u} - u^*\|$ enters an $\mathcal{O}(\frac{1}{\omega}) + \mathcal{O}(\frac{1}{\omega a \tau_I}) + \mathcal{O}(\frac{a}{\tau_I \omega})$ of the origin.

Proportional Integral ESC

- Proof of theorem demonstrates that:
  - the proportional action minimizes the impact of the time scale separation
  - the integral action acts as a standard perturbation based ESC
  - Combined action guarantees stabilization of the unknown equilibrium
  - With fast convergence
- Impact of dither signal is inversely proportional to the frequency
- Size of ROA is proportional to $\frac{a}{k}$.
- **PIESC acts as a dynamic output feedback nonlinear controller.**

Example 1

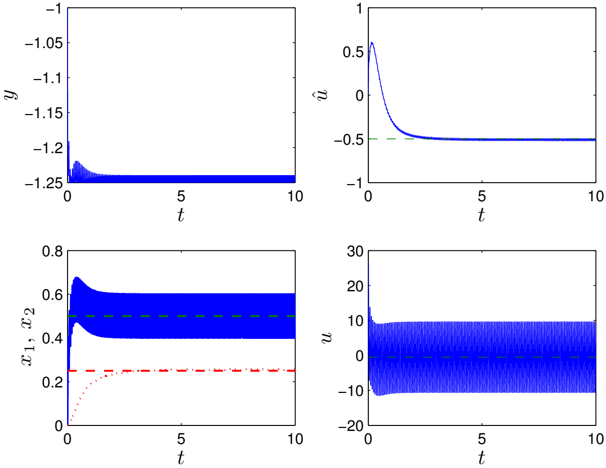
We consider the following dynamical system taken from Guay and Zhang [2003]:

$$\dot{x}_1 = x_1^2 + x_2 + u$$
$$\dot{x}_2 = -x_2 + x_1^2$$

The cost function to be minimized is given by: $y = -1 - x_1 + x_1^2$.

- the optimum cost is $y^* = -1.25$ and occurs at $u^* = -0.5$, $x_1^* = 0.5$, $x_2^* = 0.25$
- The tuning parameters are chosen as: $k = 10$, $\tau_I = 0.1$, $a = 10$, $\omega = 100$ with $\omega_h = 1000$.
- Outperforms the model-based approach of Guay and Zhang [2003]

Example 1



−1
−1.05
−1.1
−1.15
−1.2
−1.25
y
0
5
10
t
1
0.5
0
−0.5
−1
û
0
5
10
t
0.8
0.6
0.4
0.2
0
x 1 , x 2
0
5
10
t
30
20
10
0
−10
−20
u
0
5
10
t

RLS Proportional Integral ESC

- Parameterize $\dot{y}$ as:

$$\dot{y} = \theta_0 + \theta_1 u = \phi^T \theta \tag{4}$$

where $\phi = [1,\, u^T]^T$ and $\theta = [\theta_0, \theta_1^T]^T$.

- θ_0 and θ_1 are unknown time-varying parameters.
- Proposed PI-ESC given by:

$$u = -k\widehat{\theta}_1 + \widehat{u} + d(t)$$

$$\dot{\widehat{u}} = -\frac{k}{\tau_I}\widehat{\theta}_1$$

where

- $\widehat{\theta}_1$ is the estimation of θ_1.
- k is the proportional gain
- τ_I is the integral time constant.

Parameter Estimation

The proposed time-varying parameter estimation scheme consists of an output prediction mechanism.

$$\begin{aligned}
\dot{\widehat{y}} &= \phi^T\widehat{\theta} + Ke + c^T\dot{\widehat{\theta}} && (5)\\
\dot{c}^T &= -Kc^T + \phi^T && (6)\\
\dot{\widehat{\eta}} &= -K\widehat{\eta}. && (7)
\end{aligned}$$

where

- $\widehat{\theta}$ are parameter estimates
- $e = y - \widehat{y}$ and $\widetilde{\theta} = \theta - \widehat{\theta}$
- K is a positive constant to be assigned
- $c \in \mathbb{R}^p$ is the solution of the differential equation:

Parameter Estimation

The parameter estimation law is given by:

$$\dot{\Sigma}^{-1} = -\Sigma^{-1} c c^T \Sigma^{-1} + k_T \Sigma^{-1} - \delta \Sigma^{-2} \tag{8}$$

with initial condition $\Sigma^{-1}(t_0) = \frac{1}{\alpha} I$, and the parameter update law:

$$\dot{\widehat{\theta}} = \mathrm{Proj}(\Sigma^{-1}(c(e - \widehat{\eta}) - \frac{\delta}{2}\widehat{\theta}), \Theta^0), \qquad \widehat{\theta}(t_0) = \theta^0 \in \Theta^0, \tag{9}$$

where δ is a positive constant. $\mathrm{Proj}\{\phi, \widehat{\theta}\}$ denotes a Lipschitz projection operator Krstic et al. [1995] such that

$$-\mathrm{Proj}\{\phi, \widehat{\theta}\}^T \widetilde{\theta} \leq -\phi^T \widetilde{\theta}, \tag{10}$$

$$\widehat{\theta}(t_0) \in \Theta^0 \implies \widehat{\theta} \in \Theta, \forall t \geq t_0 \tag{11}$$

where $\Theta \triangleq B(\widehat{\theta}, z_\theta)$,

- **Assumption 4:** There exists constants $\alpha_1 > 0$ and $T > 0$ such that

$$\int_{t}^{t+T} c(\tau)c(\tau)^T d\tau \geq \alpha_1 I \tag{12}$$

$\forall t > 0$. $\square$

Theorem 1

Let Assumptions 1 to 4 hold. Consider the extremum-seeking controller and the parameter estimation algorithm. Then there exists tuning parameters k, k_T, K and τ_I^* such that for all $\tau_I > \tau_I^*$. the system converges exponentially to an $\mathcal{O}(D/\tau_I)$ neighbourhood of the minimizer x^* of the measured cost function y.

Example 2

Consider the following system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - x_2 + u$$

with the following cost function:

$$y = 4 + (x_1 - 1.5)^2 + x_2^2.$$

- Tuning parameters: $k_T = 20$, $K = 20I$, $k = 0.25$ and $\tau_I = 0.15$.
- $d(t) = 0.1\sin(10t)$.
- The initial conditions are $\widehat{\theta}(0) = [0,\ -1]^T$, $x_1(0) = x_2(0) = u(0) = 0$.

Example 2

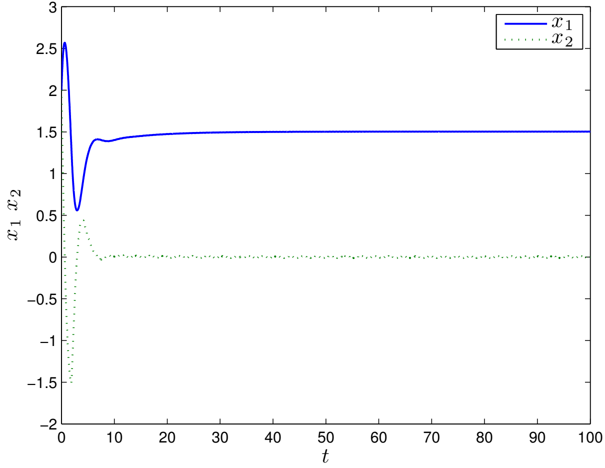


Figure : State trajectories as a function of time.

Example 2

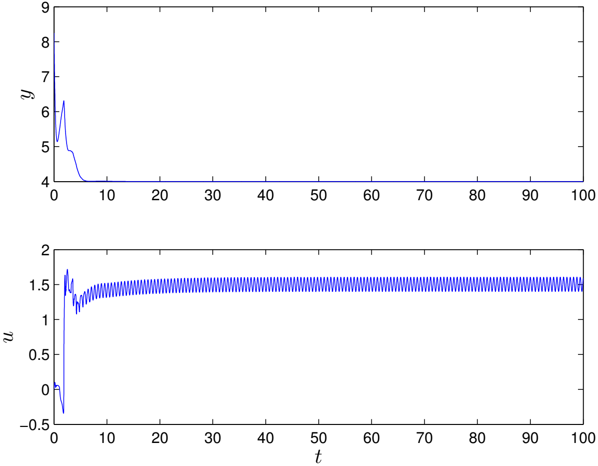


Figure : Input $u(t)$ and output $y(t)$.

Discrete-time ESC

- Design of discrete-time ESC systems is not as prevalent:
  - Discrete-time ESC [Ariyur and Krstic, 2003], [Choi et al., 2002] with application to PID tuning in [Killingsworth and Krstic, 2006].
  - Adaptive estimation approach [Guay, 2014]
  - Discrete-time ESC subject to stochastic perturbations [Manzie and Krstic, 2009] and [Liu and Krstic, 2014b].
  - Approximate parameterizations of the unknown cost function [Ryan and Speyer, 2010].
  - Analysis of nonlinear-optimization algorithms [Teel and Popovic, 2001].
  - Global sampling methods [Nesic et al., 2013].

Discrete-time techniques cannot be derived directly from continuous-time techniques.

Problem Definition

ESC objectives:

- Given an (unknown) nonlinear discrete-time dynamical system and (unknown) measured cost function:

$$
\begin{aligned}
x_{k+1} &= x_k + f(x_k) + g(x_k)u_k && (13)\\
y_k &= h(x_k) && (14)
\end{aligned}
$$

- The objective is to steer the system to the equilibrium x^* and u^* that achieves the minimum value of $y(= h(x^*))$.

Proportional Integral ESC

- The cost function dynamics are parameterized as follows:

$$y_{k+1} = y_k + \theta_{0,k} + \theta_{1,k}^T(u_k - \widehat{u}_k)$$

  where
- $\theta_{0,k}$ and $\theta_{1,k}$ are the time-varying parameters, $\theta_{0,k} = \Psi_{0,k}$ and $\theta_{1,k} = \Psi_{1,k}^T$.
- Proposed PI-ESC given by:

$$u_k = -k_g\widehat{\theta}_{1,k} + \widehat{u}_k + d_k$$
$$\widehat{u}_{k+1} = \widehat{u}_k - \frac{1}{\tau_I}\widehat{\theta}_{1,k}$$

  where
  - $\widehat{\theta}_{1,k}$ is the estimation of $\theta_{1,k}$.
  - k_g is the proportional gain
  - τ_I is the integral time constant.
  - d_k is the dither signal.

Proportional Integral ESC

- Proposed parameter estimation routine given by:

$$
\begin{aligned}
\widehat{y}_{k+1} =& \widehat{y}_k + K(y_k - \widehat{y}_k) + \phi_k^T \widehat{\theta}_k + \omega_k^T (\widehat{\theta}_{1,k+1} - \widehat{\theta}_{1,k}) \\
\Sigma_{k+1} =& \alpha \Sigma_k + \omega_k \omega_k^T + \sigma I \\
\widehat{\theta}_{k+1} =& \text{Proj}\{\widehat{\theta}_k + (\alpha \Sigma_k + \sigma I)^{-1} \omega_k Q_k (e_k - \widehat{\eta}_k), \Theta_k\} \\
Q_k =& (1 + w_k^T (\alpha \Sigma_k + \sigma I)^{-1} w_k)^{-1} \\
\omega_{k+1} =& \omega_k - K \omega_k + \phi_k, \ \widehat{\eta}_{k+1} = \widehat{\eta}_k - K \widehat{\eta}_k
\end{aligned}
$$

- $\phi_k^T = [1, \ (u_k - \widehat{u}_k)^T]^T$, $\widehat{\theta}_k = [\widehat{\theta}_{0,k}, \widehat{\theta}_{1,k}^T]^T$.
- Proj represents an orthogonal projection onto the surface of the uncertainty set $\Theta_k = B(\widehat{\theta}_c, z_{\widehat{\theta}c})$.
- Tuning parameters are α, σ and K.

Proportional Integral ESC

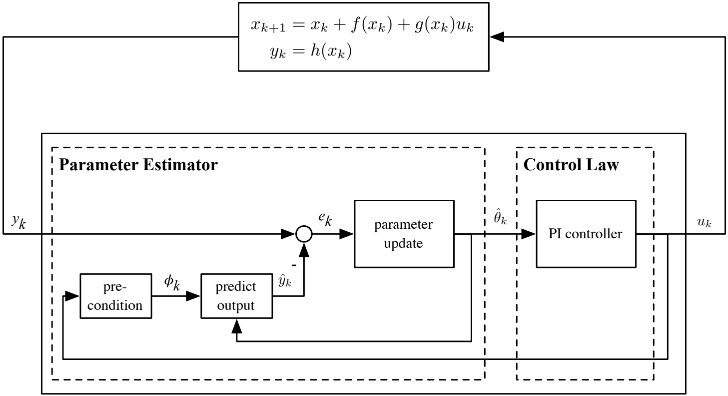


Figure : Schematic representation of the PI-ESC approach.

Proportional Integral ESC

Assumption 4 [Goodwin and Sin, 2013]

There exists constants $\beta_T > 0$ and $T > 0$ such that

$$\frac{1}{T} \sum_{i=k}^{k+T-1} \omega_i \omega_i^T > \beta_T I, \; \forall k > T. \tag{15}$$

Theorem 2

Consider the nonlinear discrete-time system (13) *with cost function* (14)*, the extremum seeking controller and parameter estimation scheme. Let Assumptions 1-6 be fulfilled. Then there exists positive constants* α*,* K*,* $k_g (> k_g^*)$ *and* τ_I *such that for every* $\tau_I \geq \tau_I^*$*, the states* x_k *and input* u_k *of the closed-loop system enter a neighbourhood of the unknown optimum* (x^*, u^*)*.*

Simulation: Example 1

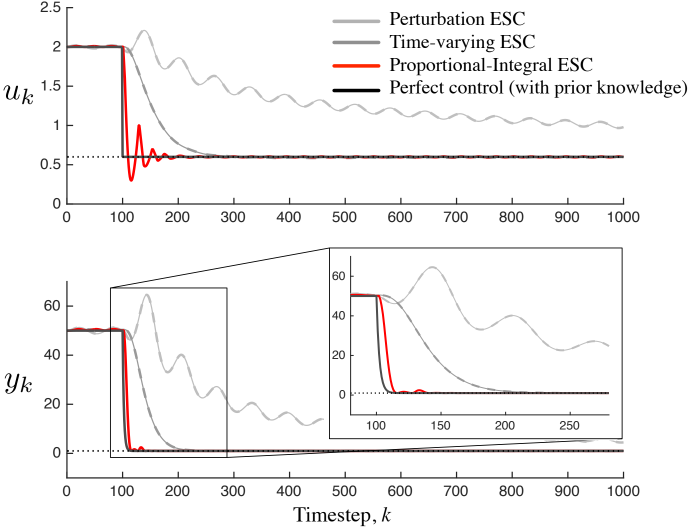
Consider a simple, 1st order, dynamical system:

$$
\begin{aligned}
x_{k+1} &= 0.8x_k + u_k \\
y_k &= (x_k - 3)^2 + 1
\end{aligned}
$$

The steady-state optimum occurs at

$$
\begin{aligned}
u^* &= 0.6 \\
y^* &= 1.
\end{aligned}
$$

A.


Perturbation ESC
Time-varying ESC
Proportional-Integral ESC
Perfect control (with prior knowledge)
u_k
y_k
Timestep, k

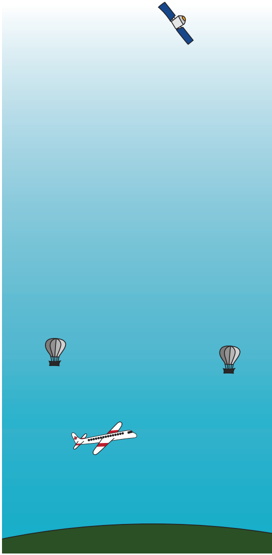
B.

- The discrete-time ESC approach can be generalized for the design of distributed optimization and control of complex unknown networks
- ESC can adjust local actions in the absence of any knowledge about the underlying dynamics and network interactions
- Application to air-based (balloon) internet system design

Air-based internet

Why balloons?

- Float in the stratosphere (10–50 km altitude)
- High enough to avoid weather and airplanes
  - Airplanes typically fly below 15 km altitude
- Low enough for fast connections without lag
  - Satellites fly in low-earth orbit at around 1200 km altitude
- Float passively to minimize energy costs
- Solar panels help balloons stay up for hundreds of days



Modeling balloon dynamics

- Each balloon moves in a spherical shell
- Altitude is limited to 10—50km
- Earth's radius is 6371km so we can neglect altitude
- i^{th} balloon's position can be represented by a point, $q_i \in \mathbb{S}^2$.
- Altitude, u_i, will be used as an input parameter

Assumption 1

The balloons move exactly with the wind currents and assume dynamics characterized by local wind patterns.

- For each altitude, u_i, let $f_{u_i} : \mathbb{R} \times \mathbb{S}^2 \to \mathsf{T}\mathbb{S}^2$ be a time varying vector field on the sphere. Then the balloon's dynamics are:

$$\dot{q}_i = f_{u_i}(t, q_i) \tag{16}$$

- *For simulation:* An approximate model of f_{u_i} can be created by interpolating gridded wind data from the NOAA

Assumption 2

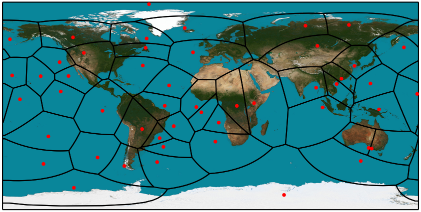
The time-varying vector fields $f_{u_i} \in \mathfrak{X}(\mathbb{R}, \mathbb{S}^2)$ *are smooth and the map* $u_i \mapsto f_{u_i}$ *is smooth*

- Let Γ_i be the region of Earth where users are connected to balloon i
- What should the regions Γ_i look like?
- Define the Voronoi partition by:

$$\Gamma_i = \left\{ q \in \mathbb{S}^2 \mid \mathbb{G}(q, q_i) < \mathbb{G}(q, q_j) \forall j \neq i \right\} \tag{17}$$

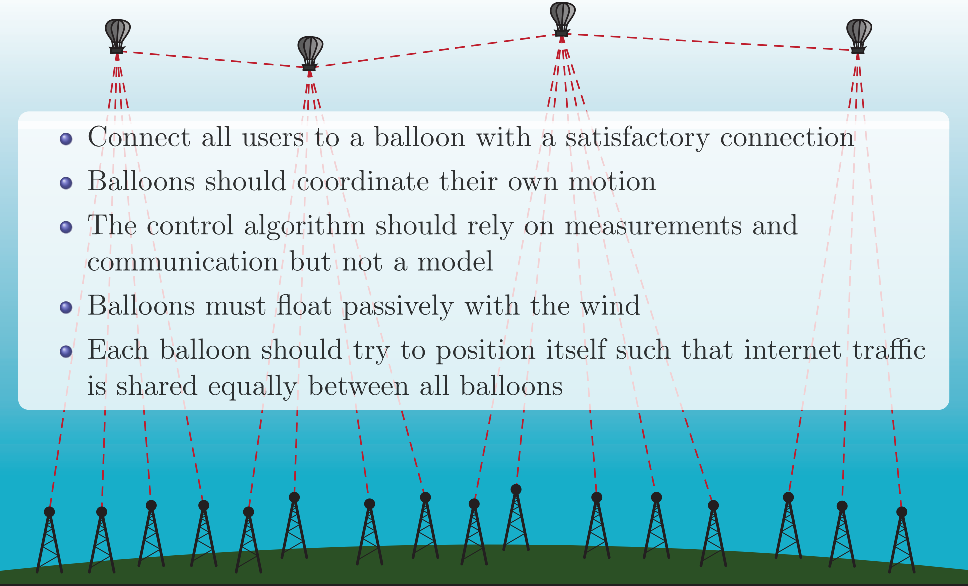
where $\mathbb{G}(\cdot, \cdot)$ is the round metric on $\mathbb{S}^2$.

- Voronoi partitions ensure each user is connected to the nearest balloon



Controller design

Control objectives



- Connect all users to a balloon with a satisfactory connection
- Balloons should coordinate their own motion
- The control algorithm should rely on measurements and communication but not a model
- Balloons must float passively with the wind
- Each balloon should try to position itself such that internet traffic is shared equally between all balloons

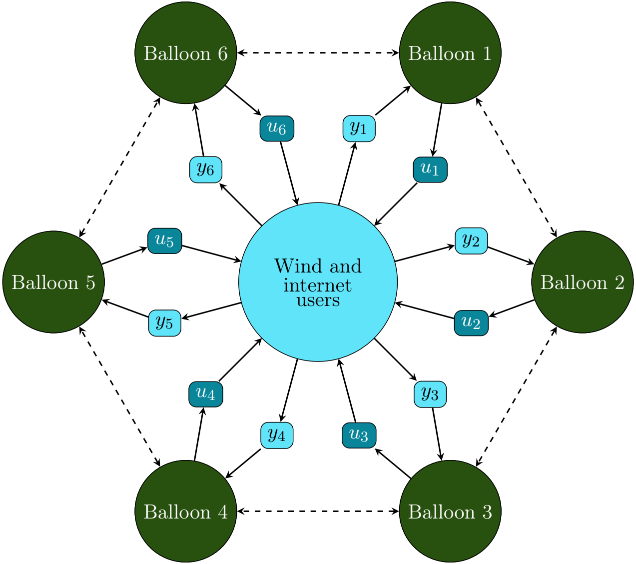
Controller design

Existing approaches

- Google intends on using "some complex algorithms and lots of computing power" (Official Google Blog [2013])
- Sniderman showed that lots of computing power is unnecessary (Sniderman et al. [2015])
  - Uses a geometric, block-circulant approach
  - Algorithms rely on a linear model of wind currents
  - Simulations only performed on a circle and do not generalize to a sphere
- Can we solve control a non-linear 2-dimensional system without lots of computing power?

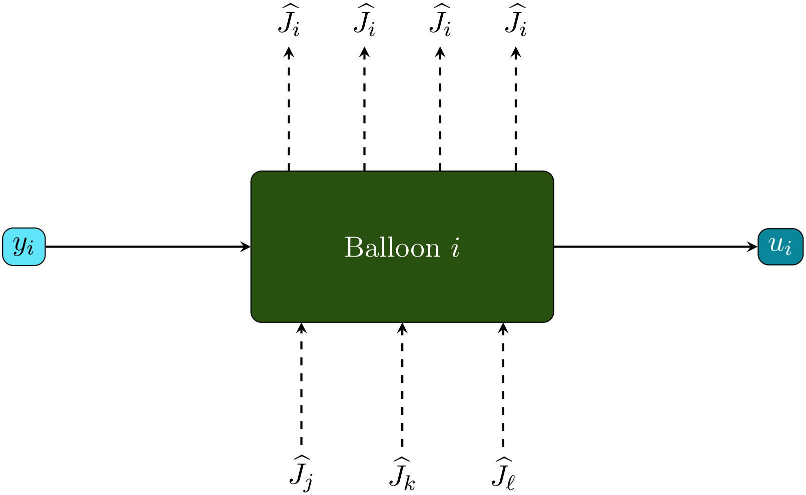
Controller design

Distributed architecture



Controller design

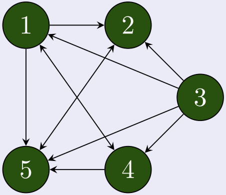
Distributed architecture as seen by one balloon



Each balloon measures y_i and estimates $\frac{1}{p}J$ by a consensus algorithm

$$\begin{bmatrix}\widehat{\boldsymbol{J}}[k+1]-\widehat{\boldsymbol{J}}[k]\\ \boldsymbol{\rho}[k+1]-\boldsymbol{\rho}[k]\end{bmatrix} = \begin{bmatrix}-\kappa_P\boldsymbol{I}-\kappa_I\boldsymbol{L} & -\boldsymbol{I}\\ \kappa_P\kappa_I\boldsymbol{L} & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}\widehat{\boldsymbol{J}}[k]\\ \boldsymbol{\rho}[k]\end{bmatrix}\Delta t + \begin{bmatrix}\kappa_P\boldsymbol{I}\\ \boldsymbol{0}\end{bmatrix}\boldsymbol{y}[k]\Delta t + \begin{bmatrix}\boldsymbol{I}\\ \boldsymbol{0}\end{bmatrix}\Delta\boldsymbol{y}[k] \tag{18}$$

Example (Laplacian matrix)



$$\boldsymbol{L} = \begin{bmatrix}3 & -1 & 0 & -1 & -1\\ 0 & 1 & 0 & 0 & -1\\ -1 & -1 & 4 & -1 & -1\\ -1 & 0 & 0 & 2 & -1\\ 0 & -1 & 0 & 0 & 1\end{bmatrix}$$

Controller design

Extremum seeking control (ESC)

- The objective of ESC is to minimize a measured cost
- Balloons estimate the gradient of the $\widehat{J}_i$ with respect to u_i and move in that direction
- We will use the PI form of ESC:

$$u_i[k] = -k_g \widehat{\theta}_{1,i}[k] + \widehat{u}_i[k] + d_i[k] \tag{19}$$

$$\widehat{u}_i[k+1] = \widehat{u}_i[k] - \frac{1}{\tau_I} \widehat{\theta}_{1,i}[k] \tag{20}$$

- $\widehat{\theta}_{1,i}$ is the gradient estimate, k_g and τ_I are tuning parameters, and d_i is a dither signal
- Dither signals must all have different frequencies

- The total cost dynamics can be parameterized as:

$$\frac{1}{p}\Delta J[k+1] = \theta_{0,i}[k] + \theta_{1,i}[k]u_i[k] = \boldsymbol{\theta}_i^\top[k]\boldsymbol{\phi}_i[k] \tag{21}$$

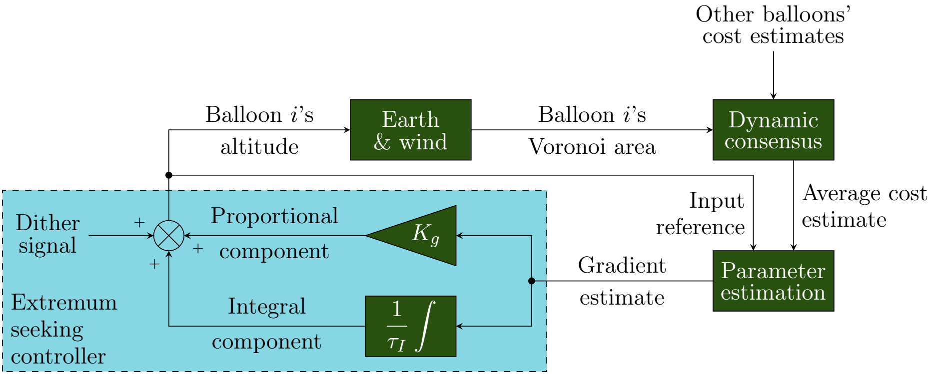
- The parameter vector, $\boldsymbol{\theta}_i$, can be estimated using a variation of recursive least squares Adetola and Guay [2008]:

$$\boldsymbol{\Sigma}_i[k+1] = \alpha\boldsymbol{\Sigma}_i[k] + \boldsymbol{w}_i[k]\boldsymbol{w}_i^\top[k] \tag{22}$$

$$\widehat{\boldsymbol{\theta}}_i[k+1] = \mathrm{Proj}_{\gamma_\theta}\left(\widehat{\boldsymbol{\theta}}_i[k] + \frac{\boldsymbol{\Sigma}_i^{-1}[k]\boldsymbol{w}_i[k]\left(e_i[k] - \widehat{\eta}[k]\right)}{\alpha + \boldsymbol{w}_i^\top[k]\boldsymbol{\Sigma}_i^{-1}[k]\boldsymbol{w}_i[k]}\right) \tag{23}$$

Controller design

Single balloon block diagram



- 1200 balloons floating between 10 kPa and 1 kPa (15–26 km altitude)
- Wind model is an interpolation of wind data on March 8, 2016 at 17:00 UTC from the NOAA National Oceanic and Atmospheric Administration [2016]
- Cost function depends on Voronoi area, A_i, and distance from centroid, $q_{c,i} \in \Gamma_i$

$$y_i = \left(A_i - \frac{A_t}{p} \right)^2 + \mathbb{G}\left(q_i, q_{c,i}\right)^2 \tag{24}$$

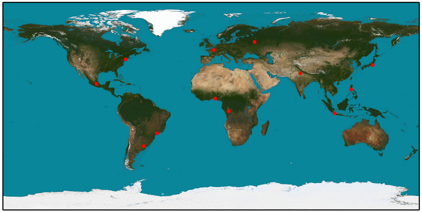
- Each balloon communicates with its Delaunay-neighbours and implements identical discrete-time distributed ESC

Δt	κ_P	κ_I	K	α	τ_I	K_g	D	γ_θ
0.1 h	1	0.5	0.8	0.8	10	1	0.1	1

Simulation results

Launch sites

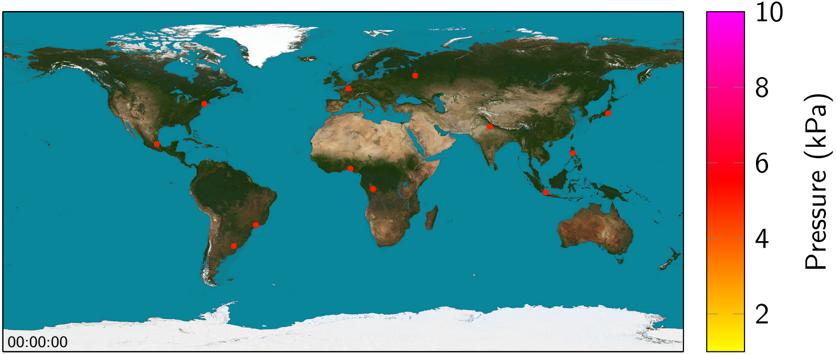
- Many balloons must all start at one of several launch sites
- For simulation, we have chosen 12 large cities around the world as launch sites

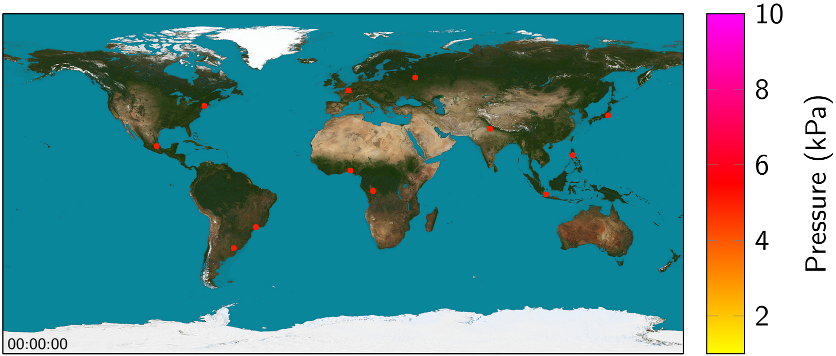


City	Country
New York	USA
Mexico City	Mexico
São Paulo	Brazil
Buenos Aires	Argentina
Paris	France
Moscow	Russia
Lagos	Nigeria
Kinshasa	DR Congo
Tokyo	Japan
Delhi	India
Jakarta	Indonesia
Manila	Philippines

Simulation results

Balloons without controllers launched from cities

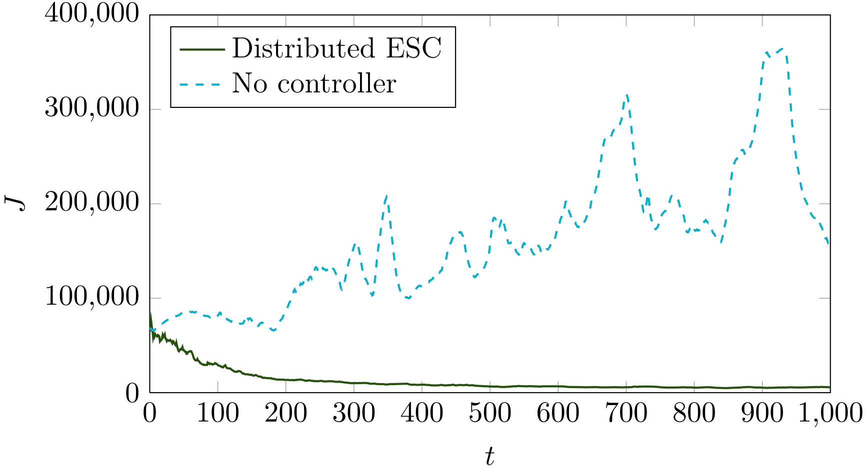




00:00:00
10
8
6
4
2
Pressure (kPa)

Simulation results

Cost function trajectories for balloons launched from cities



Concluding Remarks

- ESC can be used to solve of number of problems where:
  - Exact mathematical nature of the input-output dynamics are unknown
  - Cost function can be measured or inferred
- Useful for the development of a wealth of new tools in PSE
  - Feedback stabilization
  - Observer design
  - Large scale system optimization
  - Systematic design of RTO systems

Outlook

- Beyond existing techniques there are a wealth of new tools that are emerging:
  - ESC-based MPC
  - Machine Learning
  - Large optimization on clouds, etc...
- It is an adaptive, robust, real-time optimization technique with strong potential in many areas:
  - Automotive
  - Building Systems Management
  - Petroleum Production Technologies
  - Industrial energy management

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